Constrained Model Predictive Control of Low-power Industrial Gas Turbine

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1. INTRODUCTION

Gas turbine is a continuous combustion engine which converts fuel energy into required power [1]. This power can be particularly used as propulsion force in a jet engine or mechanical power required for one axis rotation (in turbo generator [2, 3], a turbo pumps or turbo compressor [4]). Gas turbine performance is expressed as follows: the air is drawn into the engine through inlet duct by compressor and compressed, then transferred into combustion chamber. In the combustion chamber, air is mixed with fuel and ignited causing an enhancement in temperature and expansion of the gases. These gases are passed through the turbine blades which are designed to convert kinetic energy of gases into mechanical energy and also engine gases are discharged through a nozzle or diverter [5, 6]. In general, there are many approaches applied as gas turbine fuel control system that can be divided into two categories: linear and nonlinear methods. It is obvious that applying nonlinear control methods can reveal more precise results than linear methods due to the nonlinear nature of gas turbines [7]. Among non-conventional methods, model predictive control techniques will be a sufficient alternative due to high flexibility, such as indicating the dynamic constraints imposed on state and control variables and nonlinear behavior of gas turbine system [8-10]. Model predictive control algorithm is an efficient method to deal with such a complex system in which future states of the system are already known according to prediction horizon value. Now, controllers output should be so determined that predicted process output reasonably track reference set point as much as possible [10-13]. Evidently, MPC is capable to control and handle a single-input-single-output and control multiple-input-multiple-output systems for continuous and discrete state space models. This algorithm deals with states and control variables in such a nonlinear system including various constraints and cost functions. Moreover, its capabilities for applying in nonlinear control systems which are time-variant subject to penetrations of various restrictions expected for system demonstrates that this approach is a distinct and superior method with respect to other methods [12]. In every step control, model predictive control calculates an open-
loop sequence of decision variables settings in order to optimize the future behavior of the system so that entirely a sequence of valid input variables are obtained within predefined control horizon to be implemented to the system [14-16].

This paper is organized as follows:

In the first section an introduction is presented for gas turbine and then gas turbine control problem has been addressed with various performance metrics. In section II literature reviews of researchers about gas turbine design and control methods have been addressed and then in the third section the structure of gas turbine model and its dynamic characteristics and basic concepts of model-based predictive control is described. Simulation, discussion and compassions are expressed in section 4 and 5 respectively. Finally conclusion and recommendations are described. Simulation results show that the performance of proposed approach is reasonable.

2. LITERATURE REVIEW

In reference [17] two types of fuzzy controller to control pressure in the combustion chamber are designed and evaluated. The first type of controller is designed based on the settings of membership functions and fuzzy rules (based on trial and error) and the second controller is developed based on a definite geometric method. Finally, both controllers are applied into nonlinear model of gas engine and their results are compared with Proportional-integral controller. But it is important to note that the provided model was not a fairly accurate model and on the other hand due to robustness of fuzzy controller against nonlinear dynamic, the importance of this issue is unclear. The most important thing for dynamic model simulation is dependence to understanding the process performance. Therefore, accurate dynamic and thermodynamic equations governing the gas turbine system should correctly be extracted. In references [5, 6], a convenient model of a gas turbine with low power is developed that is proper for simulation purposes. Meanwhile, after system modeling corresponding constraints should be clearly identified and also applied to the control system. In references [8, 14, 18], system constraints that are required for better understanding and control system design, have been noted. After system modeling and constraints identification, an efficient controller is required and as mentioned in reference [19], MPC controller will be a sufficient approach due to high flexibility, consideration of dynamic constraints such as indicating state and control variables constraints and capabilities to handle severe nonlinearity behavior of gas turbine system and has been considered in this paper as a control algorithm [20-36].

