Optimization of Time, Cost and Quality in Critical Chain Method Using Simulated Annealing

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1. INTRODUCTION

In the early years of the 21st century, developing countries prioritized growth, construction, and operation of large-scale important projects in different industrial and economic fields. Increased duration of implementation of projects as compared to preliminary estimates, lack of powerful strategies for completion of projects, and failure of the education system in operating ongoing projects could be considered as some of the important problems with projects in different countries. It should be noted that any delay in enforcement of development plans and lack of operation of construction projects are examples of loss of natural resources, and the most important natural resource that is lost in this process is time, which is the most valuable capital of nations in today’s rushing world [1]. One of the important factors leading to elongation of construction projects in developing countries is lack of utilization of project control and planning methods [2].

One of the methods used for project control and planning is the critical chain method, which is discussed here.

1.1. Critical Chain Philosophy  
In 1984, Goldratt made an evolution in the science of operation management by publishing a scientific novel called “Goal”. In this novel, Goldratt introduces the Theory of Constraints (TOC) in the field of operation management. TOC, which is normally classified into five categories, states that to solve a system’s problem, first its main problem has to be solved [3]. Later on in 1997, Goldratt pointed to the application of this theory to project planning in his “Critical Chain” [4]. In this book, he describes how TOC can be utilized for improving performance of projects. Critical Chain Project Management (CCPM) is an extension of the Theory of Constraints (TOC) which is specifically designed for project management. CCPM is introduced as an alternative to classic and conventional project control and planning methods in most references [5, 6].
1.2. Literature Review

Min and Rongqiu (2008) introduced a fuzzy method for determining project buffer size in critical chain planning. Since buffers are used as risk management instruments to protect the project completion time, uncertainty and ambiguity of plans should be taken into consideration in assessing buffer size [7]. Huang et al. [8] compared the CCPM and PERT methods in 3 different projects and concluded that due to appropriate use of safety times available for activities with reasonable changes and inappropriate human behavior changes, the CCPM method in multi-project environments could achieve reliable and on-time delivery within a short period. However, if the inappropriate human behavior factor is ignored and the reasonable change is just considered, the CCPM would not surpass the PERT method in terms of the average time of the project implementation. With respect to the project reliability, the CCPM achieved better results than PERT. Georgy et al. [9] examined critical chain planning applicability for construction projects in the Middle East (Egypt and Saudi Arabia). They found that linear projects have proper potential for the critical chain trial application. Afruzi et al. [10] presented a Multi-Objective Imperialist Competitive Algorithm (MOICA) to solve Discrete Time–Cost–Quality Trade-off Problem (DTCQTP). Due to real world resource-constrained situations in projects, this model is expanded to mode-identity resource-constrained DTCQTP model. Monghasemi et al. [11] proposed an evidential reasoning (ER) approach for the first time in the context of project scheduling to identify the best Pareto solution for discrete time-cost-quality trade-off problems. They developed a multi-objective genetic algorithm (MOGA) incorporating the NSGA-II procedure in order to identify all global Pareto optimal solutions. Ghoddousi et al. [12] studied a multi-attribute buffer sizing method in order to maximize the robustness of the buffered schedule generated. The methodology presented in this study is based on the critical chain buffer management methodology. In order to prove the effectiveness of the proposed method, a simulation approach is applied. Zhang et al. [13] studied a project scheduling problem under resource tightness. They focused on project buffer sizing of a project critical chain. The design structure matrix (DSM) is then adopted to analyze the information flow between activities and calculate the rework time resulting from the information interaction and the information resource tightness. Following the review of related articles on critical chain, it was found that only one study had simultaneously a multi-objective model with multi projects and optimized the model using the cloud genetic algorithm [14]. Based on what was stated, the innovatory aspect of this research lies in items such as difference in the numbers of activities of different projects and analysis of the weights of different objectives for the purpose of examining the three objectives of time, cost, and quality and the utility function. The values of expected time and cost sensitivity with respect to the objective function have also been analyzed.

