



## Vibration of Train-Rail-Bridge Interaction Considering Rail Irregularity with Arbitrary Wavelength

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### PAPER INFO

#### Paper history:

Received 27 December 2013

Accepted in revised form 13 March 2015

#### Keywords:

Train-Rail-Bridge Interaction  
Rail Random Irregularity  
Wavelength Interval  
Sensitive Wavelength  
Dynamic Response  
High-Speed Railway Bridge

### ABSTRACT

A generation method for the rail random irregularity with arbitrary wavelength interval (WI) is developed, and its accuracy and efficiency are demonstrated. Then a moving wheel-rail-bridge interaction element is derived to establish the finite element equations of motion for the train-rail-bridge interaction system, and the flow chart of assembly and calculation for the system equations is given. According to the sub-interval principle, the influences of the irregularities with the large WI and the sub-intervals on the dynamic responses are analyzed by a numerical example, and the sensitive WI of each response is discussed. The results indicate that the bridge acceleration and the contact force are both more sensitive to the irregularity with WI (1~5 m). The irregularity with WI (0.1~1 m) has less influence on the car body acceleration but mainly contributes to the rail acceleration. However, all the irregularities with wavelengths at interval (1~150 m) can have significant influences on the car body acceleration. Meanwhile, the transient jump of wheel can be simulated and should be taken into account for the derailment risk assessment.

doi: 10.5829/idosi.ije.2015.28.04a.04

## 1. INTRODUCTION

The geometry of a track in service can deviate from its initial designed geometry as a response to the effects of track settlement, rail defect and errors in surveying during design and construction. These deviations in the track geometry are defined as rail irregularities. Generally, there are two types of rail irregularities existing at the wheel-rail system. One type is the specific irregularities, caused by some specific factors, such as the wheel flat [1] and rail welds [2], which have been widely discussed in the literatures. With rail irregularities assumed as half-sine-wave, Choi et al. [3] investigated the effects of wavelengths and amplitudes on the high-speed train. The other type is the random irregularities. In order to get the statistical characteristics of the random irregularities, lots of field measurements have been made early by many countries,

and then the track spectra have been presented, such as the Chinese Railway track spectrum [4] and the German high-speed track spectrum [5]. The track spectrum is usually expressed in frequency domain by a one-side power spectrum density (PSD) function and has been widely discussed.

Because the track spectrum cannot be inputted directly into the train-rail-bridge interaction system, the PSD function should be transformed into time domain firstly. Many numerical simulation methods have been adopted in the literatures [6]. The white noise filtration method has been used to generate the time domain models of roads [7]. However, the generality of this method is poor [6]. Also, the spectral representation method has been introduced to generate the rail irregularity [8]. Although this method has an exact theory foundation, it is rather time-consuming [7]. Moreover, the method based on inverse fast Fourier transform (IFFT) [9, 10] has been proven to be more simple, efficient and suitable for arbitrary PSDs. The continuous wavelength interval (WI) of the track

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spectrum has big influences on the responses [5], and therefore it is necessary to generate the irregularity with arbitrary WI.

The rail random irregularity can lead to the random vibration of the train-rail-bridge interaction system. Yang et al. [11] established the train-bridge interaction system to investigate the effects of the irregularities. With the wheel-rail interaction implemented by a Hertz contact spring, a semi-analytical solution for the vehicle-bridge interaction was derived [12] to simulate the transient jump of the wheel. Zhang et al. [8] developed a space train-bridge interaction model. A three-dimensional dynamic interaction was developed to analyze the effects of the irregularities with various wavelength ranges [13]. In these models, the elastic effects of rail structure were neglected for the sake of brevity. However, the maintenance workers of French and German railways reported destabilization of the ballast on small and medium span bridges [14], meaning that the rail structure should be taken into account. In addition, as a threat to the running safety, the transient jump of the wheel can occur due to the rail irregularity and high speed [4], and should be taken into consideration. The sensitive WI of the rail irregularity for the dynamic response has not been thoroughly investigated in the literatures.

In this paper, a generation method, based on IFFT, for the rail random irregularity with arbitrary WI is developed, and according to the sub-interval principle, the irregularity samples with the large WI as well as the sub-intervals are generated. Then a wheel-rail-bridge interaction element considering the rail irregularity and the transient jump is derived, and in combination with the equations of motion for the classical vehicle [11], the equations of motion for the train-rail-bridge interaction system are established. The flow chart of assembly and calculation is given, and the computational program is coded in MATLAB. Finally, the accuracy and efficiency of the developed generation method are discussed, and the influences of the generated irregularities on the dynamic responses are analyzed. Meanwhile, the effects of the irregularity with the large WI are compared with the irregularities with the sub-intervals to investigate the sensitive WI of the dynamic responses.

