Two-tier Supplier Base Efficiency Evaluation Via Network DEA: A Game Theory Approach

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Abstract

In today's competitive markets, most firms try to reduce their supply costs by selecting efficient suppliers using different techniques. Several methods can be applied to evaluate the efficiency of suppliers. This paper develops generalized network data envelopment analysis models to examine the efficiency of two-tier suppliers under cooperative and non-cooperative strategies where each tier has its own inputs/outputs and some outputs of the first tier can be fed back to the second tier. Since the proposed models become nonlinear, an efficient heuristic method is proposed as an alternative solution, which can be used instead of existing time consuming methods like parametric linear programming approach. A numerical example is presented to exhibit the implementation of the proposed models. Also, for simulated data, results of proposed heuristic method and parametric linear programming are compared to demonstrate the validity and efficiency of the proposed approach.


1. INTRODUCTION

Supply chain management (SCM) is considered as the management of upstream/downstream relationships with suppliers and customers to deliver superior customer value to the supply chain as a whole by spending minimum expenses [1]. One of the key aspects of SCM, which plays an important role in supply chain costs reduction is supplier management. Supplier management means “organizing the optimal flow of high-quality, value-for-money materials or components to manufacture companies from a suitable set of innovative suppliers” [2]. Indeed, supplier management is the management of the upstream part of a supply chain.

Supplier evaluation is one of the most important tasks in supplier management that refers to methods, modals and techniques that firms apply to assess and select their suppliers. Supplier evaluation problem has been widely studied and various approaches have been presented to tackle the problem. Some of the basic and popular supplier evaluation and selection techniques are: data envelopment analysis, analytical hierarchical process, linear weighting models, outranking, expert systems and portfolio analysis [3].

The main objective of this paper is to develop general data envelopment analysis models for efficiency evaluation of suppliers. Data envelopment analysis (DEA) is an effective method proposed by Charnes et al. [4] to evaluate relative efficiency of a set of decision making units (DMU) that use multiple inputs to produce multiple outputs. DEA has been used in variety of industries and many theoretical developments have been reported. For example, recently Hatefi et al. [5] developed a common weight multi criteria decision analysis-data envelopment analysis (MCDA-DEA) method with assurance region for weight derivation in a pair wise comparison matrix (PCM). Torabi and Shokr applied the approach proposed by Hatefi et al. [5] for material selection problem [6]. A comprehensive review of DEA models and applications were presented by Cook and Seiford [7] and Liu et al. [8].

Traditional DEA was developed to evaluate the efficiency of a DMU as a black box without considering
its internal structure. Some studies indicate that to investigate the efficiency of a DMU, we may not always ignore internal processes [9, 10]. In the related literature, DEA models that consider the internal structure of DMU’s are called “network DEA” models [11]. Kao [12] reviewed different studies on network DEA and classified them based on the type of model used or developed and structures of considered network.

Application of game theory approach to model the relationship between components in network DEA originates from the work of Liang et al. [13]. They developed DEA-based models to evaluate the efficiency of seller-buyer supply chains when intermediate measures are considered. In their paper the relationship between buyer and seller is treated as leader-follower and also cooperative game. They developed non-linear programming models to evaluate the efficiency of two stage supply chain. The models can be treated as parametric linear programming models to obtain the best solution.

In existing literature, three types of games are applied to model the relationship between sub-DMUs, non-cooperative game [13-17], cooperative game [13-17] and bargaining game [18, 19].

First probable relationship between suppliers is to consider them as leader and follower, i.e. non-cooperative game, which is also known as Stackelberg game. In this paradigm the leader supplier maximizes its profit and efficiency and the follower supplier tries to maximize its profit subject to consider a fixed value for the efficiency of leader. Another formal relationship among suppliers is a cooperative one to improve the overall efficiency. In this approach, efficiencies of suppliers are evaluated, simultaneously. For cases where all outputs of the first stage are inputs of the second stage, Liang et al. [16] developed both cooperative and non-cooperative models. Li et al. [15] extended the models proposed by Ling et al. by assuming that inputs of the second stage include outputs of the first stage as well as additional inputs. They developed a linear model for non-cooperative game and also a non-linear model for cooperative approach. The non-linear model can be solved globally using a parametric linear programming algorithm. Chen and Yan [17] introduced three mathematical models under the concept of centralized (cooperative), decentralized (non-cooperative), and mixed centralized situation to evaluate the performance of a supply chain with one supplier and two manufacturers.

