Investigation of Charged Particles Radiation Moving in a Homogeneous Dispersive Medium


Abstract

In this work, we use Drude-Lorentz model description to study the radiation of a charged particle moving in a homogeneous dispersive medium. A suitable quantized electromagnetic field for such medium is utilized to obtain proper equations for energy loss of the particle per unit length. The energy loss is separately calculated for transverse and longitudinal components of the field operators. The calculations show that the longitudinal component of the field operators contributes in electron radiation, when dielectric function is exceedingly dependent on the frequency. It is also shown that when the dispersion is not included, the obtained equations are in a good agreement with previous results. For negligible dispersion, the contribution of the field's longitudinal component tends to zero and at the end the results are in agreement with Ginsberg's calculations. This calculation can reveal a development for the fields' quantization for permeable dielectric background medium.


1. INTRODUCTION

The Cherenkov radiation is known as electron emission (electromagnetic radiation), when a charged particle passes through a medium with a velocity larger than the phase velocity of the light in that specific medium [1, 2]. There are several mechanisms involved in Cherenkov radiation by which a high-energy subatomic particle—e.g. electron—can reach to an energetic equilibrium with its surrounding environment and determine how radiation is produced from these interactions. The Cherenkov radiation has been highly regarded at the beginning of the twentieth century for its importance in nuclear physics and cosmic ray [3]. Recently, the Cherenkov radiation has drawn attention in several fields and applications. In combination with electromagnetic simulations in electronics [4-6], it has several applications in meta-material optoelectronics, photonic nanostructures, imaging of medical isotopes and ultra-fast laser pulses [7-11]. The behavior of electromagnetic radiations can often be incorporated into continuous, effective material parameters [12] or ultra-small scale materials [13] with different approaches [14, 15]. As a recent comprehensive study, Shaffer et al. presented a review study about different aspects of utilizing Cherenkov light in nanotechnology [16].

Frank and Tamm [17] have investigated classical calculations of Cherenkov radiation, by deriving radiation intensity per unit time from the solution of electrodynamics equations in a particular medium. In their calculations, radiation intensity is considered as the Poynting vector flux [18] across the cylindrical surface, which hypothetically surrounds the electron trajectory [19].
On the other hand, Ginsberg reported the quantum computing of Cherenkov radiation [20, 21] with a good agreement with Pavel Cherenkov observation, who had originally discovered and analyzed the nature of this particular radiation. In this quantum mechanical approach, the theory is based on the conventional framework of quantum electrodynamics. Due to the limitation in fields’ quantization calculation at the same time, dissipative property of the background medium is not taken into account. It should be noted that the Ginsberg’s calculations have been presented within quantized fields whose dielectric permittivity variation with frequency was insignificance. Recently, the fields’ quantization for casual permeable dielectric background medium has been developed [22, 23], given to that it can be interesting to recalculate the Ginsberg quantum calculation for the fields with quantized disperse medium.

In this letter, first the Drude-Lorentz model is reviewed and since quantum mechanical methods are offered for our calculations, then we take a glance at the electromagnetic field quantization in dispersive medium. By using the applicable model and electromagnetic fields calculation for such a medium, the behavior of the medium will be explained. To perform such analyses, the Fermi golden rule is implemented to calculate the rate of energy loss per unit length with the same method that was used by Ginsberg. Both longitudinal and transverse components of the electric field are shown to contribute in the radiation process. Unlike the previous classical and quantum mechanical calculations, the field’s longitudinal component has important role in the rate of energy loss. However, it is observed that for negligible dispersion, this contribution tends to zero and at the end the results are in agreement with Ginsberg’s calculations.

2. DRUDE- LORENTS MODEL

The Drude-Lorentz model for a dielectric function in environments where damping is negligible, predicts following equation [24]:

\[ \varepsilon(\omega) = \frac{\omega_p^2 - \omega^2}{\omega_0^2 - \omega^2} \] (1)

when \( \omega_0^2 = \omega_0^2 + \omega_p^2 \), \( \omega_0 \) is inherent frequency and \( \omega_p \) is plasma frequency. In some cases where the environment has negligible damping, this equation can also be written as [24, 25]:

\[ \varepsilon(\omega) = \frac{k^2 \varepsilon_0^2}{\omega^2} \] (2)

where \( k \) is the wave-vector and \( C \) is the speed of light. When the right side of these two previous equations are equated with each other, it can be seen that for each \( k \) value there are two values for \( \omega \), as \( \omega_+ \) and \( \omega_- \), which are shown in Figure 1.