2. MATERIALS AND METHODS

2.1. Simulated Annealing

The simulated annealing method was for the first time introduced by Patrick et al. in 1983 and has been used since to solve many problems with high computational complexities [15]. The simulated annealing (SA) method is an intelligent and advanced random search method, which has been and increasingly being utilized successfully for solving mixed optimization problems and optimizing complicated objective functions in different scientific fields [16]. In this algorithm, material is simulated as a system of components. This algorithm simulates the process of cooling down by gradually reducing temperature until a thermal balance is obtained [15].

3. RESULT

3.1. Critical Chain Multi-objective Multi-project Scheduling Model

In this study, three objectives, namely time, cost, and quality, were assumed for assessing a critical chain multi-project schedule. The time objective was to minimize project duration and the critical chain method was used to perform project calculations. In addition, the cost objective was to minimize the final project cost, and the direct costs method was used in this project. Direct costs include the sum of costs of renewable and non-renewable resources of all project activities. Finally, the quality objective for this project involves maximization of quality of accomplishment of project tasks. Before presenting the proposed objective function its notations are introduced in the following.

**Notation:**

N: Total number of projects \(i=1, 2, 3, \ldots, N\)

J: Project activities \(j=1, 2, 3, \ldots, J\)

K: Renewable resources \(k=1, 2, 3, \ldots, K\)

rijk: Renewable resources, \(K\), required for accomplishing activity \(j\) in project \(i\)

Ck: Unit cost of renewable resources \(k=1, 2, 3, \ldots, K\)

tij: Duration of activity \(j\) in project \(i\)

P: Non-renewable resources

\(\text{nrij}\): The renewable non-renewable resource, \(P\), required for accomplishing activity \(j\) in project \(i\)

Cp: Unit cost of non-renewable resources \(p\)

\(p=1, 2, 3, \ldots\)
3. 1. 1. Analysis of Time Objective Function in Critical Chain Multi-Project Schedule

The critical chain multi-project control and planning process is extremely complicated and the project owner and executive’s concentration on the entire project duration is time consuming. Hence, the goal of multi-objective planning in the critical chain method is to minimize time, which is calculated through the following relation.

\[
\min T = E_{\theta} + PB
\]  

In the above relation, project buffer is calculated using the cut and paste method. To clarify the way in which project buffer is calculated and the difference between the critical path and critical chain methods, let us consider an example with three activities A (14 day), B (10 day), and C (4 day). The prerequisite relationships between the activities are such that activity A has no prerequisites, activity B depends on activity A, and activity C depends on activity B. If the project is scheduled with the critical path method, total project time will equal the sum of the times of activities A, B, and C, which will finally be 28. In the critical chain method, based on the philosophy behind the method, which states that 50 percent of activity time consists of uncertainty, this time is eliminated from the initial time of the activity, and half of the eliminated time is allocated to project buffer (This method of calculating project buffer is referred to as the cut and paste method). Thus, the times of activities A, B, and C have been assumed to be 7, 5, and 2, respectively, and project buffer time has been assumed to be 7. Figure 1 display how project time is calculated using the critical path and critical chain methods.

![Critical Path Method](image)

**Figure 1. Project buffer calculation**

3. 1. 2. Analysis of Cost Objective Function in Critical Chain Multi-project Scheduling

The total cost of multi-objective management includes costs of renewable and non-renewable resources. According to the above notation, the project cost objective function in critical chain multi-project scheduling is obtained through the following relation.

\[
\min C = \sum_{i=1}^{N} \sum_{j=1}^{J} (\text{Cost} \times \text{Activity Time}) + \sum_{p=1}^{P} \text{Project Buffer}
\]  

3. 1. 3. Analysis of Quality of Objective Function in Critical Chain Multi-project Scheduling

To conduct systematic and deep research on critical chain multi-project scheduling, quality of projects needs assessment. In this research, to indicate the quality obtained (EQVij) for each activity, the following relation is used.

\[
EQV_{ij} = EV_{ij} \times q_{ij}
\]

In the above relation, qij is the qualitative index of activity j during project i, it is used to calculate the actual quality of activities as shown in the following.

\[
q_{ij} = \frac{\text{actual quality of activity } j}{\text{predetermined quality of activity } j} \times 100
\]

Hence, the qualitative level of the multi-project scheduling can be expressed as the mean weight of qualitative level of all activities and calculated as follows.