Existing studies for evaluating the efficiency of suppliers are under following criticisms:
1) Most of the methods developed in previous researches are appropriate for evaluating the efficiency of one-tier suppliers, i.e., internal relationships between suppliers of different layers in a supplier base are ignored. Hence, existing approaches cannot be used to evaluate the efficiency of real-world cases with multi-tier (multi-stage, multi-layer) supplier bases.
2) In a few number of studies in the literature, a simple form of two-tier supplier base is assumed. This simple form is shown in Figure 1.

In this structure, the only outputs of stage 2 are those that are inputs of stage 1, i.e. stage 2 does not have additional outputs and stage 1 does not have additional inputs either. Furthermore, all outputs of stage 1 are sold to customer.

This simple structure may not be practical in real world applications. In many practical cases, stage 2 and stage 1 have additional outputs and inputs, respectively. Furthermore, some outputs of stage 1 may be fed back to stage 2. For example, in Hi-Tech industries, because of rapid growth, some unsold products are fed back to stage 1 for upgrading. Another example of such structure is automobile industry, where there is a close relationship between different suppliers of an automobile manufacturer. Supplier base of an automobile manufacturer mainly consists of raw material producer and automobile parts manufacturer. Raw material producer, sales metal plates in standard forms to parts manufacturer and scraps are flew back from parts manufacturer to raw material producer for remanufacturing (see illustrative example).
3) In papers that multi-tier supplier base and cooperation between suppliers are assumed, parametric linear programming is the common approach to solve the resulted non-linear models. This approach is time consuming especially when the number of DMUs is large.

This paper contributes to the current strand of literature in the field of supplier evaluation by developing general network DEA models under cooperation and non-cooperation conditions. The contributions of this paper can be categorized as follows:
1) Mathematical models based on network DEA approach are developed to evaluate the efficiency of two-tier supplier bases with input and output data in tier 2, input and output data in tier 1, intermediate measures between tier 2 and tier 1 and feedback measures from tier 1 to tier 2. All or some of these measures are ignored in literature for simplicity [15, 16].

Hence, proposed models are general forms that can be used for a wide range of applications. Models are developed under cooperation between suppliers and non-cooperation conditions.

\[ x_{g,1}^d = \begin{cases} z_{g,1} & \text{if } 1 \leq g \leq m \\ y_{g,2} & \text{if } 1 \leq g \leq R \\ \end{cases} \]

**Figure 1.** Common form of two-stage structures
2) For cases that developed models are in nonlinear form, a novel efficient heuristic method is proposed that can be used instead of parametric linear programming method.

This paper is structured as follows: problem definition and network DEA models under cooperative and non-cooperative situations are presented in section 2. The proposed heuristic method to solve the cooperative model is illustrated in section 3. In section 4, the relationship between efficiencies in different mechanisms and also efficiencies of tiers are presented through two theorems. The models are verified via numerical examples in section 5. In last section conclusions are given.

2. PROBLEM STATEMENT AND MODEL

In this paper it is considered that a company wishes to evaluate relative efficiency of \( n \) two-tier supplier bases and chooses the most efficient one as its supplier base. Each supplier has its inputs, outputs, and also intermediate flow between suppliers in different tiers is considered. Furthermore, the situation that outputs of first tier can be fed back to second tier as input is considered. This structure is depicted in Figure 2.

It is assumed that in each two-tier supplier base \( j \) \( (j = 1, \ldots, n) \) supplier of tier \( k \) \((k=1,2)\) has \( m^i \) inputs and \( R^k \) outputs. Also supplier of tier 2 has \( D \) intermediate outputs that are inputs of supplier of first tier. Furthermore, first tier supplier has \( G \) outputs that flow back to the tier 2. Parameters and variables used in presented models are as follows:

\[
\begin{align*}
0 & \quad \text{index of under evaluation supplier base} (0=1, \ldots, n) \\
k & \quad \text{index of tiers number in supplier base} (k=1,2) \\
m^i & \quad \text{number of inputs of tier k} \\
R^k & \quad \text{number of outputs of tier k} \\
D & \quad \text{number of intermediate outputs between tier 2 and tier 1} \\
G & \quad \text{number of feedbacks from tier 1 to tier 2} \\
x_{ij}^k & \quad \text{i th output of tier k for jth supplier base} \\
y_{ij}^k & \quad \text{i th input of tier k for jth supplier base} \\
z_{ij} & \quad \text{dth intermediate measure for jth supplier base} \\
f_{ij} & \quad \text{dth feedback for jth supplier base} \\
u_j^k & \quad \text{weight assigned to the rth output of tier k} \\
v_j^k & \quad \text{weight assigned to the rth input of tier k} \\
h_d & \quad \text{weight assigned to the dth intermediate measure} \\
w_{ij} & \quad \text{weight assigned to the gth feedback} 
\end{align*}
\]

