A Two Stage Stochastic Programming Model of the Price Decision Problem in the Dual-channel Closed-loop Supply Chain

A. Ahmadi Yazdi, M. Honarvar*

Department of Industrial Engineering, Yazd University, Yazd, Iran

PAPER INFO

ABSTRACT

In this paper, we propose a new model for designing integrated forward/reverse logistics based on pricing policy in direct and indirect sales channel. The proposed model includes producers, disposal center, distributors and final customers. We assumed that the location of final customers is fixed. First, a deterministic mixed integer linear programming model is developed for integrated logistics network design. Then, the stochastic counterpart of the proposed mixed integer linear programming model is developed by using scenario-based stochastic approach. We use the value of the stochastic solution (VSS) as a measure to evaluate the accuracy of stochastic programming approach. VSS value showed that using stochastic approach for solving the proposed model is sufficient. Moreover, we could obtain optimal values of sale prices in direct and indirect sale channel and service level by considering forward and reverse flow together.


1. INTRODUCTION

Logistic network design problem that takes into account the facility locations and the shipment of product flows have been an area of increasing attention during the last decade in both practice and academia [1]. While the traditional supply chain network design, namely forward logistic, considers the direct flow from producer to the customer in logistic networks, Nowadays, the emphasis on environmental, economic, legislative reasons and potentials of value recovery from the used products have caused many industries to focus on reverse logistic and recovery activities [2-8]. Reverse logistics (RL) is the process of planning, implementing, and controlling the cost effective flow of raw materials, in-process inventory, finished goods, and related information from the point of consumption to the point of origin in order to reuse of products and raw materials or dispose them [9]. The literature of represented models concerning the logistics network design problem dividing into forward logistics network, reverse logistics network, and finally integrated forward/reverse logistics network [1]. A comprehensive review of reverse and integrated logistics can be found in (Fleischmann [2]; Pokharel and Mutha [6]; Govindan [8]). In this paper, we survey specific network design problems for reverse and integrated logistics network design problems. Some authors like Üster et al. [10], Amiri [11], Patia et al. [12], and Aras et al. [13] are those ones, who carried out investigations about the integrated logistics. Keyvanshokooh [1] presented a mixed-integer linear programming to consider dynamic pricing approach for used products, forward/reverse logistics network configuration and inventory. Kamali et al. [14] proposed a single product, multi-echelon, multi-period closed loop supply chain for high-tech products (which have continuous price decrease). Four heuristics-based methods including genetic algorithm (GA), particle swarm optimization (PSO), differential evolution (DE), and artificial bee colony (ABC) are proposed for solving their model. Sahraeian et al. [15] introduced a supply chain network design problem which contains environmental concerns in arcs and nodes of network and there are some routes such as road, rail and etc. in each pair of nodes. Demand uncertainty and uncertainty...

*Corresponding Author’s Email: Mhonarvar@yazd.ac.ir (M. Honarvar)
in the quality of returned products is an important factor to be considered in designing supply chain networks. Some researchers consider the uncertainty in reverse logistics. Pishvaae et al. [5] proposed an integrated logistic model. They presented a single-period and single-product Mixed Integer Linear Programming (MILP) for integration of their logistic model. Then, they developed the model using scenario-based stochastic approach. Their model was aimed at minimizing of total costs. Pishvaae et al. [16] proposed a robust optimization model to design a closed-loop supply chain network. With presentation of single-product and single-period model, Pishvaae and Torabi [17] proposed a bi-objective possibilistic mixed integer-programming model. They considered two groups of customers: the recycled material customer and the product customers. To solve the presented possibilistic optimization model, an interactive fuzzy solution approach is used. Pishvaae et al. [18] suggests a dual-objective credibility-based fuzzy mathematical programming model to design a green logistic network under uncertainty conditions. Their model is aimed at minimizing of environmental effects and total cost for creating network synchronously. Vahdani et al. [19] proposed a bi-objective model to design a reliable network of hybrid reliable facilities in logistic network under uncertainty conditions. They solved the model by combining queuing theory, fuzzy possibilistic programming and fuzzy multi-objective programming. Optimal decisions on price play a critical role in revenue management of supply chain.

