A Comparative Study of Extreme Learning Machines and Support Vector Machines in Prediction of Sediment Transport in Open Channels

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\textbf{PAPER INFO}

\textbf{Paper history:}
Received 10 August 2016
Received in revised form 27 September 2016
Accepted 30 September 2016

\textbf{Keywords:}
Extreme Learning Machines (ELM)
Non-deposition
Open channel
Sediment transport
Support Vector Machines (SVM)

\textbf{ABSTRACT}

The limiting velocity in open channels to prevent long-term sedimentation is predicted in this paper using a powerful soft computing technique known as Extreme Learning Machines (ELM). The ELM is a single Layer Feed-forward Neural Network (SLFNN) with a high level of training speed. The dimensionless parameter of limiting velocity which is known as the densimetric Froude number ($Fr$) is predicted using ELM and the results are compared to those obtained using a Support Vector Machines (SVM). The comparison of the ELM and SVM methods indicates a good performance for both methods in the prediction of $Fr$. In addition to being computationally faster, the ELM method has a higher level of accuracy ($R^2=0.99$, $MAE=0.10$, $MAPE=2.34$, $RMSE=0.14$, $CRM=0.02$) compared with the SVM approach.

\textbf{NOMENCLATURE}

\begin{tabular}{ll}
$A$ & cross-sectional area of flow (m$^2$/s) \\
$b$ & bias terms of the equation \\
$b_i$ & threshold of the $i$th hidden neuron \\
$C_s$ & volumetric sediment concentration \\
$D$ & pipe diameter (m) \\
$d$ & median particle diameter (m) \\
$D_p(=((d_s-1)/\nu^2)^{1/3})$ & dimensionless particle size \\
$Fr(=V/(g(s-1)/d)^{0.5})$ & densimetric Froude number \\
g & gravitational acceleration \\
g(x) & membership function \\
$H$ & neural network output matrix \\
$K(x-x_i)$ & kernel function \\
$N$ & number of hidden neurons \\
$R$ & hydraulic radius (m) \\
$s$ & specific gravity of sediment \\
$V$ & flow velocity (m/s) \\
w & weighting vector \\
y & flow depth (m) \\
$\lambda_s$ & sediment friction factor \\
$\nu$ & kinematic viscosity (m$^2$/s) \\
$\zeta, \zeta^*$ & slack variables \\
$\rho$ & water density (kg/m$^3$) \\
$\rho_s$ & sediment density (kg/m$^3$) \\
$\varphi$ & a nonlinear function \\
$s$ & sediment \\
\end{tabular}

\textbf{Greek Symbols}

\begin{tabular}{ll}
$\alpha$ & neural network output matrix \\
$\alpha^*$ & kernel function \\
$\mu$ & number of hidden neurons \\
$\nu$ & kinematic viscosity (m$^2$/s) \\
$\zeta, \zeta^*$ & slack variables \\
$\rho$ & water density (kg/m$^3$) \\
$\rho_s$ & sediment density (kg/m$^3$) \\
$\varphi$ & a nonlinear function \\
$s$ & sediment \\
\end{tabular}

\textbf{Subscripts}

\begin{tabular}{ll}
$N$ & number of hidden neurons \\
$R$ & hydraulic radius (m) \\
\end{tabular}

\textbf{1. INTRODUCTION}

One of the most important issues in open channel design is the economic and optimized planning of it. Due to the through path of flow before reaching the channel, the inflow may erode and suspend sediments which are then transported with the flow into the open channel. If the flow velocity for a given channel slope (limiting velocity) is insufficient to transport the sediment in the flow, the sediment will be deposited within the channel. In the case of fine sediment, the
longer it remains on the bed, the more likely consolidation will occur which may lead to a permanent reduction in channel depth and a reduction in the flow cross section and changes to the velocity and shear stress in the channel. Sediment deposition occurs more often in dry weather, when the discharge flows are low or at a minimum. Hence, in the design of open channel systems sediment deposition should be avoided as much as possible; as this will minimize maintenance and operational costs.