Based on the constant return to scale CCR model [4], models 1 and 2 can be established to compute the efficiency of supplier 1 \( (e^1) \) and supplier 2 \( (e^2) \), respectively.

**max** \( e^1 = \sum_{i=1}^{m^1} u_j^k y_{ij}^k + \sum_{g=1}^{G} w_{ij} f_{ij} \)

subject to:

\[
\begin{align*}
\sum_{i=1}^{m^1} h_d z_{ij} + \sum_{j=1}^{n} \sum_{g=1}^{G} w_{ij} f_{ij} & \leq 1 \quad j = 1, \ldots, n \\
u_j^k, w_{ij}, h_d, v_j^k & \geq 0 \quad \forall r, g, d, i
\end{align*}
\]

**max** \( e^2 = \sum_{i=1}^{m^2} u_j^k y_{ij}^k + \sum_{g=1}^{G} w_{ij} f_{ij} \)

subject to:

\[
\begin{align*}
\sum_{i=1}^{m^2} h_d z_{ij} + \sum_{j=1}^{n} \sum_{g=1}^{G} w_{ij} f_{ij} & \leq 1 \quad j = 1, \ldots, n \\
u_j^k, w_{ij}, h_d, v_j^k & \geq 0 \quad \forall r, g, d, i
\end{align*}
\]

2. 1. Non-cooperative Model (Stackelberg Game Model)

In cases that one supplier is more important than the other one, the leader optimizes its profit and the follower determines its efficiency based on the information from leader. Considering supplier of tier 1 (supplier 1) as leader and supplier of tier 2 (supplier 2) as follower, the efficiency of supplier 1 can be computed using the following linear programming model. In a similar manner supplier 2 can be considered as leader.

**max** \( e^1 = \sum_{i=1}^{m^1} u_j^k y_{ij}^k + \sum_{g=1}^{G} w_{ij} f_{ij} \)

subject to:

\[
\begin{align*}
\sum_{i=1}^{m^1} h_d z_{ij} + \sum_{j=1}^{n} \sum_{g=1}^{G} w_{ij} f_{ij} & \leq 1 \quad j = 1, \ldots, n \\
u_j^k, w_{ij}, h_d, v_j^k & \geq 0 \quad \forall r, g, d, i
\end{align*}
\]

The follower (supplier 2) will maximize its efficiency such that efficiency of supplier 1 remains at \( e^1 \). Hence, model (4) can be applied to measure the efficiency of follower.

![Figure 2. Structure of supplier base j](image)
max \( e^*_o = \sum_{i \in I} \frac{h_i^o z_i^o}{w_{i,j}^f f_{j,0}} + \sum_{j \in J} h_j^o z_j^o \)

subject to:
\[
\begin{align*}
\sum_{i \in I} h_i^o z_i^o + \sum_{j \in J} h_j^o z_j^o & \leq 1 & j = 1, \ldots, n \\
\sum_{i \in I} w_{i,j}^f f_{j,0} + \sum_{i \in I} v_i^o x_i^o & \leq 1 & j = 1, \ldots, n \\
\sum_{i \in I} u_i^o w_{i,j}^f f_{j,0} + \sum_{i \in I} u_i^o v_i^o x_i^o & = e^*_o \\

u_i^o, w_{i,j}^f, h_j^o, v_i^o, u_i^o \geq 0 & \forall r, g, d, i
\end{align*}
\]  

Model (4) is a nonlinear model and can be converted to the following linear programming model:

max \( e^*_o = \sum_{i \in I} u_i^o r_i^o + \sum_{j \in J} b_j^o z_j^o \)

subject to:
\[
\begin{align*}
\sum_{i \in I} u_i^o r_i^o + \sum_{j \in J} b_j^o z_j^o - \sum_{i \in I} z_i^o f_{j,0} - \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq 1 & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq 1 & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} - \sum_{i \in I} x_i^o f_{j,0} - \sum_{j \in J} y_j^o z_j^o & = 0 \\
u_i^o, w_{i,j}^f, h_j^o, v_i^o, u_i^o \geq 0 & \forall r, g, d, i
\end{align*}
\]  

The efficiency of the whole supplier base will be: \( e^**_o = e^*o^* + e^**_o^* \). In situation that supplier 2 is leader and supplier 1 is follower, the modeling process will be the same.