Cattani et al. [20] and Tsay and Agrawal [21] presented comprehensive reviews on multi-channel models. Balasubramanian [22] considered competition in the multiple-channel environment from a strategic viewpoint. Tsay and Agrawal [23] considered the channel conflicts in dual channel supply chain. Chiang et al. [24] studied a price-setting game between a manufacturer and its independent retailer in a dual channel based on the consumer choice model. Mirzahosseiniyan et al. [25] presented a dual channel inventory model based on queuing theory in a manufacturer-retailer supply chain, consisting of a traditional retail. They used simulated annealing to find a solution for inventory level in each echelon. The service level is rate of increase needed for the distributor to exert in order to raise the profit, demand, and customer’s satisfaction. Defining the service level depends on type of production process and method of sale. Service level has significant effects on demand, profit, and pricing strategy. Ahmadavanda et al. [26] investigates the impact of provided service by the retailers and manufacturers on customers’ demand and members’ profit in a supply chain. In addition, in the most studied related to multi sales channel, the prices in each channel are determined without considering the production and transportation constraints. The main advantage of our model could be summarized as follows: 1) Designing a dual sales channel supply chain considering important constraints like production and transportation constraints, 2) the concept of dual sales channel is considered in integrated supply chain, beside the prices defined in each channel, 3) the service level of retailers is determined by optimization of model.

The proposed model may be widely employed in some industries such as manufacturing of electronic devices and personal computers (PCs), etc., in which there are both indirect and direct selling channels. Several famous companies like Dell, Sony and Apple have use multi sale channels. Therefore, this model may be a useful tool for such companies. An alternative way to incorporate more information about the demand uncertainty into the model is by formulating a stochastic linear program. To the best of our knowledge from a review of the literature, there is no existing joint pricing network design decision model in dual channel integrated supply chain based uponstochastic programming and scenario generation method. In the rest of this paper, in section 2 we develop our stochastic model. The index VSS is calculated for proposed stochastic model in section 3. Finally, concluding remarks are presented in section 4.

2. MODEL DEVELOPMENT

The proposed integrated logistic model in this paper is a single-period, single-product, and multi-stage model including producers, distributors, customers, collection centers, recovery centers, and disposal centers. For saving cost, it is assumed in this model that collection process is done through reverse flow by distribution centers and the producers in forward flow will carry out recovery and repairing operations. These centers are called hybrid centers in logistic networks, which are utilized for cost-saving. In Figure (1), a general view of the presented logistic network is shown. As it characterized in Figure (1), the final products are sent from the producer to distribution centers. After inspecting, the distribution centers returns the repairable products to the producers. The percentage of products that are not repairable, are sent to disposal centers. The rest products will be shipped to end users (customers). The returned products from customers are sent to collection-distributor centers. After inspection, the recoverable products are shipped to producers/ recovery centers and the rest irreparable products will be sent to disposal centers. In addition to a regular forward flow where the producers sell the products to retailers via retail channel, the producers can select direct flow and sale product via e-tail channel to customers.
The defective or breakdown products received by customers in direct channel are also returned to collection-distributer centers as returned products in retail channel.

In the presented model, the subject of pricing is addressed within the close-loop integrated logistic network. The concept of pricing has been independently studied by various authors but it has been so far less proposed in the integrated logistic network and its relevant mathematical optimal model. The proposed model will be able to determine producer’s pricing policy in both retail and direct channels. The producers must set a direct channel price $p_m$ and sell the product directly through the direct channel and the retailer competes against the direct channel by offering a mix of added service $ss$ and retail price $r_p$ to customers. We consider a centralized dual-channel supply chain in which an integrated manufacturer controls all three decisions: the traditional retail price, the direct sale price, and the service level in retail channel. Some parameters in logistics network design such as demand of customers, transportation costs and resource capacities are quite uncertain [27]. To handle this uncertainty and have a robust logistics network, we develop a stochastic integrated supply chain by incorporating demand uncertainty.

