Design of Direct Exponential Observers for Fault Detection of Nonlinear MEMS Tunable Capacitor

H. Mobki\textsuperscript{a}, M. H. Sadeghi\textsuperscript{a}, G. Rezazadeh\textsuperscript{b}

\textsuperscript{a}Department of Mechanical Engineering, University of Tabriz, Tabriz, Iran
\textsuperscript{b}Department of Mechanical Engineering, Urmia University, Urmia, Iran

\textbf{Abstract}

In this paper a novel method for construction of an exponential observer for nonlinear system is proposed. The method is based on direct solution of dynamic error without any linearizing of nonlinear terms. Necessary and sufficient conditions for construction of direct observer are presented. Stability of the observer is checked using Lyapunov theorem. Also, the ability of this observer is checked by implementing the observer for fault detection of micro tunable capacitor subjected to nonlinear electrostatic force.

\textbf{1. Introduction}

Fault expresses deviation of one or more system characteristics from their ideal measure. Besides decreasing efficiency, accuracy, and velocity of the system, fault is responsible for casualty and financial loss on occasions. So, quick and precise fault detection of a system is significant in function of a process.

Fault detection and isolation (FDI) techniques can be broadly classified into two categories. These include Model-based FDI and Signal processing based FDI. In signal processing based FDI, some mathematical or statistical operations are performed on the measurements, or some neural network is trained using measurements to extract the information about the fault [1]. Model based techniques are based on comparison of the output of the system with the estimated one obtained from the mathematical model. There are three types of model based approaches, namely: observer-based, parity-space and the parameter identification. Recently, observer-based methods have received considerable interests. This particular attention is mainly due to the associated advantages of observer-based approaches, such as quick detection, requiring no excitation signal, possibility of on-line implementation, etc. Moreover, control engineers are more familiar with the concepts of observer design [2].

One of the biggest challenges of observer based FDI and mathematical control theory has been the problem of constructing state observers for nonlinear systems. The problem of designing observers for linear control systems was first introduced by Luenberger [3] and it was extended for nonlinear control systems by Thau [4]. Over the past three decades, considerable attention has been paid in the literature to the design of observers for nonlinear systems. Xia and Gao obtained a necessary condition for the existence of an exponential observer for nonlinear systems [5]. They showed that an exponential observer exists for a nonlinear system only if the linearization of the nonlinear system is detectable. On the other hand, necessary conditions for nonlinear observers have been obtained from an impressive variety of points of view. Lyapunov-like method was used for design of nonlinear exponential observer by Kou et al. [6]. Nonlinear adaptive observers have been studied for the nonlinear systems whose dynamics can be linearized by coordinate change and output injection [7-10]. In some literatures [11-17], observer design
were carried out using proper coordinate transformations. In other researches [18, 19] observer designs are implemented based on quadratic Lyapunov function and solving the algebraic Riccati equation. Most of the MEMS systems are nonlinear in nature. Nowadays these systems have shown remarkable popularity in engineering and industry for their significant advantages [20]. MEMS tunable capacitors are the main parts of RF integrated circuits such as tunable filters and resonators [21]. Also, electrostatically actuated MEM devices are widely designed, fabricated, used and analyzed in: micro actuators [22, 23] mems capacitive microphone [24-26], sensors [27], capacitive micro-plate [28, 29] and micro capacitors [30, 31]. In spite of the numerous works accomplished on MEMS, there are not enough and basic studies, concentrating on fault detection of such systems. In the development of MEMS fabrication, failure and fault analysis play a major role both in development time reduction, and in reliability evaluation.

Some works have been done on fault detection of these structures. Asgarya et al. [32] studied fault detection of nonlinear MEMS devices using neural network. Reppa and Tzes [33] applied set membership identification for micro electrostatic actuators. They assumed that the system is linearly parameterizable and the parameter vector contains the quantities that are susceptible to faults. Zahidul Islam et al. [34] studied fault detection of MEMS using frequency response. Izadian and Famouri [35] studied fault diagnosis of MEMS lateral comb resonators. Their valuable works are based on using multiple model adaptive estimators for fault detection of the resonators, which have linear electrostatic force proportional to state space.

In this paper, a novel method is proposed for construction of an observer for nonlinear systems. The method is based on direct solution of dynamic error without any linearizing of nonlinear terms. Stability of the observer is checked using Lyapunov theorem. Also, the ability of this observer is checked with implementation of observer for fault detection of micro tunable capacitor subjected to nonlinear electrostatic force.