3. ELECTRIC FIELD QUANTIZATION EQUATIONS

All of the field operators are typically categorized into two separate parts as the transverse and longitudinal, and each of them should have a vector potential. The transverse component of the vector potential in dispersive medium is as following [26, 27]:

\[
\hat{A}^T(\tau, \vec{r}) = \left( \frac{\hbar}{16\pi^2\epsilon_0} \right)^{\frac{1}{2}} d^3\hat{\kappa} \sum_{\pm} \int d^3k \left\{ \frac{v_p(\omega_k)}{v(g(\omega_k))} \right\} \left[ \hat{e}_{k} \left( \frac{\omega - \omega_k}{\omega_0} \right) \right] \hat{\kappa} \cdot \left( \hat{\kappa} - k \right) \] (3)

when \( v_p(\omega) \) and \( v(g(\omega)) \) are the phase velocity and group velocity, respectively. \( \epsilon_r = \eta^2 + H.C \) referring to the Hermit conjugate of the function. It should be noted that \( \hat{e}_{k} \) is the polarization vector in the plane perpendicular to \( \hat{\kappa} \), when \( \hat{e}_{k} \) is a boson operator and both are applicable to the following equation:

\[
\hat{e}_{k} \left( \frac{\omega - \omega_k}{\omega_0} \right) \hat{e}_{k}^\dagger \left( \frac{\omega - \omega_k}{\omega_0} \right) = \delta_{AA} \delta(\hat{\kappa} - \hat{\kappa}^\dagger) \] (4)

The longitude operator of the quantized vector potential is equal to:

\[
\hat{A}^L(\tau, \vec{r}) = \left( \frac{\hbar}{16\pi^2\epsilon_0\omega_0^2} \right)^{\frac{1}{2}} \int d^3\hat{\kappa} \left\{ \hat{e}_{k} \left( \frac{\omega - \omega_k}{\omega_0} \right) \right\} \hat{\kappa} \cdot \left( \hat{\kappa} - k \right) \] (5)

where \( \omega_0 \) is the only root of the equation \( \eta^2(\omega) = 0 \), and \( \hat{\kappa} \) is the unit vector along the wave vector.

**Figure 1.** The graph of \( k \) vs \( \omega \), when imaginary part of the dielectric function tends to zero.
4. FINDING THE RATE OF ENERGY LOSS FOR A MOVING CHARGE PARTICLE IN THE FIELD TRANSVERSE COMPONENT

To find the rate of energy dissipation, the Ginsberg method is implemented [28]. First, the Fermi golden rule is used to find the probability of emitted photons per unit time for each moving charged particle, which is expressed as the following:

$$\Gamma = \frac{2\pi}{\hbar} \left| f_{f} \right|^2 \delta(E_f - E_i)$$ \hspace{1cm} (6)

$|i\rangle$ and $|f\rangle$ are the initial and final state of the system, respectively. It is assumed that the charged particle is initially moving with the momentum of $h\tilde{q}$ or velocity of $v = \frac{hq}{m}$, which leads to the emission of a photon with the momentum of $\hbar\tilde{k}$. Accordingly, it can be written:

$$\langle f \mid \rho \rangle \langle \tilde{k} \mid i \rangle = \langle 0 \mid \tilde{q} \rangle$$ \hspace{1cm} (7)

The term in the bracket of Equation (6) is the energy difference between the first and final states for a particle travelling with the mass of m and charge of Q, which means:

$$E_f - E_i = \frac{h^2}{2m} \left| q - \tilde{k} \right|^2 + h\omega_k - \frac{h^2q^2}{2m}$$ \hspace{1cm} (8)

Then, the perturbation potential is only considered as the first-order terms of vector potential [29], and we have:

$$V_{fi} = \langle f \mid -\frac{Q}{m} \tilde{p} \cdot \tilde{A} \rangle$$ \hspace{1cm} (9)