In the conducted studies regarding the pricing such as Chiang et al. [24] and Bin et al. [28], demand function is indicated by a linear function of base demand $(d_{m_k}, d_{r_k})$, direct and indirect sale prices $(p_r, p_m)$ and service level $(ss)$, as follows:

\[ D_{r_k} = d_{r_k} - b_{r} p_r + b_{m} p_m + \psi r^* ss + \xi_k \]  

\[ D_{m_k} = d_{m_k} - b_{m} p_m + b_{r} p_r - \psi r^* ss + \xi_k' \]  

where, the coefficients $b_{r}$ and $b_{m}$ are the coefficients of the price elasticity in the retail channel and direct channel demand functions, respectively. The cross-price sensitivities $b_{r}$ and $b_{m}$ reflect the degree to which the goods are sold via two channels are substitutes, and $\psi m$ and $\psi r$ are the service sensitivity of the demand in the direct channel. If the service level $ss$ increases by one unit, $\psi m$ units of demand (customers) will be lost from the direct channel, of which $\psi r$ units of demand will transfer to the retail channel. The total demand of the two channels should be downward sloping in the retailer’s price, direct sale price and upward sloping in the service level. Thus, we have $bm_i < br_i$, $br_i < bm_i$ and $\psi m < \psi r$. $\xi_k$ and $\xi_k'$ are random variable with PDF (probability density function) $f(\cdot)$ and CDF (cumulative distribution function) $F(\cdot)$, that does not depend on the price and service level and shift the demand randomly about the mean. The related works that considered linear-additive functional demand form include (Chen & Simchi-Levi [29]; Dana & Petruzzi [30]; Federgruen & Heching [31]).

We will formulate this problem as a two-stage stochastic recourse model. Such a model includes primary decisions at first stage according to the related decisions to opening or non-opening the centers, prices, and service level at any channel, and quantity of products that transfers from producer to any retailer in such a way that after their determination, the values of the stochastic events may be characterized and decisions
are made at the second stage. The decisions at second stage include rate of supply by retailers, supply through direct channel and rates of the returned products. We employ the scenario-tree method to solve the stochastic model. For this purpose, suppose the given stochastic vector \( \xi = (\xi_1, \ldots, \xi_1, \xi_1', \ldots, \xi_1'') \) with certain number scenarios \( \xi^s = (\xi_1^s, \ldots, \xi_1^s, \xi_1'^s, \ldots, \xi_1''^s) \) and probability \( \delta^s \) while \( s = 1, \ldots, S \). With respect to the above-said definitions, the given indices, parameters, and the variables at first and second stage are defined according to the scenario as follows:

**Sets**

- \( I \): Set of potential production/recovery center locations
- \( J \): Set of hybrid distribution-collection center locations
- \( K \): Set of fixed locations of customer zones
- \( M \): Set of potential disposal center locations

