Numerical Study of Natural Convection Heat Transfer in a Horizontal Wavy Absorber Solar Collector Based on the Second Law Analysis

B. M. Ziapour*, F. Rahimi

Department of Mechanical Engineering, University of Mohaghegh Ardabili, Ardabil, Iran

**ABSTRACT**

Literature about entropy generation analysis of a wavy enclosure is surprisingly scarce. In this paper, a FORTRAN code using an explicit finite-volume method is provided for estimating the entropy production due to the natural convection heat transfer in a cosine wavy absorber solar collector. The volumetric entropy generation terms -both the heat transfer term and the friction term, were straightly calculated. The solution was conducted assuming the isothermal boundary conditions of the absorber and the cover of solar collector. The results were obtained for Rayleigh numbers from $10^2$ to $10^5$. The simulation results were compared with a flat plate absorber. It was found that, with increasing the cosine wave amplitude, the collector enclosure irreversibility decreases.

**NOMENCLATURE**

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
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<tbody>
<tr>
<td>$A_r$</td>
<td>wave amplitude ratio</td>
</tr>
<tr>
<td>$A_s$</td>
<td>enclosure aspect ratio ($A_s = L/H$)</td>
</tr>
<tr>
<td>$B_e$</td>
<td>dimensionless Bejan number</td>
</tr>
<tr>
<td>$\overline{H}$</td>
<td>dimensionless mean value of the enclosure height</td>
</tr>
<tr>
<td>$k$</td>
<td>thermal conductivity ($W/mK$)</td>
</tr>
<tr>
<td>$L$</td>
<td>dimensionless absorber length</td>
</tr>
<tr>
<td>$n$</td>
<td>wave numbers</td>
</tr>
<tr>
<td>$P$</td>
<td>dimensionless static pressure</td>
</tr>
<tr>
<td>$P_r$</td>
<td>Prandtl number</td>
</tr>
<tr>
<td>$R_a$</td>
<td>Rayleigh number</td>
</tr>
<tr>
<td>$\dot{S}$</td>
<td>entropy generation ($W/m^3K$)</td>
</tr>
<tr>
<td>$t$</td>
<td>Time (s)</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature (K)</td>
</tr>
<tr>
<td>$U,V$</td>
<td>components of dimensionless velocity in $X,Y$ direction</td>
</tr>
<tr>
<td>$X,Y$</td>
<td>dimensionless Cartesian coordinates</td>
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**Greek Symbols**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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<tbody>
<tr>
<td>$\alpha$</td>
<td>thermal diffusivity ($m^2/s$)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>artificial compressibility parameter</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>dimensionless time</td>
</tr>
<tr>
<td>$\phi$</td>
<td>ratio between the viscous and the thermal irreversibility</td>
</tr>
<tr>
<td>$\theta$</td>
<td>dimensionless temperature ($\theta = (T - T_c)/(T_h - T_c)$)</td>
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**Subscripts**

<table>
<thead>
<tr>
<th>Subscript</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>$a$</td>
<td>dimensionless</td>
</tr>
<tr>
<td>$av$</td>
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</tr>
<tr>
<td>$c$</td>
<td>cold</td>
</tr>
<tr>
<td>$h$</td>
<td>heat transfer effect, hot</td>
</tr>
<tr>
<td>$f$</td>
<td>fluid viscous effect</td>
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<tr>
<td>$l$</td>
<td>Local</td>
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<tr>
<td>$T$</td>
<td>total</td>
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* Corresponding Author’s Email: bmziapour@gmail.com (B.M. Ziapour)
1. INTRODUCTION