One of the easiest approaches used in open channel design is constant velocity or constant shear stress. In this method of design, the minimum velocity value (in the range of 0.3-0.9 m/s) or shear stress (in the range of 1-2.5 N/m²) is determined [1]. However, using this approach is not considered to be a good practice, due to the lack of consideration of other hydraulic parameters including the sediment and the flow and channel characteristics. Therefore, numerous experimental and theoretical studies to evaluate the flow hydraulic in the open channels were conducted by many researchers [2-6]. From this research a range of different relationships were presented, which were mostly based on regression analysis. The main problem of using regression equations is they generally perform well for data upon which they have been derived, however, for other datasets the performance is often less good leading to limiting velocity predictions which are either an underestimate or overestimate with large errors.

In recent years the use of artificial intelligence techniques has increased due to their good performance in identifying relationships between the parameters in non-linear systems and across a range of different engineering fields, but particularly in hydraulics and hydrology where the results have often been remarkably good [7-13]. Kumar et al. [14] presented predictor models based on genetic programming for incipient motion, sediment transport in vegetated flow and total bedload. Kumar et al. developed their models and compared with several previous regression models and found the accuracy of the results to be better than these earlier models. Bonakdari & Ebtehaj [15] compared two different data driven methods, namely Gene-Expression Programming (GEP) and Group Method of Data Handling (GMMDH) for the prediction of sediment transport in pipe channels. They presented two equations which were derived from a wide range of hydraulic parameters for use in practical design. Azamathulla et al. [16] proposed a functional relationship to predict sediment transport in pipes using Adaptive Neuro-Fuzzy Inference Systems (ANFIS) as an alternative approach obtaining results with high accuracy. Najafzadeh et al. [17] predicted critical velocity for preventing sedimentation by Evolutionary Polynomial Regression (EPR) and the Model Tree (MT). The authors compared the results of proposed technique with benchmark equations and found that the new artificial intelligence methods (MT and EPR) are more stronger than others method.

One of the newest soft computing approaches is Extreme Learning machines (ELM). ELM is a Single-Layer Feed-Forward Neural Network (SLFNN) which removes the problems of general neural networks such as computational time and overfitting. The use of this method in different fields of science such as feature selection [18], non-linear time-series data analysis [19], bioinformatics [20], and environmental engineering [21,22] indicated a high level of accuracy.

In this study the ELM approach is developed to predict sediment transport in open channels. The performance of the ELM is compared with another powerful techniques used in soft computing, namely the Support Vector Machines (SVM) method. For this purpose, it is first necessary to determine the effective dimensionless parameters to represent sediment transport without deposition in open channel flow using dimensional analysis. Following this, the ELM and SVM methods are used to predict the limiting velocity.

2. METHODS

2.1. Extreme Learning Machine (ELM) One of the classical neural networks (NN) problems is the computational time taken to perform the calculations due to using gradient-based learning algorithms and iterative tuning parameters. Therefore to overcome this problem, Huang et al. [23] introduced a new training algorithm, a single-hidden layer feed-forward neural network (SLFNN), with random determination of the hidden layer neurons to establish the output weights. Unlike gradient-based training algorithms, which only minimize the model training error, the ELM method, in addition to considering this issue, also randomly assigns weights connecting inputs to the hidden nodes. In addition, ELM solves the classic gradient-based algorithm problem that are used only for differentiable activation functions, and in a SLFNN they can be trained with a non-differentiable activation function as well [23]. Also this method avoids the problems associated with the gradient method such as overfitting, local minimum, and improper learning rate [24].

With \( N \) samples defined as \((x_j,t_j)\) where \(t_j=[t_{j1},t_{j2},\ldots,t_{jm}]^T \in \mathbb{R}^m \) \(x_j=[x_{j1},x_{j2},\ldots,x_{jm}]^T \in \mathbb{R}^m \), a standard neural network with a hidden layer, membership function \((g(x))\), and the number of hidden neurons \(N\) is defined as follows:

\[
\sum_{i=0}^{\hat{N}} \beta_i g(w_i,x_j+b_i) = O_j \quad j=1,2,\ldots,N
\]  

(1)

where \(w_i = [w_{i1},\ldots,w_{im}]^T\) and \(\beta_i = [\beta_{i1},\beta_{i2},\ldots,\beta_{im}]^T\) are the vector weights that connect the input and output neurons
to $i^{th}$ neuron of the hidden layer, $b_i$ is the threshold of the $i^{th}$ hidden neuron and the ‘*’ in $w_1, x_1$ is the inner product of $w_1$ and $x_1$.