2. CONSTRUCTION OF DIRECT OBSERVER FOR NONLINEAR SYSTEM WITH LINEAR OUTPUT:

Consider the nonlinear system with form:

\[
\begin{align*}
\dot{x} &= Ax + \varphi(x, u) + \eta(x, u) \\
y &= Cx
\end{align*}
\]  

(1)

where \( x \in R^m \), \( u \in R^n \) and \( y \in R^m \) are state, input and output vectors, respectively. \( A \) and \( C \) are known system matrices, \( \varphi(x, u) \) represents the nonlinear function and \( \eta(x, u) \) is unknown nonlinear function which contains noises and uncertainties. It must be noted that \( \eta(x, u) \) is a bounded function. For implementation of direct observer, the following conditions must be satisfied:

1. Matrices \( A \) and \( C \) are observable.
2. The nonlinear function \( \varphi(x, u) \) is continuously differentiable and satisfies the Lipschitz condition locally with constant \( \gamma \), i.e. \[ \|\varphi(x, u) - \varphi(\hat{x}, u)\| \leq \gamma \|x - \hat{x}\| \] (2)

The following observer is proposed for state reconstruction of system (1).

\[
\begin{align*}
\dot{\hat{x}} &= A\hat{x} + \varphi(\hat{x}, u) + K(y, \hat{y})(y - \hat{y}) \\
\hat{y} &= C\hat{x}
\end{align*}
\]  

(3)

where \( \hat{x} \) and \( \hat{y} \) represent estimated state and output and \( K(y, \hat{y}) \) is the unknown gain of observer, which must be obtained. Defining the observer error as \( e = x - \hat{x} \), we have:

\[
\dot{e} = (A - KC)e + \varphi(x, u) - \varphi(\hat{x}, u) \]  

(4)

where \( e \) is the dynamic error. We want to find an observer gain, \( K(y, \hat{y}) \), such that the observer error dynamics is asymptotically stable. The dynamic errors are nonlinear and its stability is now unclear. To check the stability of the nonlinear dynamic error in (4), Lyapunov stability theory is employed. This approach leads to results giving sufficient conditions for existence of observers for nonlinear system (3). If we can find continuous and differentiable function \( f(x, \hat{x}, u) \) in which \( f(x, \hat{x}, u)(x - \hat{x}) = \varphi(x, u) - \varphi(\hat{x}, u) \), Equation (4) can be rewritten as:

\[
\dot{e} = [A - KC + f(x, \hat{x}, u)]e \]  

(5)

Theorem 1. Consider the nonlinear system (1), the nonlinear observer (3) and the dynamic error of (4). The observer error dynamics (4) is (globally) asymptotically stable. If there exists a constant \( n \times m \) matrix \( K(y, \hat{y}) \) and a positive definite, symmetric \( n \times n \) matrix \( P \) such that:

\[
P[A - KC + f(x, \hat{x}, u)] < 0 \]  

(6)

i.e., it is uniformly negative-definite for all values of \( x \) and \( \hat{x} \). Proof: consider the following Lyapunov function candidate:

\[
V = e^TPe
\]  

(7)

Then, time derivative of \( V \) for system (4) is:

\[
\dot{V} = 2e^TP\dot{e} = 2e^TPf(x, \hat{x}, u) \]  

(8)

With attention to Equation (5) above equation can be rewritten as:
\[ \dot{V} = 2 \left( e^T P (A - KC) e + e^T P f(x, \dot{x}, u) e \right) \]
\[ \dot{V} = 2 e^T P (A - KC + f(x, \dot{x}, u)) e \]  
(9)

Since from Equation (6): 
\[ P [A - LC + f(x, \dot{x}, u)] < 0 \]
so \[ V < 0 \], \[ V \] is a Lyapunov function for system (4).

**Theorem 2.** The dynamic error of the observer is (globally) asymptotically stable if eigen values of \[ A - KC + f(x, \dot{x}, u) \] have negative real part. 

**Proof:** Theorem 2 can be proved easily, if matrix \( P \) set equal with identity matrix.

### 3. NUMERICAL EXAMPLE

In this section, ability of the direct observer is examined by implementing it for fault detection of micro tunable capacitor subjected to nonlinear electrostatic force. To this end, geometrical and mathematical model of the capacitor are presented.