\[
\text{max } Q = \frac{1}{\sum_{i=1}^{N} \sum_{j=1}^{J} EV_{ij}} \times \sum_{i=1}^{N} \sum_{j=1}^{J} EQV_{ij}
\]

\[
= \frac{1}{\sum_{i=1}^{N} \sum_{j=1}^{J} EV_{ij}} \times \sum_{i=1}^{N} \sum_{j=1}^{J} EV_{ij} \times q_{ij}
\]

3. 1. 4. Proposed Model’s Utility Function

As mentioned, T, C, and Q shows time, cost, and quality for multi-objective optimization in critical chain multi-project scheduling. Considering the decomposition of the multi-objective utility function, it can be decomposed to a weighted polynomial shown by the following relation.

\[
u(T, C, Q) = \alpha_T \cdot u(T) + \alpha_C \cdot u(C) + \alpha_Q \cdot u(Q)
\]

\[
\alpha_T, \alpha_C, \alpha_Q \geq 0
\]

\[
\alpha_T + \alpha_C + \alpha_Q = 1
\]
The second-order utility function can be used as the solution space where all utility functions are convergent. The total project’s utility (D) is assumed to be 1, therefore:

$$U(T) = \begin{cases} \phi_T - \beta_T (T - D), & T \in [0,2D] \\ 0, & T \notin [0,2D] \end{cases}$$

(9)

The total multi-project management cost includes costs of renewable and non-renewable resources and its utility is equal to 1, therefore:

$$U(C) = \begin{cases} \phi_C - \beta_C (C - (1 - \eta)) , & C \in [0,2(1 - \eta)].U \\ 0, & C \notin [0,2(1 - \eta)].U \end{cases}$$

(10)

If the quality utility equals 1, then:

$$U(Q) = \begin{cases} \phi_Q - \beta_Q (Q - 1) , & Q \in (0,1) \\ 0, & Q \notin (0,1) \end{cases}$$

(11)

3.1.5. Multi-Objective Optimization Model To solve the proposed model, a multi-objective optimization model was introduced. Considering the three mentioned characteristics, namely time, cost, and quality, which are mainly considered by organizations and contractors, a utility function was developed for these objectives. Therefore, the proposed optimization model is as follows:

$$\text{max} \ u(T, C, Q)$$

(12)

$$s.t. E_{ij} - E_{(j-1)i} \geq t_{ij}$$

(13)

$$PB = \frac{\sum t_i}{2}$$

(14)

Relation (12) shows the problem objective function, which is focused on maximizing the multi-objective optimization utility in critical chain multi-project scheduling. Relation (13) shows the pre- and post-requisite relation in this project which calls for cessation of a post-requisite activity until completion of current activities. Moreover, when an activity starts, it cannot be stopped due to continuation of the activity. Relation (14) shows calculation of the project buffer.

3.2. Design of Simulated Annealing Algorithm The SA (simulated annealing) method was used in this research for optimization of critical chain multi-objective scheduling. In this algorithm, the decrease in temperature can be used to completely search the problem solution space and find the optimal solution. In addition, using this method it was tried to prevent local optima. To solve this model using the SA method the following hypotheses were formulated.

- Total Number of Iterations: \( N = 100 \)
- Number of iteration at each temperature: \( \text{It}=15 \)
- Initial temperature: \( T0=1000 \)
- Final temperature: \( T\text{f}=0 \)

The cooling method in this research was also calculated as follows:

$$Ti = (T_{i-1} - T_f) \times \frac{N+1}{N} + T_0 - (T_{i-1} - T_f) \times \frac{N+1}{N}$$

(15)

3.2.1. Numerical Example To assess validity and accuracy of the model an example is presented in the following. In this example, three projects are simultaneously programmed, and these three projects involve 7, 10, and 8 activities. The information on activities and prerequisite relations of each project are presented in the following.

- Project One:

This project includes 7 activities and Table 1 presents prerequisite relations of the activities.

- Project Two:

This project involves 10 activities and Table 2 presents prerequisite relations of these activities.

- Project Three:

This project consists of 8 activities and prerequisite relations for these activities are shown in Table 3.