3. STRUCTURE OF THE PROPOSED MODEL

3.1 Dynamic Modeling of Gas Turbines

The main part of a gas turbine consists of inlet duct, compressor, and combustor, turbine and gas nozzle or gas diverter (Figure 1). The interaction between these components is being fixed by physical structure of the engine.

3.2 Mathematical Model of Gas Turbine

In this section, relevant equations for the combustion chamber as a volume control is written as follows [37-39]:

\[
\frac{dm_{\text{comb}}}{dt} = C_v + V_{\text{ref}} - V_T
\]

\[
\frac{dp}{dt} = \frac{1}{V_{\text{comb}}} \left( 1 - \frac{R}{\rho} \right) - \frac{V_T}{\rho} \left( 1 - \frac{R}{\rho} \right)
\]

\[
\frac{ds}{dt} = \frac{1}{4 T_\theta \rho}
\]

3.3 Model Predictive Control

Model predictive control is a sufficient controller for nonlinear systems including uncertainty and constraints in their control and state variables because it provides an explicit model of the system in order to form control calculation as well as considering behavior of system at a future time horizon and also taking into account the limitations of inputs and outputs of a control system [20].

The MPC approach is a class or classes of predictive algorithms that will forecast future behavior of a system control in a certain horizon through an explicit model of process control within two phases: prediction and optimization process. Accurately, in every step control model predictive control calculates an open-loop sequence of manipulated variable settings (MV) in order to optimize the future behavior of system (CV), so that entirely a sequence of valid input variables are obtained within predefined control horizon to be implemented to the system.

![Figure 1. The main parts of a gas turbine](image-url)
The first element of this sequence and in some practical cases multi-element or fitted elements are applied to the system and prediction and optimization phases are re-executed at each control step that may be sampling time of system.

For more details representation, consider Figure 2.

As it is clear in the above figure, MPC attempts to follow the reference trajectory defined within system by outputs signals in the predefined prediction horizon. For this purpose, future outputs will be predicted through previous input-output data and finally a sufficient control sequence with explicit defined horizon is calculated according to these new data and considering reference trajectory, disturbance and control strategy. If disturbance is negligible, model-plant mismatching is not considerable and also optimization problem is solved in infinite horizon, we can consider the obtained input sequence at the present time to be employed into next time instants. But this is an ideal case and generally the obtained input sequence should be applied into system only when next measurements are available.

3.4. Problem Solving Process (Concept and Methodology) The first step: convert MPC problem to a nonlinear programming problem for applying into solver function:

The main approach for solving MPC controller problems rely on converting them into a nonlinear optimization problem. The fundamental problem in this conversion is dynamic nature of MPC control problems. Indeed, the dynamic nature of predictive control problems causes cost function and constraint reveal as differential and integral expression and problems must be defined in time domain.

While in nonlinear optimization problems, objective functions and constraints are formed as nonlinear algebraic expressions and essentially dynamic and time are meaningless.

Here, conversion process of a model predictive control problem into a nonlinear optimization problem will be adopted through conversion of predictive control dynamic expressions (differential expressions in state space) into simple arithmetic expressions. In summary, it can be said that a model predictive control problem includes the following components:

State variables equations (gas turbines nonlinear equations)

\[
\dot{x} = f(x(t), u(t), t)
\]

State and control variables

\[ [x, u] \]

Initial and terminal constraint

\[
\psi_x(x(t_0), t_0) \geq 0 \quad \psi_f(x(t_f), t_f) \geq 0
\]

Feasible constraint during process running

\[
g(x(t), u(t), t) \geq 0
\]

States variables limits

\[
u_L \leq u(t) \leq u_U, \quad x_L \leq x(t) \leq x_U
\]

Objective function

\[
J = \varphi(x(t_f), t_f) + \int_{t_0}^{t_f} L(x(t), u(t), t) \, dt
\]

The second step: discretizing problem and time into smaller time intervals:

For this purpose, whole time intervals are divided into smaller time sub intervals where length of these sub intervals can be same or different. The initial and final points of these sub periods are called temporal nodes.