**Parameters**

- \( \phi \): The Cost of providing a unit of service level
- \( br_1 \): The coefficient of price elasticity in retail channel
- \( br_2 \): The coefficient of cross-price sensitivities in retail channel
- \( bm_1 \): The coefficient of price elasticity in direct channel
- \( bm_2 \): The coefficient of cross-price sensitivities in direct channel
- \( \psi r \): The coefficient of service elasticity in retail channel
- \( \psi m \): The coefficient of service elasticity in e-tail channel
- \( dr_k \): Base level of direct channel demand for customer \( k \)
- \( ri_k \): Rate of return of used products from retail channel customer \( k \)
- \( ri_k' \): Rate of return of used products from direct channel customer \( k \)
- \( \gamma_j \): Rate of return of products by distributer \( j \) to producers
- \( \lambda \): Average disposal fraction in direct and indirect channels
- \( \lambda' \): Average disposal fraction for returned products by distributors
- \( f_i \): Fixed cost of opening production/recovery center \( i \)
- \( g_j \): Fixed cost of opening hybrid distribution-collection center \( j \)
- \( h_m \): Fixed cost of opening disposal center \( m \)
- \( c_{ij} \): Shipping cost per unit of products from production/recovery center \( i \) to hybrid distribution-collection center \( j \)
- \( a_{jk} \): Shipping cost per unit of products from production/recovery center \( i \) to direct channel customer \( k \)
- \( bj_k \): Shipping cost per unit of products from customer \( k \) to hybrid distribution-collection center \( j \)
- \( ej_i \): Shipping cost per unit of products from hybrid distribution-collection center \( j \) to production/recovery center \( i \)
- \( \pi_{jm} \): Shipping cost per unit of products from hybrid distribution-collection center \( j \) to disposal center \( m \)
- \( \rho_i \): Manufacturing cost per unit of product at production/recovery center \( i \)
- \( \theta_i \): Recovery cost per unit of product at production/recovery center \( i \)
- \( v_j \): Processing cost per unit of product at hybrid distribution-collection center \( j \)
- \( \eta_m \): Disposal cost per unit of product at disposal center \( m \)
- \( \tau_i \): Penalty cost per unit of non-utilized capacity at production/recovery center \( i \)
- \( \beta_j \): Penalty cost per unit of non-utilized capacity at hybrid distribution-collection center \( j \)
- \( \alpha_m \): Penalty cost per unit of non-utilized capacity at disposal center \( m \)
- \( cw_i \): Capacity of production/recovery center \( i \)
- \( cy_j \): Capacity of handling products in forward flow at hybrid distribution-collection center \( j \)
- \( cz_m \): Capacity of handling products at disposal center \( m \)
- \( cy_{rj} \): Capacity of handling products in reverse flow at hybrid distribution-collection center \( j \)
- \( cwr_i \): Capacity of recovery for production/recovery center \( i \)
- \( m_{\text{service}} \): Upper limit of service level in retail channel.

The first stage variables are \( X_{ij}, N_{ym}, p_{aq}, p_{r}, W_{j}, Y_{j}, Z_{m} \) and the second stage variables are as follows:
A. Ahmadi Yazdi and M. Honarvar / IJE TRANSACTIONS B: Applications  Vol. 28, No. 5, (May 2015)  738-745                                742

$Q_{ik}$: Quantity of products in direct channel shipped from production/recovery center $i$ to customer zone $k$ under scenario $s$

$U_{jk}$: Quantity of products shipped from hybrid distribution-collection center $j$ to customer zone $k$ under scenario $s$

$Q_{kj}^s$: Quantity of returned products shipped from customer zone $k$ to hybrid distribution-collection center $j$ under scenario $s$

$V_{ji}^s$: Quantity of returned products shipped from hybrid distribution-collection center $j$ to production/recovery center $i$ under scenario $s$

$T_{jm}$: Quantity of irreparable products shipped from hybrid distribution-collection center $j$ to disposal center $m$ under scenario $s$

$M_{ij}$: Quantity of returned products from direct channel customer zone $k$ to hybrid distribution-collection center $j$ under scenario $s$

The formulation of proposed stochastic model is as follows;

$$\text{Max} \{ \sum_{i} \sum_{j} \sum_{m} \sum_{s} \theta_i U_{jk} + \sum_{i} \sum_{j} \sum_{m} \sum_{s} \rho_i U_{jk} - \sum_{i} \sum_{j} \sum_{m} \sum_{s} \gamma_i V_{ji}^s - \sum_{i} \sum_{j} \sum_{m} \sum_{s} \mu_i T_{jm} - \sum_{i} \sum_{j} \sum_{m} \sum_{s} \theta_i \cdot \left( W_i \cdot \sum_{j} V_{ji}^s + \sum_{m} T_{jm} \right) - \sum_{i} \sum_{j} \sum_{m} \sum_{s} \theta_i \cdot \left( (Y_j - X_j) + (Y_j - \sum_{i} \sum_{s} \theta_i Q_{ij}^s + \sum_{i} \sum_{s} \theta_i M_{ij}^s) \right) \cdot ss^s \phi \}
$$

s.t.