SLFNN aims to minimize the difference between the predicted values ($e_j$) and actual values ($t_j$) which is defined as follows:

$$\sum_{j=1}^{N} \beta_j (w_j, x_j + b_j) = t_j \quad j = 1,2,...N$$  \hspace{1cm} (2)

which can be present as a compact form as follows:

$$H^T \beta = T$$  \hspace{1cm} (3)

where

$$H(w_1,...w_N, b_1,...b_N, x_1,...x_N) =$$

$$\begin{bmatrix}
g(w_1, x_1 + b_1) & \cdots & g(w_N, x_N + b_N) \\
\vdots & \ddots & \vdots \\
g(w_1, x_N + b_1) & \cdots & g(w_N, x_N + b_N)
\end{bmatrix}$$  \hspace{1cm} (4)

$$\beta = \begin{bmatrix}
\beta_1^T \\
\beta_2^T
\end{bmatrix}$$  \hspace{1cm} (5)

$$T = \begin{bmatrix}
T_1^T \\
T_2^T
\end{bmatrix}$$  \hspace{1cm} (6)

where $H$ is known as a neural network output matrix.

According to the provided description, the training process in an ELM algorithm can be explained in a general stage: In the first stage, random values are dedicated to weights and bias in the hidden layer neurons, and the output value of the hidden layer using matrix $H$ is estimated. In the second stage, the output weights using matrix $H$, and the desired values (target) for different samples are calculated. Using matrix $H$ to determine the weights gives much higher computational speeds than existing methods such as Levenberg-Marquards [23, 24].

The number of hidden neurons not only affects the network complexity in order to model a nonlinear system but also affects the ability of network to generalize and learning. Considering many number of hidden neurons will lead to overfitting. Due to no existence of unique relation to calculate the number of hidden neurons before training it should be determine through trial and error. In this study, trial and error is utilized to determine the maximum permissible number of ELM hidden neurons. It is clear that increasing the number of hidden neurons results in higher prediction performance with the training dataset. However, overfitting should also be considered. Increasing the number of hidden neurons may lead to a model that predicts the training dataset very well but has high error in predicting the testing dataset. In such cases, overfitting occurs.

In the present case, the number of hidden neurons are increased considering that the difference between the training prediction accuracy and the testing prediction accuracy is very low. So that, it could be mentioned that there is no overfitting here. The number of hidden neurons in the ELM models were considered as 15. Also, the used activation function was sigmoidal.

### 2.2. Support Vector Machine (SVM)

SVM is a new modelling technique that uses the statistical learning theory principles [25]. This modelling technique applies an optimized linear regression model in a feature space to estimate the unknown values. The feature space is defined using input data mapping from the main space in an m-dimensional space. For a given observational dataset with an input vector as p-dimensional and the target vector as one dimensional, the relationship between the input and output can be expressed as follows:

$$f(x) = w^T \varphi(x) + b$$  \hspace{1cm} (7)

where $\varphi$ is a nonlinear function and $b$ and $w$ are the bias terms of the equation and weighting vector, respectively. Optimal values of these parameters whilst minimizing the risk function using variables $\zeta_i$ and $\zeta_i^*$ known as the slack variables, are calculated as follows:

$$R(f) = \frac{1}{2} ||w||^2 + C \sum_{i=1}^{l} (\zeta_i + \zeta_i^*)$$  \hspace{1cm} (8)

Subjected to:

$$\begin{aligned}
  d_i - w^T \varphi(x_i) - b &\leq \varepsilon + \zeta_i & i = 1,2,...l \\
  w^T \varphi(x_i) + b - d_i &\leq \varepsilon + \zeta_i^* & i = 1,2,...l \\
  \zeta_i, \zeta_i^* &\geq 0 & i = 1,2,...l
\end{aligned}$$  \hspace{1cm} (9)

where $C$ is a constant parameter defining the trade-off between the determination error and flatness. Equation (9) is solved based on defining Lagrange multipliers $(a_i, a_i^* \in [0, C])$ and the dual problematic formulation. The solution of this equation is presented as follows:

$$f(x) = \sum_{i=1}^{l} (a_i - a_i^*) K(x_i - x_i^*) + b$$  \hspace{1cm} (10)

where $K(x_i - x_i^*)$ is called a kernel function and $x_i$ and $x_i^*$ are two vectors in the input space (training or testing). The Radial Basis Function (RBF) kernel function has been applied to problems in a number of fields and has shown good performance due to features such as computational efficiency and reliability [26, 27]. Therefore, in this study, the kernel function is applied and calculated as follows:
\[ K(x, x') = \exp(-\gamma|x - x'|^2) \]  
(11)

It should be noted that the good performance of SVM modeling is dependent on accurate determination of three parameters \( \gamma, \varepsilon \) and \( C \). In this study, the values of these parameters were considered as 0.45, 0.05 and 0.95, respectively, through trial and error.