According to the above equations, the probability of emitted photons per unit time for the transverse component of vector potential ($\Gamma^T$) for the Equation (6) can be written as:

$$\Gamma^{(T)} = \frac{Q^2}{8\pi^2m^2\tilde{e}_\omega} \frac{\tilde{e}_\omega}{\tilde{e}_\omega} \Gamma^T \left( \frac{\omega}{\Omega} \right) \left( \frac{\omega}{\Omega} \right) \left( \frac{\omega}{\Omega} \right)$$ \hspace{1cm} (10)

where $\theta$ is the angle between the initial velocity of the particle, before emission, and the direction of the emitted photon. Now it is time to calculate the energy loss of the particle per unit length, from the following relation:

$$\frac{dW}{dx} = \frac{1}{v} \sum_{k} d^3k \delta\omega_k \Gamma$$ \hspace{1cm} (11)

After substituting the Equation (10) into the above relation for transverse component of $\Gamma$ (as $\Gamma^T$), and using the definition of the volume element in $k^2$ space, the energy loss is calculated as follows [30]:

$$\frac{dW}{dx} = \frac{Q^2}{4\pi\omega_0^2} \sum_{\omega} d\omega \left( \frac{k\left(\omega\right)}{\Omega\left(\omega\right)} \right) \left( \frac{k\left(\omega\right)}{\Omega\left(\omega\right)} \right) \left( \frac{k\left(\omega\right)}{\Omega\left(\omega\right)} \right) \left( \frac{k\left(\omega\right)}{\Omega\left(\omega\right)} \right)$$ \hspace{1cm} (12)

4.1. Finding the Integration Limits and Minimum of Speed

By means of using Dirac delta function term in Equation (12), the integration limits can be determined. According to the concept of delta function and the value of the cosine, which is between +1 and -1, one can write:

$$\omega_0 \leq -\frac{\hbar}{2m} k^2 + vk$$ \hspace{1cm} (13)

From the intersection of the curve in Figure 1 with the parabola formula, from the right side of Equation (13), we reach to an inequality to determine the value of $k$ (Figure 2).

The numerical calculations of Equation (13) are as following:

i) Several numerical calculations show that the parabola from the right side of the Equation (13) does not intersect with the $\omega_0$ curve. Then for dispersive media, $\omega_0$ is the only term which is entered in the calculations.

ii) For the mediums with $\frac{\omega_{p}}{\omega_0} < 1$, i.e. condensed environments, the $\omega_0$ curve quickly reaches its limit value ($\omega_0$ in Figure 2), and this helps to be able to find the value of $k$.

By having $\omega_0 = \omega_0$, the intercept of $-\frac{\hbar}{2m} k^2 + vk$ (parabola) and $\omega_0$ (curve), as $k_+$ and $k_-$, can be obtained as following:

$$k_{\pm} = \frac{mv}{\hbar} \left( 1 \pm \sqrt{1 - \frac{2\hbar\omega_0}{mv^2}} \right)$$ \hspace{1cm} (14)

![Figure 2](image-url)
These two values define the integration limits in Equation (12). It should be noted that according to the existing tables, $k_v$ and $k_c$ are always real [25]. Now, Equation (12) can be rewritten as following:

$$
\frac{dW^{(v)}}{dx} = \frac{Q^2}{4\pi\epsilon_0} \times 
\int_{\omega_v}^{\omega_c} \frac{dk}{\eta^2(\omega_m)} \left( \frac{\eta^2 - \left( \frac{k_c}{\omega} \right)^2}{\omega - \omega_m} \right)
\left( 1 - \left( \frac{hk}{2mv + kv} \right)^2 \right)
$$

(15)

Regarding the fact that $\omega_m = \omega_0$, Equation (13) leads to determination of the minimum speed of the charged particle, at which a photon emission can be occurred.

$$
v \geq \frac{c}{n} + \frac{\hbar}{2m}
$$

(16)