<table>
<thead>
<tr>
<th>Activity name</th>
<th>Prerequisite relations</th>
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<td>A</td>
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<td>B</td>
<td>A</td>
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<td>C</td>
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<td>D-E</td>
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In the present research, to solve the optimization problem, 4 renewable and 2 non-renewable resources were used. The range of variations of these resources is shown in the following:

\[ C_{K1} = [2000 - 4000] \quad C_{K2} = [3000 - 6000] \]
\[ C_{K3} = [4000 - 8000] \quad C_{K4} = [2500 - 5000] \]
\[ C_{P1} = [3000 - 5000] \quad C_{P2} = [5000 - 7000] \]

On the other hand, to calculate the utility function it is necessary to ask the experts about the favorable time, cost, and quality. Therefore, the desired values proposed for the aforementioned three parameters are as follows:

\[ Q = 1 \quad U = 20000 \quad D = 80 \]

In addition, in view of the characteristics of each objective, weight coefficient were also assigned to each characteristic as:

\[ \alpha_T = 0.5 \quad \alpha_C = 0.3 \quad \alpha_Q = 0.2 \]

After solving the example with the proposed algorithm in MATLAB, the following result (solution) was obtained.

4. DISCUSSION

In this section, calculation of results for assessment of the Simulated Annealing algorithm are described. Afterwards, results are presented and analyzed. To this end, the three-objective scheduling problem which was introduced in the previous section, is used to simulate the optimization model.

First, the weight coefficient of one of the objectives is assumed to be invariant and the effect of the other two objectives on output parameters is analyzed.

4.1. Fixed Quality Coefficient

In this section, the weight coefficient of the quality objective function is assumed to be 0.1, and values of weight coefficients of the other two objectives are assumed to vary between 0.1 and 0.8, so that the sum does not exceed 0.9. The following table shows values of weight coefficients and their effect on the ultimate utility, project final cost, project completion time, and quality.

According to Figure 2, as the quality weight coefficient remains invariant, the weight coefficient of time and ultimate utility increase and the cost weight coefficient decreases.

According to Figure 3, when the weight coefficient of quality remains invariant, time weight coefficient increases and cost weight coefficient decreases but the total project cost does not change. It shows that impressive decrease in total cost would happen when time and cost weight factor have least difference from each other. This is because increasing these two factors are significant in this case compared with other cases which causes more effort to minimize them.

<table>
<thead>
<tr>
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<th>Prerequisite relations</th>
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<td>A</td>
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<td>-</td>
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<td>C</td>
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**TABLE 3. Prerequisite relations of the third project**

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<tr>
<th>Activity name</th>
<th>Prerequisite relations</th>
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<td>B</td>
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<td>C</td>
<td>-</td>
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<td>D</td>
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<td>H</td>
<td>E-C</td>
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</table>

**TABLE 4. Time, cost, and quality of three projects both separately and concurrently**

<table>
<thead>
<tr>
<th></th>
<th>First project</th>
<th>Second project</th>
<th>Third project</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost</td>
<td>5953000</td>
<td>10281000</td>
<td>7666000</td>
</tr>
<tr>
<td>Time</td>
<td>61.5</td>
<td>50.25</td>
<td>42</td>
</tr>
<tr>
<td>Quality</td>
<td>0.551</td>
<td>0.5598</td>
<td>0.339</td>
</tr>
</tbody>
</table>

**Figure 2. Changes in ultimate utility when the quality coefficient is invariant**

**Figure 3. Variations of total cost by assuming an invariant quality coefficient**
According to results shown in Figure 4, when the quality weight coefficient remains invariant, the time weight coefficient increases and the cost weight coefficient decreases, and thus the final project time increases in most cases.

According to results shown in Figure 5, when the quality coefficient remains unchanged, the time weight coefficient rises and the cost weight coefficient decreases, but the final project time shows an increasing trend in most cases.

4.2. Fixed Cost Coefficient

In this section, the weight coefficient of the objective function of cost is assumed to be 0.1, and weight coefficients of the other two objectives vary between 0.1 and 0.8 so that their sum does not exceed 0.9. The following table shows the weight coefficients and their effect on the ultimate utility, final project cost, project completion time, and quality.

According to Figure 6, as the weight coefficient of cost remains invariant, the quality weight coefficient grows and the time weight coefficient decreases. Finally, the ultimate utility decreases. According to Figure 7, when the cost weight coefficient remains invariant, the quality weight coefficient increases and time weight coefficient decreases.