#### 3. 1. Model Description

Figure 1a shows schematic view of a parallel plate capacitor, which consists of a movable electrode suspended over a stationary conductor plate, having primary distance, \( G_0 \), between two electrodes. Attractive electrostatic force due to the applied bias voltage \( u \) pulls movable electrode down towards the stationary plate. Figure 1b shows top view of the movable electrode, which is suspended by four supporting beams (two at each side). The area and thickness of movable electrode are \( S \) and \( h \), respectively. All supporting beams are identical, having width, thickness and length of \( b, h, \) and \( L \), respectively. The effective stiffness of each beam is \( k = \frac{12EI}{L^3} \), in which \( EI \) is the flexural modulus. The movable electrode is considered isotropic with density \( \rho \).

![Figure 1. Parallel plate tunable capacitor](image)

#### 3. 2. Mathematical Modeling

The governing equation of motion of an electro-mechanical tunable capacitor such as the one in Figure 1a can be described as:

\[ m \ddot{z} + c \dot{z} + k z = q_{elec} \]  
(10)

where \( z, m, c, \) and \( k \) are the deflection, mass, damping coefficient, and equivalent stiffness \((k_{eq} = 4k)\) of the movable electrode, respectively. Also, \( q_{elec} \) represents electrostatic force.

When the actuating voltage \( u \) is applied between the movable and stationary electrodes, the electrostatic force is computed using a standard parallel capacitance model, which yields [37]:

\[ q_{elec} = \frac{c_s S u^2}{2(G_0 - z)} \]  
(11)

where \( c_s = 8.854 \times 10^{-12} C^2 N^{-1} m^{-2} \) is the permittivity of the vacuum within the gap. For convenience, Equation (10) can be rewritten in a non-dimensional form by defining the following parameters:

\[ w = \frac{Z}{G_0}, \quad \tau = \frac{t}{t^*}, \quad \epsilon = \frac{c_s S}{2k_{eq} G_0} \]  
(12)

where \( \tau \) is the dimensionless time, and \( \epsilon \) represents electrostatic force.

Therefore, Equation (10) may be written as:

\[ \frac{d^2 w}{dt^2} + \frac{c'}{\epsilon} \frac{dw}{dt} + w = \frac{\alpha u^2}{(1 - w)^2} \]  
(13)

where \( c' \) and \( \alpha \) are dimensionless damping and electrostatic coefficients, respectively, defined as:

\[ c' = \frac{c}{\epsilon k_{eq}}, \quad \alpha = \frac{c_s S}{2k_{eq} G_0} \]  
(14)

#### 3. 2. 1. Mathematical Model in State Space Form

Consider \( x_1 = w \) and \( x_2 = \frac{dw}{dt} \); so, Equation (13) can be rewritten in the state space form as:

\[
\begin{bmatrix}
\frac{d}{dt} x_1 \\
\frac{d}{dt} x_2
\end{bmatrix} =
\begin{bmatrix}
0 & 1 \\
-1 & -\epsilon c'
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
\alpha u^2 \\
(1 - x_1)^2
\end{bmatrix}
\]
\[ y = \begin{bmatrix}
1 & 0
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]  
(15)

---

**TABLE 1.** Spatial properties of the micro parallel plate capacitor

<table>
<thead>
<tr>
<th>Properties</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of movable electrode (S)</td>
<td>400( \mu m \times 400( \mu m )</td>
</tr>
<tr>
<td>Thickness of movable electrode</td>
<td>1( \mu m )</td>
</tr>
<tr>
<td>Thickness of beam</td>
<td>1( \mu m )</td>
</tr>
<tr>
<td>Length of beams</td>
<td>100( \mu m )</td>
</tr>
<tr>
<td>Young’s modulus of beams</td>
<td>169GPa</td>
</tr>
<tr>
<td>Density</td>
<td>2300( kg/m^3 )</td>
</tr>
</tbody>
</table>

---

\[ 10854.8 \]

---

In this paper, output is considered as the non-dimensional deflection \( w \). Pursuing the same procedure, the structure of the direct observer may be rewritten as:

\[
\begin{bmatrix}
\dot{\hat{x}}_1 \\
\dot{\hat{x}}_2
\end{bmatrix} + \begin{bmatrix}
1 & -c_1 \\
-1 -c_2
\end{bmatrix} \begin{bmatrix}
\hat{x}_1 \\
\hat{x}_2
\end{bmatrix} + \begin{bmatrix}
a_1 \\
a_2
\end{bmatrix} + \begin{bmatrix}
0 \\
a_1 u - a_2 u
\end{bmatrix} \\
K_1 y - \hat{y}
\end{equation}
\]