3. METHODOLOGY

3.1 Dimensional Analysis  From the assessment of experimental and analytical studies in the field of sediment transport in open channels \([2, 5, 28]\) a number of different parameters such as hydraulic radius \( R \), flow depth \( y \), cross-sectional area of flow \( A \), flow velocity \( V \), water density \( \rho \), sediment density \( \rho_s \), kinematic viscosity \( \nu \), pipe diameter \( D \), median particle diameter \( d \), sediment friction factor \( \lambda_s \) and volumetric sediment concentration \( C_v \) were considered to be important to estimate the minimum velocity to prevent sediment deposition (limiting velocity). The functional equation of limiting velocity is presented as follows:

\[ V = \Phi \left( \frac{V}{g(s-1)/d^{0.5}}, \frac{D}{d}, \frac{R}{D}, \frac{A}{D}, \lambda_s \right) \]  
(12)

In the above equation, the volumetric sediment concentration \( C_v \) and sediment friction factor \( \lambda_s \) parameters are dimensionless parameters. Using dimensional analysis, the effective dimensional parameters in the relationship are represented by different dimensionless parameters as follows: densimetric Froude number \( Fr = V/(g(s-1)/d^{0.5}) \), dimensionless particle size \( D_s = (d/(s-1)/v^{0.5}) \), the ratio of median diameter particle size to hydraulic radius \( d/R \), the ratio of median diameter to pipe diameter \( D/d \), the ratio of hydraulic radius to pipe diameter \( R/D \) and the square pipe diameter to the cross-sectional area of flow \( D^2/A \) \([8, 29, 30]\). Regarding the nature of each dimensionless parameter, Ebtehaj and Bonakdari \([8]\) categorized the dimensionless parameters into five different groups.

The five groups are “movement” \( Fr \), “flow resistance” \( \lambda_s \), “transport” \( C_v \), “transport mode” \( d/R \), \( R/D \), \( D^2/A \) and “sediment” \( D_{dp} \) and \( d/D \). The \( Fr \) parameter provides the limiting velocity as a dimensionless value and is the only member of the “movement” group and is considered as the target parameter. Among the residual four groups, the “flow resistance” and “transport” groups only have one parameter whilst the “sediment” and “transport mode” groups, respectively, have 2 and 3 different dimensionless parameters. Hence, to consider the parameters of all four groups, 6 different combinations are required to calculate the limiting velocity, which can be expressed as a dimensionless parameter, \( Fr \), as follows:

\[ Fr_1 = \Phi_1(C_v, D_{dp}, d/R, \lambda_s) \]  
(13)

\[ Fr_2 = \Phi_2(C_v, D_{dp}, D^2/R, \lambda_s) \]  
(14)

\[ Fr_3 = \Phi_3(C_v, D_{dp}, R/D, \lambda_s) \]  
(15)

\[ Fr_4 = \Phi_4(C_v, d/D, d/R, \lambda_s) \]  
(16)

\[ Fr_5 = \Phi_5(C_v, d/D, D^2/R, \lambda_s) \]  
(17)

\[ Fr_6 = \Phi_6(C_v, d/D, R/D, \lambda_s) \]  
(18)

Recent study of the authors \([30]\) showed that among the different combinations of the above relationships, the relationship \( Fr_5 \) shown in Equation (16) provides the best results compared to the other relationships. Therefore, in this study, the performance of the ELM and SVM method is evaluated utilizing Equation (16).