4.3. Fixed Time Coefficient

In this section, the weight coefficient of the objective function of time is assumed to be 0.1 and weight coefficients of the other two objectives vary between 0.1 and 0.8, so that the sum does not exceed 0.9. The following table presents these weight coefficients and their effect on ultimate utility, final project cost, project completion time, and quality.

According to Figure 8, when the cost weight coefficient remains invariant, the quality weight coefficient increases and the time weight coefficient decreases. Therefore, the total project cost remains unchanged and displays no significant change. According to Figure 9, when the cost weight coefficient remains invariant, the quality weight coefficient grows and the time weight coefficient decreases. However, no change is observed in the ultimate project completion time. According to Figure 10, when the cost weight coefficient remains invariant, the quality weight coefficient increases and the time weight coefficient decreases. However, no change is observed in the project quality.

Therefore, the total project cost remains unchanged and displays no significant change. According to Figure 8, when the cost weight coefficient remains invariant, the quality weight coefficient increases and the time weight coefficient decreases. However, no change is observed in the ultimate project completion time. According to Figure 9, when the cost weight coefficient remains invariant, the quality weight coefficient grows and the time weight coefficient decreases. However, no change is observed in the project quality.

4.3. Fixed Time Coefficient

In this section, the weight coefficient of the objective function of time is assumed to be 0.1 and weight coefficients of the other two objectives vary between 0.1 and 0.8, so that the sum does not exceed 0.9. The following table presents these weight coefficients and their effect on ultimate utility, final project time, project completion time, and quality.

According to Figure 10, when the time weight coefficient is invariant, the quality weight coefficient increases and the cost weight coefficient decreases. Consequently, the ultimate utility is relatively invariant in the beginning but grows to the end.
Figure 8. Variations of project completion time by assuming an invariant cost coefficient

Figure 9. Changes in project quality by assuming an invariant cost coefficient

Figure 10. Variations of ultimate utility of the project when time remains invariant

According to Figure 11, when the time weight coefficient remains invariant, quality weight coefficient escalates and the cost weight coefficient declines. Eventually, the total project cost remains relatively unchanged, but a slight growing trend can be observed in the figure in general.

According to Figure 12, when time weight coefficient remains unchanged, quality weight coefficient grows and the cost weight coefficient decreases, but no significant change is observed in the final project time.

According to Figure 13, when the time weight coefficient remains invariant, quality weight coefficient grows and the cost weight coefficient decreases. However, no significant change is observed in project quality. Sensitivity analysis and computational result indicated the proposed model performance for three objectives project scheduling problem in different states so that it can create different situation in problem by various parameters variation and then can obtained desire outputs.

4. Analysis of expected time and cost sensitivity

Next, we have examined the expected time and cost, and analyzed the effect of their changes on the objective (utility) function. In one case, expected cost has been assumed to be constant, and the effect of time on the utility function has been demonstrated. The results obtained based on Figure 14 demonstrate that the final utility value for the designed problem decreases as expected time increases. In the other case, the process has been repeated while expected time has remained constant, and expected cost has increased. The results obtained from this section are also displayed in Figure 15.
In this paper, multi-objective optimization for multi-project scheduling on critical chain is studied, which considers time, cost and quality as an objective functions. Utility function approach was employed to determine the amount of objectives. In order to solve the problem, the simulated annealing algorithm was tested many times in different conditions. Sensitivity analysis was performed based on various values of each objective weights. To achieve this purpose, one of the objective weights is assumed constant while two others are changed in pre-determined interval, and then the effect of these variation is investigated on utility function. In addition, the number of activity in each project is not considered the same unlike previous research.

5. REFERENCES

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In the late 80s, the idea for a new way of managing projects was born. In this article, a multi-objective multi-project scheduling model in the critical chain framework is considered. The objectives include time, cost and quality. The model is developed and then the solution method is applied. The number of tasks in each project is not considered in this study. The quantities of time, cost and quality are obtained by solving the model and then a measure is calculated. The sensitivity of the model is performed on different weights of objectives. In addition, it is shown that the developed algorithm can solve the problem in higher dimensions. doi: 10.5829/idosi.ije.2017.30.05b.01