With attention to Equations (15) and (16), dynamic error can be obtained as:

\[
\begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2
\end{bmatrix} = \begin{bmatrix}
-1 -c_1 & \hat{x}_1 \\
-1 -c_2 & \hat{x}_2
\end{bmatrix} \begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2
\end{bmatrix} + \begin{bmatrix}
0 \\
a_1 u - a_2 u
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix}
\end{equation}

The above equation can be rewritten as:

\[
\begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2
\end{bmatrix} = \begin{bmatrix}
0 & \hat{x}_1 \\
0 & \hat{x}_2
\end{bmatrix} \begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2
\end{bmatrix} + \begin{bmatrix}
0 \\
a_1 u - a_2 u
\end{bmatrix}
\end{equation}

or

\[
\begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2
\end{bmatrix} = \begin{bmatrix}
0 & \hat{x}_1 \\
0 & \hat{x}_2
\end{bmatrix} \begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix}
\end{equation}

The simulated faults are

\[
\begin{bmatrix}
-1 & \hat{x}_1 \\
-1 -c_2 & \hat{x}_2
\end{bmatrix} \begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2
\end{bmatrix} + \begin{bmatrix}
0 \\
a_1 u - a_2 u
\end{bmatrix}
\end{equation}

be presented in compacted form as:

\[
\begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2
\end{bmatrix} = \begin{bmatrix}
-1 & \hat{x}_1 \\
-1 -c_2 & \hat{x}_2
\end{bmatrix} \begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2
\end{bmatrix} + \begin{bmatrix}
0 \\
a_1 u - a_2 u
\end{bmatrix} + \begin{bmatrix}
0 \\
0
\end{bmatrix}
\end{equation}

For the stability of dynamic error, following condition must be satisfied:

\[
\text{eig} \left[ \begin{bmatrix}
-1 & \hat{x}_1 \\
-1 -c_2 & \hat{x}_2
\end{bmatrix} \begin{bmatrix}
\hat{e}_1 \\
\hat{e}_2
\end{bmatrix} + \begin{bmatrix}
0 \\
a_1 u - a_2 u
\end{bmatrix} \right] < 0
\]

Following conditions guarantee the asymptotic stability of dynamic error.

\[
K_i > -c
\]

\[
K_i > -1 - c_2 e + \frac{2 - (\hat{x}_1 + \hat{x}_2)}{[1 - \hat{x}_1 (1 - \hat{x}_2)]} a_2 u
\]

With attention to Equation (21), it is concluded that \( K_2 \) is a variable gain and depends on actual and estimated states of system.

### 4. SIMULATION RESULTS

In this section fault detection of parallel plate capacitor with nonlinear electrostatic term has been developed. Spatial properties of the capacitor are shown in Table 1. The system output is dimensionless position of the movable electrode with respect to the stationary electrode. Residual is obtained using direct observer as the difference of system output and the estimated one. It is assumed that the applied voltage is contaminated by \pm 20\% noise, and also \pm 5\% uncertainty in determination of \( \varepsilon / \varepsilon_0 \). So applied voltage and noise vary within \( 0.8u \leq \text{voltage} \leq 1.2u \) and \( -0.2u \leq \text{noise} \leq +0.2u \) respectively. As mentioned before is a bounded and nonlinear function, which contains noise. Attention to above explanation it can be concluded that \( -a(0.2u)^2 \leq \eta(x, u) \leq +a(0.2u)^2 \). The simulated faults are suddenly decreasing of applied voltage, and change of the movable electrode mass.

Residual is the index of an observer-based fault detection, which takes the information of faults. It is generated by comparing the outputs of the system and their estimates obtained by an observer. For fault-free system, with condition of no disturbances and modeling uncertainties, the outputs are equal with their estimations, resulting in zero residual. Any deviation of residual from zero will announce of presence of a fault. Existence of the disturbances and uncertainties are unavoidable and the residual signal is not completely decoupled from the effect of them; so even for fault-free system, residual deviates from zero. Therefore, a threshold is needed to handle the effect of disturbances and uncertainties [2]. In this paper threshold is obtained based on the asymptotic results of residual with extreme magnitude of the uncertainty and noise.