2. 2. Cooperative Model  According to CCR model, the following cooperative model can be established:

max \( e^*o^* = \sum_{i \in I} u_i^o r_i^o + \sum_{j \in J} b_j^o z_j^o \)

subject to:
\[
\begin{align*}
\sum_{i \in I} u_i^o r_i^o + \sum_{j \in J} b_j^o z_j^o - \sum_{i \in I} z_i^o f_{j,0} - \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq 1 & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq 1 & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} - \sum_{i \in I} x_i^o f_{j,0} - \sum_{j \in J} y_j^o z_j^o & = 0 \\
u_i^o, w_{i,j}^f, h_j^o, v_i^o, u_i^o \geq 0 & \forall r, g, d, i
\end{align*}
\]  

Model (6) is a non-linear programming model and cannot be converted into linear form but a parametric linear programming approach can be applied to solve this model (see Appendix A). In next section an efficient heuristic method is proposed which in many cases can obtain the global solution.

3. SOLUTION METHOD

The non-cooperative model is a linear programming model and can be solved globally. But the cooperative model is in nonlinear form and optimal solution is not guaranteed. A parametric linear programming approach can be applied to obtain global solution of this model. As the parametric linear programming approach is very time consuming, in this paper a heuristic method is developed to solve model (6). This approach is an efficient method.

\textbf{Step 1}. Calculate maximum achievable efficiency of suppliers \((e^1o^*, e^{2+}o^*)\) via following linear models:

max \( e^1o^* = \sum_{i \in I} u_i^1 r_i^o + \sum_{j \in J} b_j^o z_j^o \)

subject to:
\[
\begin{align*}
\sum_{i \in I} u_i^1 r_i^o + \sum_{j \in J} b_j^o z_j^o - \sum_{i \in I} z_i^o f_{j,0} - \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq e^1o^* & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq 1 & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} - \sum_{i \in I} x_i^o f_{j,0} - \sum_{j \in J} y_j^o z_j^o & = 0 \\
u_i^1, w_{i,j}^f, h_j^o, v_i^o, u_i^1 \geq 0 & \forall r, g, d, i
\end{align*}
\]  

max \( e^{2+}o^* = \sum_{i \in I} u_i^{2+} r_i^o + \sum_{j \in J} b_j^o z_j^o \)

subject to:
\[
\begin{align*}
\sum_{i \in I} u_i^{2+} r_i^o + \sum_{j \in J} b_j^o z_j^o - \sum_{i \in I} z_i^o f_{j,0} - \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq \min(e^1o^*, e^{2+}o^*) & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq 1 & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} - \sum_{i \in I} x_i^o f_{j,0} - \sum_{j \in J} y_j^o z_j^o & = 0 \\
u_i^{2+}, w_{i,j}^f, h_j^o, v_i^o, u_i^{2+} \geq 0 & \forall r, g, d, i
\end{align*}
\]  

\textbf{Step 2}. Solve models (9) and (10).

max \( e^o^* = e^*o^* + \sum_{i \in I} u_i^o r_i^o + \sum_{j \in J} b_j^o z_j^o \)

subject to:
\[
\begin{align*}
\sum_{i \in I} u_i^o r_i^o + \sum_{j \in J} b_j^o z_j^o - \sum_{i \in I} z_i^o f_{j,0} - \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq 1 & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq 1 & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} - \sum_{i \in I} x_i^o f_{j,0} - \sum_{j \in J} y_j^o z_j^o & = 0 \\
u_i^o, w_{i,j}^f, h_j^o, v_i^o, u_i^o \geq 0 & \forall r, g, d, i
\end{align*}
\]  

max \( e^o^* = e^*o^* + \sum_{i \in I} u_i^o r_i^o + \sum_{j \in J} b_j^o z_j^o \)

subject to:
\[
\begin{align*}
\sum_{i \in I} u_i^o r_i^o + \sum_{j \in J} b_j^o z_j^o - \sum_{i \in I} z_i^o f_{j,0} - \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq \min(e^1o^*, e^{2+}o^*) & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} & \leq 1 & j = 1, \ldots, n \\
\sum_{i \in I} z_i^o f_{j,0} + \sum_{j \in J} w_{j,i}^f f_{j,0} - \sum_{i \in I} x_i^o f_{j,0} - \sum_{j \in J} y_j^o z_j^o & = 0 \\
u_i^o, w_{i,j}^f, h_j^o, v_i^o, u_i^o \geq 0 & \forall r, g, d, i
\end{align*}
\]  