Flat-plate solar collectors have simpler structure than concentrating types. They are planned to gain the solar heat at moderate temperatures below 150°C [1-4]. Although, the conduction heat loss occurs from the insulated box of a solar collector, the main heat losses take place due to the natural convection heat transfer inside the shallow enclosure of the solar collector (because of the temperature difference between the glazing cover and the absorber surface). The non-planar absorber surface such as wavy or corrugated shape may improve the rate of the absorbed solar heat. The natural convection heat transfer inside the wavy walled enclosures has been investigated by some investigators. Zemani et al. [5] analyzed natural convection of air in a cubic enclosure. The cubic vertical hot wall had a wavy geometry with partitions introduced at the ridge. The opposite cold wall was straight. The other surfaces were thermally insulated. Their results showed that the wavy wall partitions has smaller mean Nusselt number than the undulated wall without partitions. Ozotop et al. [6] studied numerically the influences of the volumetric heat sources on the natural convection flow structures for a wavy walled enclosure. The bottom and the top wavy walls enclosure were adiabatic; and the two parallel and vertical walls were assumed to be heated differentially. Their results showed that the undulation function of bottom and top walls has resulted the heat transfer to decrease in the case of (internal Rayleigh number/ external Rayleigh number)>1 and the case of (internal Rayleigh number/ external Rayleigh number) <1.

In some different works of Varol et al. [7-9], the effects of wavy walls enclosures on natural convection heat transfer were studied, especially for wavy absorber solar collector. One of their results showed that the heat transfer rate grows for wavy enclosure as compared to the flat enclosure. Also, in a tilted wavy solar collector, heat transfer rate increases due to increasing of Rayleigh number and aspect ratio, and decreases with increase of the wavelength. In horizontal and shallow wavy cavities (in the case of sinusoidal function for bottom wall and the flat surface for top wall) [9], heat transfer rate increases with decreasing the dimensionless wave length and also increases with increasing aspect ratio and Rayleigh number.

The numerical solution of the natural convection heat transfer in an enclosure bounded by horizontal bottom and the top wavy walls and other flat vertical walls was conducted by Subha [10]. The numerical approximation was implicit finite difference. The results were obtained for the fluid with Prandtl number 0.71, Rayleigh number between 10^2 and 10^6 and aspect ratio between 0.35 and 0.75. The results showed that the heat transfer increases by increasing the Rayleigh number. Also, in comparison with the rectangular and the bottom wavy enclosures, the local Nusselt number had the highest value for top and bottom wavy enclosure. The effects of the vertical wavy walls on the natural convection heat transfer inside the two dimensional enclosure was numerically studied by Tahavvor et al. [11]. The vertical wavy walls and the top wall were kept isothermal and bottom wall temperature was higher and had sinusoidal temperature distribution. Several numbers of undulations were tested. One of the results showed that with increasing the waviness, the hot region increases and covers other space of the cavity. Also, for low amplitude case, the surface waviness does not affect the Nusselt number.

In all fluid flow and heat transfer processes, the entropy is produced because of irreversibility and it is the reason that leads to low efficiency [12]. Minimization of the entropy generation of thermal systems has the useful role in energy conversion by those systems [13]. In an enclosure consisting of the non-wavy walls, there are many works to improve its heat conversion based on the entropy generation minimization [14-18]. But, the literature about wavy walls entropy generation is surprisingly scarce [19]. Natural convection heat transfer and entropy generation within an inclined wavy enclosure were numerically investigated by Mahmud et al. [20]. The solution method was an implicit finite-volume approach. Two tilted parallel and isothermal walls (with angle θ) were wavy with the specific sine function. Two other parallel and adiabatic walls were flat in shape. Results showed that for a special aspect ratio and angular position, average Nusselt number (\( \text{Nu}_{av} \)) progressively decreases with surface undulation (\( \lambda \)), except for the higher values of \( \lambda \) after which \( \text{Nu}_{av} \) starts to increase. Volume averaged entropy generation reveals the similar history θ as average Nusselt number. Bhardwaj et al. [21] numerically analyzed the effects of undulation isothermal left wall and heated sinusoidally bottom wall on the natural convection heat transfer and the entropy generation within the porous right angled triangular cavity. The right inclined wall was adiabatic. The results showed that for low value Darcy number , conduction heat transfer is dominated. Also, for higher values of Darcy number, convection heat transfer is enhanced. The results revealed that the share of fluid friction entropy generation was higher for waveness wall than the no-wavy wall, while the share of heat transfer entropy generation is closely identical for both cases.

As mentioned above discussions, there is not any works for the horizontal wavy absorber solar collector enclosure based on the second law analysis. In this paper, an explicit cell-centered finite-volume method is extended for solving the governing equations on the
natural convection heat transfer in a horizontal wavy absorber solar collector, based on the second law analysis. The artificial compressibility scheme is applied in order to couple the momentum equations to the energy equation. Since the total entropy generation components are revealed as the first order derivative terms of the fluid flow variables, then in this numerical method may obtain them directly.