The third step: formulating physical constraints into equal restrictions and less than or equal to zero:

To convert state space equations and objective functions and constraints in the form of discrete and integration of system dynamics, conventional discretization methods of differential equations, such as first-order linear approximation or other methods can be used. In this essence, differential expression of predictive control problem will be converted into simple

Figure 2. Prediction process in model-based predictive control by determining the appropriate input sequence ²

Figure 3. MPC controller block diagram

² http://math.stackexchange.com/questions/109070/model-predictive-control
algebraic equations using integration method and also are solved in terms of state and control variables at any time instants.

\[ \dot{x}_k = f_k(x_k(t), u_k(t), t_k) \]  

Integral part expressions

\[ \dot{x}_k = \frac{S_{x_{k+1}} - x_k}{dt} \]  

\[ x_{k+1} = x_k \times dt + x_k \]  

\[ \frac{dm_{comb}}{dt} = V_c + V_{fuel} - V_f \]  

\[ \frac{dP}{dt} \frac{k}{V_{comb}c} \left( \left[ \frac{P}{P_i} \right] - 1 - \frac{V_f}{V_c} \right) \left[ \frac{P}{P_i} \right] - 1 \right] = \frac{1}{419.0} \]  

Fourth step: applying cost function and constraints into problem solver:

In solving process of optimization problem, initial guesses must be determined at any moment for all optimization factors namely control variables to start solving procedure.

The objective function to track reference motor speed turn

\[ J = (n(N_p) - n_{reference})^2 \]  

But interestingly, this method is not sensitive to the initial guesses. In general, well-defined initial guesses leads to quicker resolution, while insufficient initial guesses may lead to an optimal solution in reasonable time consumption.

The fifth step: illustrating simulation results and re-executing computational process until completion of problem solving.

3.5. Objectives of Controller Design  The purpose of controller design is to guarantee system stability in all areas of its working area with respect to parameters perturbation. The main control objectives in gas turbine can be summarized as follows:

A) Gas turbine speed must be a function of fuel flow rate is not influenced by environmental conditions and load changes.

B) Temperature of turbine different points and essentially total temperature before the turbine (\(T_c\)) or after that (\(T_f\)) and axial velocity (\(n\)) should be limited to the maximum threshold. Particularly, temperature exhaust gas from the turbine is measured usually by thermocouples due to intolerance gas turbine inlet temperature and it will be considered as a measure of gas turbine heat tolerant.

The operational point considered in controller design and simulation will be as follows:

\[ n = 730.7095 \text{ 1/sec.} \]

\[ P_i = 208270 \text{  } P_o \]

\[ m_{comb} = 0.0043 \text{ kg} \]

The above operational point is shown as the following vector:

\[ X_o = [730.7095 \ 208270 \ 0.0043] \]

Values corresponding to such an operating point is declared for three disturbed inputs:

\[ T_c = 305.45 \text{ K} \]

\[ P_i = 98711 \text{  } P_o \]

\[ M_{load} = 99.2 \text{ N.M.} \]

And disturbance Vector is denoted as:

\[ d = [305.45 \ 98711 \ 99.2] \]

In this section, simulation results of MPC controller on a nonlinear gas turbine is evaluated and results are examined in several modes with different noises. The gas turbine system state variables are defined as follows: \( \tau = [m_{comb} \  P_i \  n]^T \), where \( m_{comb} \) is mass of combustion chamber inside, \( P_i \) is the total pressure of turbine inlet duct, \( n \) is dimensionless turbine speed. Accordingly, the equations have been used. In Figure 4, block diagram of problem solving through MPC approach is shown.