#### 4.1. Determination of the Static Threshold

In this part, the static thresholds are determined. Upper and lower thresholds are obtained based on asymptotic results of the residual for fault-free capacitor (ARRFFC), considering maximum amount of noise for the upper threshold and minimum amount of noise for the lower threshold. The level of uncertainty was maximum (5\%) for both of the cases. Figures 2 and 3 show the convergence of ARRFFC and determination of upper and lower threshold for \( u = 1V \). As shown in these figures upper and lower threshold equal with \( 1.22 \times 10^{-3} \) and \( -1.09 \times 10^{-3} \), respectively.

#### 4.2. Actuator and Component Fault Detection

In this section simulated results for fault detection of the capacitor are presented. The simulated fault is abrupt decreasing of the applied voltage (20\%) occurred in 4.57s. Figures 4-6 show residual versus time for various amount of applied voltage. As shown in these figures for \( \text{time} \leq 4.57s \) the residuals are within thresholds. Furthermore, as shown in these figures, direct observer has good ability in quick detection of fault. These indicate that direct observer is robust to noise and sensitive to fault. Dynamic pull-in phenomenon is an unstable condition which can occur in MEMS capacitor subjected to nonlinear electrostatic force [29]. In this
condition, movable electrode loses its stability and knocks up to the stationary electrode. Dynamic pull-in voltage for the capacitor is 4.71V. Figure 7 shows residual versus time for applied voltage $u = 4.5V$, which is in the vicinity of dynamic pull-in voltage. As shown in this figure, observer is robust to noise and sensitive to fault in the vicinity of dynamic pull-in voltage (unstable condition).
In the reset of this section, the ability of the observer for additive mass detection is examined. As mentioned before, “Fault expresses deviation of one or more system characteristics (such as mass, stiffness, and damping coefficient) from their ideal measure”. So, additive mass may be considered as existence of fault for the capacitor. MEMS capacitive sensor have been devised to perform measurements of phenomena in their surrounding environment, such as presence of humidity, mercury vapor, hazardous gas, and volatile organic compounds (chemicals) [38, 39]. The presented micro tunable capacitor may be applied as a gas sensor.

Mass-based sensors use a functional material to capture molecules of a target material. In this condition, equivalent mass of capacitor is increased and lead to exceeding of residual from thresholds. The structure is then coated with a material that the chemical being measured will adhere to resulting in an increase of mass on the structure when the chemical is present. Subsequently, the resonant frequency of the MEMS structure will change indicating the presence of the chemical [40].

Figures 8-11 show residual versus time for various amounts of applied voltages for 20% extra mass of the movable electrode. These figures show that direct observer has good ability in component fault detection. Figure 11 proves that this observer can detect fault of capacitor in the vicinity of dynamic pull-in voltage (unstable condition). The results presented in this section prove ability of direct observer as a suitable instrument for additive mass detection for micro capacitive sensors.

5. CONCLUDING REMARKS

In this paper, the observer based method for fault detection of parallel plate capacitor was accomplished. To this end, necessary and sufficient conditions for construction of direct observer have been presented. Stability of the observer was checked using Lyapunov theorem. The ability of observer for fault detection of tunable capacitor subjected to nonlinear electrostatic force was examined. To this end, governing dynamic equation of the capacitor was presented. The effects of noise and uncertainty were compensated by using thresholds. Upper and lower threshold limits were obtained based on asymptotic results of the residuals for fault-free case. The sensitivity of the observer to fault detection as well as robustness to the noise and uncertainty was examined. It was shown that direct exponential observer has good ability in fault detection of the micro tunable capacitor, even in the vicinity of dynamic pull-in voltage.

6. REFERENCES


Design of Direct Exponential Observers for Fault Detection of Nonlinear MEMS Tunable Capacitor

H. Mobki\textsuperscript{a}, M. H. Sadeghi\textsuperscript{a}, G. Rezazadeh\textsuperscript{b}

\textsuperscript{a}Department of Mechanical Engineering, University of Tabriz, Tabriz, Iran
\textsuperscript{b}Department of Mechanical Engineering, Urmia University, Urmia, Iran

\textbf{Paper Info}

\textit{Paper history:}
Received 06 September 2014
Received in revised form 06 December 2014
Accepted 29 January 2015

\textit{Keywords:}
Nonlinear Observer
Exponential Observer
Lyapunov Theorem
Micro Tunable Capacitor