2. MATHEMATICAL MODELING

The cosine wavy absorber enclosure has been shown schematically in Figure 1. As shown, the enclosure has two-dimensional geometry with a horizontal cosine wavy wall (i.e. the solar collector absorber) at hot uniform temperature \( T_h \), and a horizontal straight wall (i.e. the solar collector glazing cover) at cold uniform temperature \( T_c \). The enclosure vertical walls are adiabatic. The wavy wall curve has been obtained and drawn from the following equation as:

\[
Y = A_r \left( 1 + \cos \left( \frac{2\pi x}{A_v} \right) \right)
\]  

(1)

It is assumed that the air flow \( \text{Pr} = 0.71 \) inside the enclosure be as: steady, laminar, incompressible and viscous. Also, the fluid no-slip is considered on the enclosure walls (i.e. \( U = V = 0 \)).

The dimensionless governing equations with artificial compressibility may be written as [16]:

\[
\frac{1}{\beta} \frac{\partial P}{\partial \zeta} + \frac{\partial U}{\partial \zeta} \cdot \frac{\partial V}{\partial \eta} = 0
\]  

(2)

\[
\frac{\partial U}{\partial \zeta} \cdot \frac{\partial (U^2)}{\partial \zeta} + \frac{\partial (U V)}{\partial \zeta} = - \frac{\partial P}{\partial \zeta} + \frac{\partial \left( P \frac{\partial U}{\partial \eta} \right)}{\partial \eta} + \frac{\partial \left( P \frac{\partial V}{\partial \eta} \right)}{\partial \eta}
\]  

(3)

\[
\frac{\partial \theta}{\partial \zeta} + \frac{\partial (U \theta)}{\partial \zeta} = \frac{\partial \left( \theta \frac{\partial U}{\partial \eta} \right)}{\partial \eta} + \frac{\partial \left( \theta \frac{\partial V}{\partial \eta} \right)}{\partial \eta}
\]

(4)

\[
\frac{\partial \theta}{\partial \eta} + \frac{\partial (U \theta)}{\partial \eta} = \frac{\partial \left( \theta \frac{\partial U}{\partial \eta} \right)}{\partial \eta} + \frac{\partial \left( \theta \frac{\partial V}{\partial \eta} \right)}{\partial \eta}
\]

(5)

The local Nusselt number is obtained as:

\[ N_u = -(d\theta/dN)_\text{wave} \]. Then, the average Nusselt number is calculated as:

\[ N_u = \frac{1}{L_u} \int_0^{L_u} N_u dl \]

Dimensionless volumetric entropy generation in flow is the sum of the heat transfer and the fluid friction terms as:

\[ \dot{S}_v = \dot{S}_{v,h} + \dot{S}_{v,f} \]. Then, for them, we have the following equation as:

\[ \dot{S}_{v,h} = \left[ \left( \frac{\partial \theta}{\partial \zeta} \right)^2 + \left( \frac{\partial \theta}{\partial \eta} \right)^2 \right] \]

\[ \dot{S}_{v,f} = \left[ 2 \left( \frac{\partial U}{\partial \zeta} \right)^2 + 2 \left( \frac{\partial V}{\partial \zeta} \right)^2 + \left( \frac{\partial U}{\partial \eta} \right)^2 + \left( \frac{\partial V}{\partial \eta} \right)^2 \right] \]

(6)

(7)

where \( \phi \) is the ratio between the viscous and the thermal irreversibilities [16]. The dimensionless Bejan number is defined as:

\[ Be = \dot{S}_{v,h}/\dot{S}_v \]. Also, total dimensionless entropy generation is obtained as:

\[ \dot{S}_v = \int \dot{S}_{v,h} dV \].