\textbf{Step 3}. \( e^o^* = \max\{e^1o^*, e^{2+}o^*\} \)

4. EFFICIENCY ANALYSIS

In this section, the relationship between efficiency scores in different mechanisms and efficiency scores of tiers are investigated.
4. Efficiency In Different Mechanisms

**Theorem 1.** The efficiency of supplier base under different mechanisms can be expressed as:

\[ e^{NC} \leq e^C \]

**Proof.** Suppose \( \lambda_1 = \{ u_1^*, w_1^*, h_1^*, v_1^*, u_2^*, v_2^* \} \) is an optimal solution to model (4). Accordingly, the efficiency of the system is \( e^{NC} = e_2^* \cdot e_1^* \). Note that \( \lambda_1 \) is also a feasible solution to model (6). Thus we have \( e^{NC} \leq e^C \). Similarly, it can be proven that when supplier 2 is leader, optimal efficiency of non-cooperative model is smaller or equal to cooperative mechanism. However, the relationship between CCR model and game models depends on parameters.

4.2. Tiers Efficiency

**Theorem 2.** \( e_1^{S1} \geq e_2^{S1} \) and \( e_2^{S2} \geq e_1^{S2} \). Where \( e_1^{S1} \) and \( e_2^{S1} \) are the optimal efficiency of supplier 1 when supplier 1 is leader and when supplier 2 is leader, respectively. And \( e_2^{S2} \) and \( e_1^{S2} \) are the optimal efficiency of supplier 2 when supplier 2 is leader and when supplier 1 is leader, respectively.

**Proof.** Let \( \lambda_2 = \{ u_1^{S1}, w_1^{S1}, h_1^{S1}, v_1^{S1}, u_2^{S2}, v_2^{S2} \} \) be the optimal efficiency of supplier 1, when supplier 2 is leader. This solution is feasible for model (3). Denote that model (3) calculates the optimal efficiency of supplier 1, when its leader. Hence \( e_1^{S1} \geq e_2^{S1} \). In a similar manner we have \( e_2^{S2} \geq e_1^{S2} \).

5. ILLUSTRATIVE EXAMPLE

An automobile manufacturer wishes to choose its supplier base. 20 two tier supplier bases with same structure as depicted in Figure 3 are considered. Supplier of tier 2 produces raw materials from mineral stones in standard plate forms. Numbers of employees and operation costs are inputs of tier 2 supplier. This supplier sales steel and aluminum plates in standard form to supplier of tier 1 and resin and graphite to other customers. Supplier of tier 1 is an automobile parts manufacturer that uses steel and aluminum plates to produce automobile parts. Number of employees is another input of supplier 1. During manufacturing process, some scraps are produced. Supplier 1 flows back these scraps to supplier 2 for remanufacturing. Data related to numerical example are listed in Table 1.

In Table 2 and Figure 4, the efficiencies of 20 supplier bases under cooperative and non-cooperative mechanisms are compared and results of CCR model are illustrated. In Table 2, \( e_1^{S1} \) and \( e_2^{S2} \) are optimal efficiencies of supplier 1 and supplier 2 without considering any relationship with other supplier. As shown in Figure 4 for all supplier bases, efficiency under non-cooperative mechanism is smaller or equal to efficiency score in cooperative approach (Theorem 1).

The cooperative method can be solved globally via parametric linear programming method which is an iterative method and takes a long time (especially for large number of DMUs). Table 3 shows that in this example for 85% of DMUs the heuristic method obtains the global solution in 6.756 seconds while achieving optimal solutions taking 178.837 seconds in parametric linear programming method.