4. SIMULATION

In the simulation section, fuel mass flow rate is employed as control commands for mathematical developed model at a specified interval so that corresponding values are extracted from references [5, 6]. The model output results are stored and plotted for analysis and comparison. It should be noted that this is an open-loop evaluation of controller and there is no feedback for fuel mass flow rate input applied to the turbine. Indeed, the only feedback applied is only turbine state variables values to be used for simulation of the next step. To test controller performance, a reference speed is applied as an input to the controller in order to be tracked by obtained turbine speed. The simulation results are shown below.

Figures 5 to 7 shows the system state variables, Figure 8 shows control variable and Figures 9 to 11 indicate disturbances applied to system. Obvioucly, in these figures any changes of each variable is observable duration simulation time.
As expected, their values have not been exceeded of the specified range (according to limitations imposed on control system) and it indicates that designed controller has effectively meet desired objective by considering control and systematic constraints.

5. DISCUSSION

The functional values of state variables are described as:

\[
0.0021 < m_{fuel} < 0.011 \\
101334 < P_r < 357894 \\
650 < \alpha < 833.33
\]

And the range of only input variable \( m_{fuel} \) is defined as:

\[
0.00367 < m_{fuel} < 0.027
\]

And also functional values of perturbation parameters are declared as follows:

\[
98700 < P_i < 110000 \\
283.15 < T_i < 308 \\
0 < M_{\text{fuel}} < 365
\]
The advanced objective is associated with designing a controller for gas turbine nonlinear system. Partially, if disturbances parameters are being fixed and fuel flow rate increase, gas turbines speed will be increased and turbine outlet temperature decreases. If we maintain fuel flow rate as a constant, by increasing each of the perturbation parameters, gas turbine speed reduces and turbine outlet temperature increases. Indeed, controller must be designed in such a way that the stability of the system is realized against disturbance input variations so that simulation results reveal stability of system against disturbance inputs variations. In reference [19], adaptive control method is applied for gas turbine speed tracking. In this reference controller could well track speed proportional with load torque. The mentioned adaptive approach is simulated for our dynamic model and as it can be seen in Figure 12 for reference [19] and Figure 13 corresponding to MPC controller results, both approaches are well capable to track nominal speed but model predictive control readily can track reference speed with shortest simulation time.

6. CONCLUSIONS AND RECOMMENDATIONS

It should be noted that there are several approaches to solve gas turbine control problem such that most of them are suitable only for speed and power tracking control while method used in this paper is extremely efficient unlike other methods because it is able to define a suitable cost function contributing to achieve the most optimal conditions possible for meeting all constraints ranging from dynamic, differential, etc. Meanwhile, simulation results demonstrate effectiveness performance of proposed controller. Besides it satisfies all constraints this controller tried to hold power within acceptable range according to inlet air. Meanwhile, due to less consideration of issues relating to gas turbines in our country we recommend to researchers to apply online intelligent control solutions for turbines plant.
Moreover, a systematic cost function determination which is compromising with this research can be carried out as following: investigation of other constraints such as dynamics restrictions (for example, variations is structure, dimensions and turbines dynamics) during rotating mode and handling these values over power generation process as well as considering functional constraints that can be expressed in terms of performance. Moreover, evaluation of optimal controllers with other criteria, such as minimum required power control, minimum power consumption and so on is recommended.

7. REFERENCES


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By focusing on the importance of industrial gas turbines in various industries and the necessity of designing a suitable control system for them, extensive research has been conducted in recent decades. The design of a control system for gas turbines involves many control variables, but one of the most important variables is the fuel flow rate. The fuel flow rate affects all the parameters of turbine operation, such as pilot gas turbine, compressor pressure ratio, fuel consumption, and exhaust temperature. In this paper, a control system is designed in a closed loop form such that the parameters are maintained within acceptable limits. On the other hand, gas turbines are characterized by complex and time-varying parameters; therefore, the parameters change constantly. In this paper, the design of an industrial gas turbine control system is presented in the form of a closed-loop predictive control model, which can be considered a control method in which the control is performed in an optimal way. The goal of the research is to control the fuel flow rate according to the desired values.