4. 2. Reaching to a State with a Negligible Dispersion
In this section, the state with the negligible dispersion is considered, and it is shown that the equation obtained in Equation (15) is in well agreement with the classical calculation [17], as well as Ginzburg quantum calculations [31]. By implementing the following equation [7]:

$$
v_p(\omega_m) = \frac{c}{\eta(\omega_m)}
$$

$$
v_g(\omega_m) = \frac{2c\eta(\omega_m)}{\omega_m \frac{\partial}{\partial \omega} \left( \eta^2 - \left( \frac{k_c}{\omega} \right)^2 \right)}
$$

(17)

Then, Equation (15) can be written as:

$$
\frac{dW^{(v)}}{dx} = \frac{Q^2}{2\pi\epsilon_0} \times 
\int_{0}^{\omega_m} \frac{d\omega}{\omega} \frac{\partial}{\partial \omega} \left( \frac{k}{\eta^2(\omega) - \left( \frac{k_c}{\omega} \right)^2} \right)_{\omega=\omega_m}
\times \left( 1 - \left( \frac{hk}{2mv + kv} \right)^2 \right)
$$

(18)

where $\frac{\partial n}{\partial \omega} \approx 0$, then the value of $\omega_m$ and $\omega_0$ should get close together. In this case, the allowed region for integration is between 0 and $k_m$, and for this area (as shown in Figure 2), the relationship between $k$ and $\omega$ is almost linear, as in $\omega = \omega_c = \frac{kc}{n}$, where $c$ and $n$ are constant. Accordingly, Equation (15) can be written as:

$$
\frac{dW^{(v)}}{dx} = \frac{Q^2}{2\pi\epsilon_0} \left\{ \omega_0 \frac{\partial}{\partial \omega} \left( \frac{\omega^2 \epsilon(\omega)}{\omega} \right) \right\}
\times \left( 1 - \left( \frac{hk}{2mv + kv} \right)^2 \right)
$$

(19)

Ginzburg obtained the same expression in his calculation [31].

4. 3. Whether All the Energy Loss of the Moving Charged Particles Originated from the Transverse Fields’ Component? If we apply the previous procedure for the longitudinal component of the electric field, then equation for the energy loss of the particle per unit length can be written:

$$
\frac{dW^{(l)}}{dx} = \frac{Q^2}{2\pi\epsilon_0 \omega_L} \left( \frac{\partial}{\partial \omega} \epsilon(\omega) \right)
\times \left\{ \frac{\hbar^2k^4}{16m^2v^2} + \frac{\hbar^2k^4}{2m^2v^2} \frac{\omega_0^2}{v^2} \ln(k) \right\}
$$

(20)

The important question is why in previous calculations the contribution of such energy has not been included. The answer is that whatever obtained in previous calculations was related to the contribution of the transverse component of the field operators. Here, it is shown that the longitudinal component of field operators contributes in electron radiation, when the dielectric function is highly frequency dependent. In previous works, the dielectric function was considered with low dependency in frequency, where the frequency value was considered as $\omega_L \approx \omega_T$. At the frequencies close to $\omega_L$, the above equation will be converted to the form below:

$$
\frac{dW^{(l)}}{dx} = \frac{Q^2}{4\pi\epsilon_0} \left( \frac{\omega_0^2 - \omega_T^2}{\omega_L^2} \right)
\times \left\{ \frac{\hbar^2k^4}{16m^2v^2} + \frac{\hbar^2k^4}{2m^2v^2} \frac{\omega_0^2}{v^2} \ln(k) \right\}
$$

(21)

This shows that if the dispersion is negligible, then $\frac{dW}{dx}$ tend to zero in agreement to the previous works.

5. CONCLUSIONS
In this study, the radiation of charged particles moving in a homogeneous dispersive medium was studied. During the energy loss calculations, two frequencies were found for each $k$ value.
We presented that $\omega_e$ has important role in calculations but $\omega_v$ has no contribution in calculations. It is also found that if the dispersion in medium is significant, then not only the field’s transverse component, but also longitudinal component has contribution in energy losses. This is the case that was not derived in Frank and Tamm classical calculation or Ginsberg quantum calculations.

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7. REFERENCES


Investigation of Charged Particles Radiation Moving in a Homogeneous Dispersive Medium

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