3.2. Used Data  In this study to evaluate the \( Fr \) variable, three different datasets; Vongvissomjai et al. \([5]\), Ab Ghani \([28]\) and Ota and Nalluri \([31]\), have been applied. Vongvissomjai et al. \([5]\) conducted their experimental tests at two pipe channel with different diameters, 100 and 150 mm. The pipes length are 16 m. The authors used three slopes 0.002, 0.004 and 0.006. The uniform sands with different median particle diameter \( d = 0.2, 0.3 \) and 0.43 mm) were used. Also, the Manning roughness coefficient for clear water tests was 0.0125. Using three pipes of 154, 350 and 450 mm of diameters, 20.5 m length and maximum flow discharge of 0.04 m³/s, Ab Ghani \([28]\) conducted their experimental tests. The bed of pipes was considered smooth and rough. The test conducted on different slopes so that the maximum was 0.006. Ota and Nalluri \([31]\) conducted their experimental test using a pipe with 18 m length and 305 mm diameter. The authors surveyed the effect of sediment gradation on sediment transport by considering uniform and non-uniform conditions. The specific gravity of all used sediment was 2.65. More details of the datasets are presented in previous studies \([8, 29, 30]\).

Based on presented descriptions, the experiments have been conducted in different experimental conditions and, therefore, provide a wide range of hydraulic parameters for use in the analysis.

To train the model in this study, all the data were divided into two categories: train and test. Among the 218 different datasets, 70% of all datasets (151 samples) were selected randomly to train the model and other datasets (67 samples) were used to test the model.
3.3. Goodness of Fitness  
In this paper, different statistical indices such as the correlation coefficient ($R^2$), Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Coefficient of Residual Mass (CRM) which is an index for trend recognition of prediction are used for performance evaluation of each soft computing method (ELM & SVM). The calculation of the above mentioned indices are as follows:

$$R^2 = \left(\frac{n \sum T_{i0} T_{o} - \sum T_i \sum T_o}{\sqrt{\left(n \sum T_i^2 - (\sum T_i)^2\right) \left(n \sum T_o^2 - (\sum T_o)^2\right)}}\right)^2$$  \hspace{1cm} (19)$$

$$RMSE = \sqrt{\frac{1}{n} \sum T_i^2}$$ \hspace{1cm} (20)

$$MAE = \frac{1}{n} \sum |T_i - T_o|$$ \hspace{1cm} (21)

$$MAPE = \frac{100}{n} \sum \frac{\vert T_i - T_o \vert}{T_o}$$ \hspace{1cm} (22)

$$CRM = \left(\frac{\sum T_{i0} - \sum T_o}{\sum T_o}\right) / \sum T_o$$ \hspace{1cm} (23)

where $T_{i0}$ and $T_o$ are the measured and corresponding predicted value of the densimetric Froude number ($Fr$), respectively, and $n$ is the number of samples. The combination of these statistical indices is sufficient to evaluate model performance.

4. RESULTS AND DISCUSSION

In this section, the results of modelling the densimetric Froude number ($Fr$) using SVM and ELM artificial intelligence methods are provided. Figure 1 compares the $Fr$ modelling results using SVM and ELM methods in both the train and test modes to the observed experimental values. According to Figure 1 it can be seen that in model training mode, both the SVM ($R^2 = 0.97$) and ELM ($R^2 = 0.98$) methods have a relatively good performance, as the majority of the estimated values have errors in the range of ± 10%. The average relative error for both methods, SVM (MAPE = 5.82%) and ELM (MAPE = 5.94%) is almost equal and less than 6%. Values for the other are presented in Table 2, and also show good performance of SVM (MAE = 0.24 & RMSE = 0.36) and ELM (MAE = 0.22 & RMSE = 0.29) methods in estimating the value of $Fr$. The CRM index value for both the SVM and ELM method in model training is positive, which indicates the overestimate performance of the models. It is noteworthy that the index value is relatively small ($SVM = 0.01$ & $ELM = 0.02$). As a result, using these methods to estimate $Fr$, does not lead to a significant increase in the economic cost of the design. For small values of $Fr$ ($Fr<5$), ELM estimations are associated with a relative error of more than 10%, and for large values of $Fr$ ($Fr>5$), SVM method has estimations with errors more than 10%. But as can be seen, these method would not be reliable in Fr estimation. Test data results indicate that both methods for all $Fr$ values, estimates the variable value with less than 10% relative error, in fact using test data that have not any role in model training, not only reduce the SVM ($R^2 = 0.99$) and ELM ($R^2 = 0.99$) performance, but also increase the model accuracy as well. Table 1 shows that CRM index value for both methods is similar to model training mode with positive value. In fact, the modeling process is not changed.