$$\sum_{j} U_{jk} \leq \left( \Delta_k \cdot \left( \begin{array}{c} \beta \cdot p_{ij} + \beta_m \cdot p_{jm} + \psi \cdot \psi' \times ss \times \phi \end{array} \right) \right) \quad \forall k \in K, \quad s \in S \quad (4)$$

$$\sum_{j} Q_{ij}^s \leq \left( \Delta_k \cdot \left( \begin{array}{c} \beta \cdot p_{ij} + \beta_m \cdot p_{jm} + \psi \cdot \psi' \times ss \times \phi \end{array} \right) \right) \quad \forall k \in K, \quad s \in S \quad (5)$$

$$\sum_{j} M_{ij}^s = \gamma \cdot \sum_{j} U_{jk}^s \quad \forall k \in K, \quad s \in S \quad (6)$$

$$\sum_{j} Q_{ij}^s + \sum_{j} M_{ij}^s \geq \sum_{k} X_{jk} \quad \forall j \in J, \quad s \in S \quad (7)$$

$$\sum_{j} V_{ji}^s - \lambda \sum_{j} Q_{ij}^s - \sum_{j} M_{ij}^s = 0 \quad \forall j \in J, \quad s \in S \quad (8)$$

$$\sum_{i} T_{jm} - \lambda \sum_{i} Q_{ij}^s - \sum_{i} M_{ij}^s = 0 \quad \forall j \in J, \quad s \in S \quad (9)$$

The formulation of proposed stochastic model is as follows;

Objective function is profit maximization including total revenue and costs from two channels. Constraints (4) and (5) guarantee that the sent products to the customer may meet the demand wholly or partially. The constraints (6) and (7) define respectively the collection of returned products in direct and indirect channel. The constraint (8) is the balancing constraint at production/recovery and hybrid distribution collection centers in forward flow.

The constraint (9) insures that, the flow of repairable products existing from hybrid distribution/collection centers is equal to the flow entering to production/recovery centers. The constraint (10) indicates quantity of the transferred products to disposal centers. The constraint (11) shows the amount of products, which are returned by distributor to the producer. The constraints (12) and (13) denote the sale prices should be greater than process costs at production and distribution/collection centers.

Constraints (14-18) are capacity constraints that ensure the production capacity in each center is not exceeded. Finally, the constraint (19) identifies the upper bound for the distributor’s service level.
TABLE 1. Computational results for VSS and the stochastic programming solutions with respect to different probability distributions

<table>
<thead>
<tr>
<th></th>
<th>Uniform (-400, 400)</th>
<th></th>
<th>Uniform (-1000, 1000)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q*</td>
<td>EEV</td>
<td>VSS</td>
</tr>
<tr>
<td>Optimal values</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4410584</td>
<td>3318367</td>
<td>1092217</td>
</tr>
<tr>
<td></td>
<td>5016065</td>
<td>3277029</td>
<td>1739036</td>
</tr>
<tr>
<td></td>
<td>4739643</td>
<td>3210228</td>
<td>1529415</td>
</tr>
<tr>
<td></td>
<td>4524917</td>
<td>3182271</td>
<td>1342464</td>
</tr>
<tr>
<td></td>
<td>4663731</td>
<td>4032283</td>
<td>631448</td>
</tr>
<tr>
<td>EXP(500)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal values</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6390871</td>
<td>4331456</td>
<td>2059415</td>
</tr>
<tr>
<td></td>
<td>5943686</td>
<td>4256678</td>
<td>1687008</td>
</tr>
<tr>
<td></td>
<td>5489227</td>
<td>4633543</td>
<td>855684</td>
</tr>
<tr>
<td></td>
<td>6427444</td>
<td>4292808</td>
<td>2134636</td>
</tr>
<tr>
<td></td>
<td>6240829</td>
<td>4498743</td>
<td>1742086</td>
</tr>
<tr>
<td>Normal(100)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Optimal values</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4275990</td>
<td>3695557</td>
<td>580433</td>
</tr>
<tr>
<td></td>
<td>4391981</td>
<td>3449710</td>
<td>942271</td>
</tr>
<tr>
<td></td>
<td>4460511</td>
<td>3783662</td>
<td>676849</td>
</tr>
<tr>
<td></td>
<td>4404930</td>
<td>3737229</td>
<td>667701</td>
</tr>
<tr>
<td></td>
<td>4229052</td>
<td>4058084</td>
<td>170968</td>
</tr>
</tbody>
</table>