![Figure 3 Structure of supplier bases of numerical example](image-url)
TABLE 1. Input and output data in example

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<th>Inputs of supplier 2</th>
<th>Output of supplier 2</th>
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TABLE 2. Supply base efficiency under different relationship mechanisms

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<th>Supplier bases</th>
<th>(e^{NC}) Supplier 1 as leader</th>
<th>(e^{NC}) Supplier 2 as leader</th>
<th>(e^C)</th>
<th>(e^{CCR})</th>
<th>(e^{S_1})</th>
<th>(e^{S_2})</th>
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<td>1.0000</td>
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</tbody>
</table>

To investigate the validity of proposed heuristic method, 25 cases are randomly generated. Input and output measures are random numbers distributed uniformly between 1 and 10000.
Table 3: Results of proposed heuristic method and the parametric linear programming approach

<table>
<thead>
<tr>
<th>Supplier base</th>
<th>Proposed heuristic method</th>
<th>Parametric linear programming method</th>
</tr>
</thead>
<tbody>
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<td>1</td>
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<td>0.0153</td>
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<tr>
<td>2</td>
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<td>0.2232</td>
</tr>
<tr>
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<td>0.1304</td>
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<tr>
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<td>0.0818</td>
<td>0.0818</td>
</tr>
<tr>
<td>5</td>
<td>0.1338</td>
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</tr>
<tr>
<td>6</td>
<td>0.2069</td>
<td>0.2069</td>
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<tr>
<td>7</td>
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<tr>
<td>20</td>
<td>0.4838</td>
<td>0.4838</td>
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</tbody>
</table>

Number of DMUs in each case is a random number between 10 and 30 and number of inputs and outputs for each supplier is also a random number between 1 and 4. For each case, results of heuristic method and parametric linear programming ($\Delta \varepsilon = 0.001$) are compared. Results are shown in Table 4.

Table 4 shows that for different DMUs at least for about 85% of DMUs, the heuristic method reaches the optimal solution in a very short time.

6. CONCLUSIONS

Network data envelopment analysis can be applied by firms to evaluate the efficiency of potential supplier bases and choose the most efficient one to reduce their supply costs. The current paper develops network data envelopment analysis models to investigate efficiency of a generalized form of two tier supplier bases. Different relationship mechanisms are considered in this paper: cooperation and leader-follower game. As under cooperation mechanism the resulted model is nonlinear, a novel heuristic method is proposed that can be used instead of existing methods. It is shown that the proposed method can obtain the optimal solution for more than 85% of supplier bases.

The proposed models can be extended for other relationship mechanisms such as bargaining. Another potential extension of this paper is evaluating efficiency of supplier bases when input and output data are...
uncertain. Supplier bases with undesirable outputs provide another opportunity for extending this study.

7. REFERENCES


8. APPENDIX A: PARAMETRIC LINEAR PROGRAMMING APPROACH

Model (6) is a nonlinear model and can't be solved globally but parametric linear programming approach can be applied to solve this model. To achieve this aim, efficiency of one of suppliers can be considered as a parameter and efficiency of another supplier will be variable. Consider the following model:

\[ \text{max } e^*_i = e^*_i \sum w_j f_{ij} + \sum h_j z_{ij} \]

subject to:

\[ \sum w_j f_{ij} + \sum h_j z_{ij} + \sum x_j i_{ij} \leq 0 \quad j = L \ldots n \]

\[ \sum w_j f_{ij} + \sum h_j z_{ij} + \sum x_j i_{ij} \geq 0 \quad j = L \ldots n \]

\[ n_i, w_j, x_{ij}, e^*_i, z_{ij} \geq 0 \quad \forall r, g, d, i \]

In model (11), efficiency of supplier 1 \( e^*_1 \) is a parameter. The maximum efficiency of supplier 1 can be obtained by solving model (7). Let \( e^*_n = e^*_n - k\Delta \varepsilon \). \( \Delta \varepsilon \) is step size. To obtain more precise results, a smaller step size should be selected. \( k \) is an integer number between 0 and \( e^*_n / \Delta \varepsilon \). To solve model (11), in first iteration \( k = 0 \). Now given \( e^*_1 \), model (11) can be solved optimally as a linear programming model. In each iteration we increase \( k \) and obtain \( e^*_n(k) \). Finally \( e^*_n = \max_k e^*_n(k) \). If we consider efficiency of supplier 1 as variable and efficiency of supplier 2 as parameter, same result will be obtained.
Two-tier Supplier Base Efficiency Evaluation Via Network DEA: A Game Theory Approach

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