Table 1 presents the statistical indices which report the model performance as an average, whilst in Figure 2, the cumulative relative error value for both the SVM and ELM methods is provided. The general conclusion that can be obtained from this figure is that both the SVM and ELM methods have relatively similar performances, as both methods present about 90% of the estimated values with relative errors less than 10%. Also 60% of estimations have a relative error less than 5%. The figure shows that less than 2% of the estimated values of $Fr$ using SVM and ELM have an error of more than 15%.

According to the given description in Figures 1 and 2 and Table 1, it can be concluded that both presented models in this study have a very good performance in $Fr$ estimation. But the computational speed of the SVM and ELM methods are not comparable as the ELM, trains the model much more quickly. Also in the ELM approach only the determination of the hidden layer neuron values is needed; whilst in the SVM method the coefficients of the kernel function and the coefficient C need to be optimized simultaneously and may lack proper selection, leading to poor modelling results.

Figure 3 shows the Discrepancy Ratio (DR) for the SVM and ELM methods. The $DR$ is the average of the relative predicted value to actual value. According to the figure, most of the estimated values using the SVM method are in the range of 0.95 < $DR$ < 1.0. It is also observed that the minimum and maximum $DR$ value in the SVM method are 1 ± 0.15. The SVM and ELM methods show the same degree of scatter indicating that the minimum and maximum values of the $DR$ for both models is the same. Tables 2 and 3 indicated the results of sensitivity analysis for ELM and SVM techniques, respectively. Based on these tables, the weakest result is related to model 4-4 which removed the variable $C_1$ in comapred with Equation (16). The mean absolute relative error of model 4-4 is 6 time more than Equation (16).
The best performance of presented model in Tables 3 and 4 except model 4, is regard to model 4-2 for Elm ($R^2 = 0.96$, $MAE = -0.09$; $MAPE = 6.15$; $RMSE = 0.69$; $CRM = -0.02$) and SVM ($R^2 = 0.95$, $MAE = -0.09$; $MAPE = 6.82$; $RMSE = .047$; $CRM = -0.03$). The difference of this model with model 4 is the lack use of $d/D$ as an effective parameters in $Fr$ predicting.

Because the effect of $d$ and $D$ are considered in $d/R$ dimensionles parameters. Based on the statistical indices presented in Table 1 and 2, the lack use of each dimensionless variable presented in Equation (16) as an input parameter in predicting $Fr$ lead to reduction of modeling accuracy. Therefore, all four variables in Equation (16) are essential to reach high modeling performance.

### Table 1. Statistical indices for performance evaluation of ELM and SVM (Train & Test)

<table>
<thead>
<tr>
<th>Index</th>
<th>SVM</th>
<th>ELM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train</td>
<td>$R^2$</td>
<td>0.97</td>
</tr>
<tr>
<td></td>
<td>$MAE$</td>
<td>0.24</td>
</tr>
<tr>
<td></td>
<td>$MAPE$</td>
<td>5.82</td>
</tr>
<tr>
<td></td>
<td>$RMSE$</td>
<td>0.36</td>
</tr>
<tr>
<td></td>
<td>$CRM$</td>
<td>0.01</td>
</tr>
<tr>
<td>Test</td>
<td>$R^2$</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>$MAE$</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>$MAPE$</td>
<td>3.24</td>
</tr>
<tr>
<td></td>
<td>$RMSE$</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>$CRM$</td>
<td>0.03</td>
</tr>
</tbody>
</table>

**Figure 1.** Comparison of ELM and SVM performance in prediction of $Fr$ (Train & Test)

**Figure 2.** Error distribution for ELM and SVM methods

**Figure 3.** Histogram of $DR$ for $Fr$ predicted by ELM and SVM

Because the effect of $d$ and $D$ are considered in $d/R$ dimensionles parameters. Based on the statistical indices presented in Table 1 and 2, the lack use of each dimensionless variable presented in Equation (16) as an input parameter in predicting $Fr$ lead to reduction of modeling accuracy. Therefore, all four variables in Equation (16) are essential to reach high modeling performance.
TABLE 2. Results of sensitivity analysis for ELM