3. VALUE OF STOCHASTIC PROGRAMMING

Stochastic programs are computationally expensive and difficult to solve. Therefore, for real-world problems, simpler models have been considered. For example, some researchers have solved the deterministic program by replacing all random variables with their expected values. The Value of the Stochastic Solution (VSS) has been used as measure of the accuracy in these studies. The concept of VSS can be used to determine whether putting extra effort into modeling and solving stochastic programming can be beneficial. Let $Z(\xi)$ be the optimal decision of the first stage in deterministic problem where all random variables are replaced by their expected values. The VSS is then defined as $VSS = Q^* - EEV$, with $EEV = E_z(Q(Z(\xi), \xi))$. EEV is expected result of using EV (expected value program) solution. If this difference is large enough, it indicates that using the stochastic programming approach is beneficial. In this section, we compute this measure and study the effect of the distribution function type and variance of stochastic factor $\xi$ in demand function, on VSS. For this purpose, sensitivity analysis has been carried out for different probabilistic functions as follows: In Table 1, it can be seen that as the mean value and variance of distributions increase, the VSS also increases. In Table 1, VSS value in mode (Uniform [-1000, 1000]) is greater than mode (Uniform [-400, 400]) in all cases. The VSS values for distributions with greater mean and variance are higher than the VSS values for distributions with lower mean value and variance at all modes.

4. CONCLUSION

In this paper, we proposed a new mixed integer linear programming model for designing integrated forward/reverse logistics based on pricing policy in direct and indirect sales channel. We could calculate the optimal sale prices in direct and indirect sales channels using our model. The stochastic counterpart of the proposed MILP model was also developed by using scenario-based stochastic approach, considering uncertain demand with specific probability distribution. Value of stochastic solution (VSS) was calculated to evaluate the accuracy of stochastic programming approach. By considering different probability distribution for uncertain demands, the results showed that the VSS values for distribution with greater mean and variance are higher than the VSS values for distributions with lower mean value and variance at all cases.

5. REFERENCES


A Two Stage Stochastic Programming Model of the Price Decision Problem in the Dual-channel Closed-loop Supply Chain

A. Ahmadi Yazdi, M. Honarvar

Department of Industrial Engineering, Yazd University, Yazd, Iran

**Paper Info**

**Paper History:**
Received 27 November 2014
Received in revised form 19 January 2015
Accepted 13 March 2015

**Keywords:**
Logistic Network
Reverse Logistics
Integrated Logistics
Pricing Policy
Multi-channel Sale

**Abstract:**

In this paper, a two-stage stochastic programming model is developed to address the price decision problem in a dual-channel closed-loop supply chain. The model considers both the main channel and the reverse channel, where the main channel involves new product sales and the reverse channel involves product returns. The model is designed to optimize the overall profit of the supply chain by determining the optimal pricing strategy for both channels.

**Keywords:**
Logistic Network
Reverse Logistics
Integrated Logistics
Pricing Policy
Multi-channel Sale

**Address:**

A. Ahmadi Yazdi and M. Honarvar


**DOI:** 10.5829/idosi.ije.2015.28.05b.12