3. RESULTS AND DISCUSSIONS

To discretize the governing equations (1)-(5), an explicit cell-centered finite-volume method was extended using a FORTRAN code. To find the steady state solutions, an explicit forth-order Runge-Kutta scheme was used. In this explicit method, the initial values of both fluid flow variables (such as \( U, \theta \) and etc.) and their first derivatives (such as \( \partial U/\partial \zeta, \partial \theta/\partial \zeta \) and etc.) were estimated. Then, the guessed values were corrected through an iteration process. Therefore, in this useful method, all terms of volumetric entropy generation, both the heat transfer and the friction terms, were straightly calculated. To see more details of this method please refer to references [16]. To test a grid free solution, several grid sizes, from 50×50 to 150×150 were tested. Then, the difference between maximum and minimum value of the dimensionless Bejan number was obtained about 0.02%. Finally, 80×100 grid points were selected on the base of less computation time and non-compromising the grid reliance. Also, the convergence criterion was used for the mass conservation residue as 10⁻⁸. The trial and error was
used to find the optimal value of the artificial compressibility parameter \((\beta)\), for different Rayleigh numbers. For example, Figure 2 shows the history of the mass conservation residues (Res) for different \(\beta\) values and Rayleigh number of 100. As shown, the best convergence speed has resulted for \(\beta\) as 100.

In the case of the flat absorber geometry (i.e. \(As = 1\) and \(Ar = 0\)), the results of the present model for average Nusselt number \((Nu_{av})\), was compared with works of Mahmud and Sadrul Islam [22], Famouri and Hooman [23] and Abdelkader et al. [24], as shown in Table 1. It is seen that the results of the present model are as good as expected.

Comparisons of isotherms between the wavy walled absorber \((Ar = 0.15)\) and the planar wall absorber are shown in Figures 3-6, for different Rayleigh numbers as \(Ra = 3800\) (Figures 3a, 4a and 5) and \(Ra = 10^5\) (Figures 3b, 4b and 6). These isotherms were sketched in different cases of cavity aspect ratio \((As)\) as: one (Figure 3), two (Figure 4) and four (Figures 5, 6). As shown, with relation to the flat plate absorber, in small Rayleigh number (i.e. \(Ra = 3800\)), isotherm lines changes slowly. However, in high Rayleigh numbers (i.e. \(Ra = 10^5\)), especially with increasing of aspect ratio, the isotherms have the curved shape. It is true; since, in the case of the high Rayleigh number, the circulation form of the air natural convection heat transfer is dominated.

Also, in the flat absorber with increasing aspect ratio, the numbers of main cells are obtained. These cells are produced due to the same numbers of circulations of the fluid flow. As shown from isotherms for the wavy absorber, it is seen that the wave geometry affects the formation of isotherms. This effectiveness has been seen in both high Rayleigh number and aspect ratios. One can see that the numbers of main and identical cells are obtained due to increase in the aspect ratio (or the wave number).

In Figure 7, air velocities fields have been shown for both flat (Figure 7a and 7c) and wavy (Figure 7b and Figure 7d) absorbers. These solutions have been obtained in Rayleigh number as \(10^5\). Also, the wave amplitude ratio was selected as \(Ar = 0.15\). As shown, when the enclosure aspect ratio is one, only one clockwise circulation is produced. But for the enclosure aspect ratio as two, then two approximately similar circulations (with opposite spin) are formed due to the free convection heat transfer mechanism.

**Figure 2.** History of the mass conservation residues (Res), for different artificial compressibility parameter \((\beta)\).

**Figure 3.** Isotherms comparisons between wavy and flat absorber collectors, in the cases as \(As = 1\) and \(Ar = 0.15\), for two different Rayleigh numbers: (a) \(Ra = 3800\), and (b) \(Ra = 10^5\).

**Figure 4.** Isotherms comparisons between wavy and flat absorber collectors, in the cases as \(As = 2\) and \(Ar = 0.15\), for two different Rayleigh numbers: (a) \(Ra = 3800\), and (b) \(Ra = 10^5\).
Comparisons show that the Nusselt peak values occur in the cases of fluid flow raising or fluid flow falling from the absorber surface. The variation of $N u_{ave}$ vs. $R a$, is shown in Figure 9, for (a) flat absorber with different absorber length, (b) flat absorber with different absorber height, (c) wavy absorber with different absorber length and $A r = 0.15$, and (d) wavy absorber with different wave amplitudes ratios and $A s = 2$. In all figures, it is seen that the increasing of the Rayleigh number is resulted to increase the average Nusselt number. As shown from Figure 9a (flat absorber) and Figure 9c (wavy absorber) one can see that the increase of the absorber length is resulted to increase the average Nusselt number. Also, the increase of the absorber height increases the average Nusselt number (Figure 9b). Therefore, the shallow solar collector (i.e. smaller enclosure height) has less heat losses. The effects of the wave amplitude on the free convection heat transfer are shown in Figure 9d. From this figure, it is seen that the increase of the wave amplitude, is resulted to decreasing of the $N u_{ave}$.