<table>
<thead>
<tr>
<th>Input variables</th>
<th>$R^2$</th>
<th>RMSE</th>
<th>MAE</th>
<th>MAPE</th>
<th>CRM</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. $Fr = \Phi(C_v, d/R, d/D, \lambda)$</td>
<td>0.99</td>
<td>0.1</td>
<td>2.34</td>
<td>0.14</td>
<td>0.02</td>
</tr>
<tr>
<td>4-1. $Fr = \Phi(C_v, d/R, d/D)$</td>
<td>0.95</td>
<td>0.52</td>
<td>-0.12</td>
<td>8.11</td>
<td>-0.03</td>
</tr>
<tr>
<td>4-2. $Fr = \Phi(C_v, d/R, \lambda)$</td>
<td>0.96</td>
<td>0.36</td>
<td>-0.09</td>
<td>6.15</td>
<td>-0.02</td>
</tr>
<tr>
<td>4-3. $Fr = \Phi(C_v, d/D, \lambda)$</td>
<td>0.87</td>
<td>0.63</td>
<td>-0.15</td>
<td>11.62</td>
<td>-0.04</td>
</tr>
<tr>
<td>4-4. $Fr = \Phi(d/R, d/D, \lambda)$</td>
<td>0.75</td>
<td>0.83</td>
<td>-0.04</td>
<td>12.57</td>
<td>-0.10</td>
</tr>
</tbody>
</table>

TABLE 3. Results of sensitivity analysis for SVM

<table>
<thead>
<tr>
<th>Input variables</th>
<th>$R^2$</th>
<th>RMSE</th>
<th>MAE</th>
<th>MAPE</th>
<th>CRM</th>
</tr>
</thead>
<tbody>
<tr>
<td>4. $Fr = \Phi(C_v, d/R, \lambda)$</td>
<td>0.99</td>
<td>0.14</td>
<td>3.24</td>
<td>0.19</td>
<td>0.03</td>
</tr>
<tr>
<td>4-1. $Fr = \Phi(C_v, d/R)$</td>
<td>0.95</td>
<td>0.58</td>
<td>-0.10</td>
<td>8.84</td>
<td>-0.02</td>
</tr>
<tr>
<td>4-2. $Fr = \Phi(C_v, \lambda)$</td>
<td>0.95</td>
<td>0.47</td>
<td>-0.09</td>
<td>6.82</td>
<td>-0.03</td>
</tr>
<tr>
<td>4-3. $Fr = \Phi(C_v, d/D)$</td>
<td>0.86</td>
<td>0.69</td>
<td>-0.14</td>
<td>12.09</td>
<td>-0.03</td>
</tr>
<tr>
<td>4-4. $Fr = \Phi(d/R, \lambda)$</td>
<td>0.76</td>
<td>0.83</td>
<td>-0.05</td>
<td>12.92</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

5. CONCLUSION

Concerning the importance of sediment transport in open channels with the aim of limiting sediment deposition, this study has used a new artificial intelligence method to obtain an estimate of the limiting value of velocity to minimize sediment deposition. The numerical approach combines the fast and powerful Extreme Learning Machines (ELM) method with the Support Vector Machines (SVM) method. The key parameters used in the model were obtained using dimensional analysis. The densimetric Froude number ($Fr$) was represented by a number of different dimensionless parameters and its value was predicted by using the ELM and SVM methods. The results showed that both methods, SVM ($R^2 = 0.99$, $MAE = 0.14$; $MAPE = 3.24$; $RMSE = 0.19$; $CRM = 0.03$) and ELM ($R^2 = 0.99$, $MAE = 0.10$; $MAPE = 2.34$; $RMSE = 0.14$; $CRM = 0.02$) compared against the data used in this study to train and test the models accurately estimated the value of $Fr$. The error description for both methods showed that about 90% of the estimated values using these methods had a relative error less than 10%. Also, the calculated $DR$ value in this study for the ELM showed that the index value in the weakest condition was $1 \pm 0.15$. The results show that the ELM method, in addition to giving a good accuracy in the modelling, was computationally very efficient and, therefore, can be used as a good alternative to the classical artificial intelligence methods that are normally used to achieve the optimised solutions. The results of sensitivity analysis for ELM and SVM show that the lack of use of each dimensionless parameters which are presented in Equation (16), result in significant decrease in $Fr$ predicting. The results indicated that he $d/D$ and $C_v$ have the lower and higher impact on $Fr$ predicting.

6. REFERENCES


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Paper Info

Paper history:
Received 10 August 2016
Received in revised form 27 September 2016
Accepted 30 September 2016

Keywords:
Extreme Learning Machines (ELM)
Non-deposition
Open channel
Sediment transport
Support Vector Machines (SVM)

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Doi: 10.5829/idosi.ije.2016.29.11b.03