It is found that the quality variation of the average Nusselt number ($N u_{ave}$) is similar to the quality variation of the dimensionless entropy generation ($S_{TS}$), as shown in Figure 10. In all figures, it is seen that the increasing of the Rayleigh number has been resulted to increase the total entropy generation. In both flat and wavy absorbers, increase of the length or the height enclosure, is resulted to increase the value of $S_{TS}$. Also, the increase of the wave amplitude ratio, is resulted to decrease the $S_{TS}$ especially in the high wave amplitude ratio (see Figure 9d for $A r = 0.35$). Therefore, the absorber including high wave amplitude produces low irreversibility. The variation of $B e$ vs. $R a$ is shown in Figure 11, for (a) flat absorber with different absorber lengths, (b) flat absorber with different absorber heights, (c) wavy absorber with different absorber lengths and $A r = 0.15$, and (d) wavy absorber with different wave amplitudes ratios and $A s = 2$. As mentioned in the previous section, Bejan number is defined as: $B e = S_{TS}/S_{TS}$. It shows the share of the entropy generation due to heat transfer within a system. One can see that in all figures, the increasing of Rayleigh number is resulted to decrease the Bejan number. It must be true, since in the high Rayleigh number, the relative speed between the fluid flow layers is high, and that is resulted to increase the friction entropy generation. Also, as shown in Figure 11a and Figure 11b, the effect of increase of the absorber length on $B e$ is not sensible. But, it is observed that the enclosure with small height has the small Bejan number (Figure 11b). Also, the increase of the wave amplitude ratio is resulted to decrease the $B e$ (Figure 11d).
Figure 8. The variation of $N_u$ vs. $Ra$, for (a) flat absorber with $As = 1$, (b) flat absorber with $As = 2$, (c) wavy absorber with $As = 1$ and $Ar = 0.15$, and (d) wavy absorber with $As = 2$ and $Ar = 0.15$

Figure 9. The variation of $N_u$ vs. $Ra$, for (a) flat absorber with different absorber length, (b) flat absorber with different absorber height, (c) wavy absorber with different absorber length and $Ar = 0.15$, and (d) wavy absorber with different wave amplitudes ratios and $As = 2$
Figure 10. The variation of $\tilde{S}_{Ta}$ vs. $Ra$, for (a) flat absorber with different absorber length, (b) flat absorber with different absorber height, (c) wavy absorber with different absorber length and $Ar = 0.15$, and (d) wavy absorber with different wave amplitudes ratios and $As = 2$.

Figure 11. The variation of $Be$ vs. $Ra$, for (a) flat absorber with different absorber length, (b) flat absorber with different absorber height, (c) wavy absorber with different absorber length and $Ar = 0.15$, and (d) wavy absorber with different wave amplitudes ratios and $As = 2$. 
4. CONCLUDING REMARKS

The waviness effects of the absorber in a horizontally solar collector enclosure on the natural convection heat transfer and the entropy generation were numerically studied and compared with a flat absorber using an explicit cell-centered finite-volume method. Some important results are as follows:

1. In all conditions, the increasing of the Rayleigh number is resulted to increase the local Nusselt number.
2. The increase of both absorber length and height is resulted to increase the average Nusselt number.
3. The increase of the wave amplitude ratio, is resulted to decrease the $Nu_{ave}$.
4. The quality variation of the average Nusselt number is similar to the quality variation of the dimensionless entropy generation.
5. The absorber including high wave amplitude produces the low irreversibility.
6. In all the conditions, the increasing of the Rayleigh number is resulted to decrease the Bejan number.

5. REFERENCES

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B. M. Ziapour, F. Rahimi

Department of Mechanical Engineering, University of Mohaghegh Ardabili, Ardabil, Iran

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