## 2.GENERATION OF RAIL RANDOM IRREGULARITY

Rail random irregularity represents an important excitation source of the train-rail-bridge interaction, and is usually characterized in frequency domain, namely track spectrum. According to the German track spectra [5], the PSD function for elevation irregularity, i.e.,  $S_v(\Omega)$ , is expressed as:

$$S_v(\Omega) = \frac{A_r \Omega_c^2}{(\Omega^2 + \Omega_r^2)(\Omega^2 + \Omega_c^2)} \quad (1)$$

where  $\Omega$  denotes the spatial frequency;  $A_r=4.032 \times 10^{-7}$  m<sup>2</sup>·rad/m;  $\Omega_c=0.8246$  rad/m;  $\Omega_r=0.0206$  rad/m.

The relationships among the spatial frequency  $\Omega$ , the time frequency  $f$ , the train speed  $v$  and the wavelength  $\lambda$  can be written as:

$$\omega = 2\pi f = \Omega v; f = \frac{v}{\lambda} \quad (2)$$

Based on Equation (2), Equation (1) can be transformed into time frequency domain as:

$$S_v(f) = \frac{v^3 A_r \Omega_c^2}{2\pi(4\pi^2 f^2 + v^2 \Omega_r^2)(4\pi^2 f^2 + v^2 \Omega_c^2)} \quad (3)$$

The PSD function should be transformed into time domain to input the track spectrum into the interaction system, and a time-frequency transformation technique, based on IFFT, is therefore developed. The total analysis length of the rail is divided into  $N$  sections. The relationship between the PSDs at discrete sampling points and the signal spectrum can be written as:

$$X(k) = N \varepsilon(k) \sqrt{S_v(k\Delta f) \Delta f} \quad (4)$$

where  $X(k)$  ( $k = 0, 1, \dots, N-1$ ) denotes the Fourier frequency spectrum;  $\varepsilon(k)$  denotes the random phase series;  $S_v(k\Delta f)$  denotes the discrete PSDs.

The discrete frequency range ( $\Delta f$ ,  $N\Delta f$ ) is taken as the total interval, and let it includes the arbitrary WI ( $\lambda_{\min}$ ,  $\lambda_{\max}$ ) for consideration. It is assumed that if the frequencies are out of consideration, their PSDs are 0. Then the rail random irregularity with arbitrary WI in time domain can be generated directly by IFFT. The wavelength  $\lambda$  in the above German track spectrum is generally longer than 1 m [4]. However, the German track spectrum expressed as Equation (1) is still adopted for the irregularity with wavelength shorter than 1m. According to the sub-interval principle, the irregular samples with the large WI (0.1~150 m) and the sub-intervals (0.1~30 m), (30~60 m), (60~90 m) and (90~150m) are generated with the same random phase series. The generated irregularity with the large WI is shown in Figure 1, where the SUM represents the summation of all the four irregularities with the sub-interval wavelengths. The irregularity with the large WI shows the characteristics of both long and short wavelengths and agrees very well with the SUM, indicating that the generated irregularities are reasonable. The generated irregularities with WIs (0.1~30 m) are shown in Figure 2, the irregularity changes seriously that can lead to the high-frequency excitation. As shown in Figure 3, the PSDs in theory determined by Equation (3) agree well with those of the generated irregularities, indicating that the generation method has great precision.

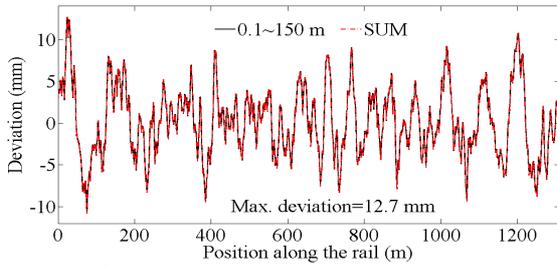


Figure 1. Irregularity with the large WI (0.1~150 m).

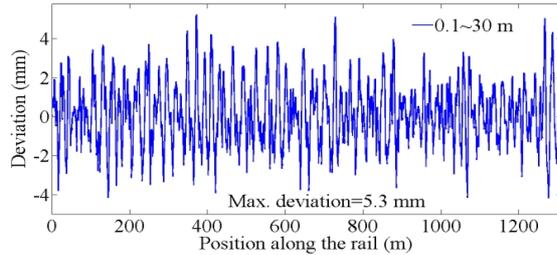


Figure 2. Irregularity with the WI (0.1~30 m).

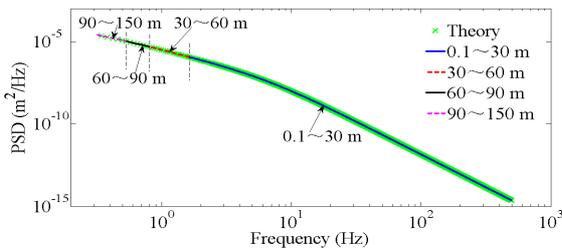


Figure 3. Comparisons of the PSDs in theory and those of the generated irregularities

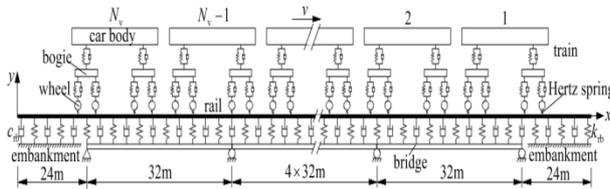


Figure 4. A typical train-rail-bridge interaction system.

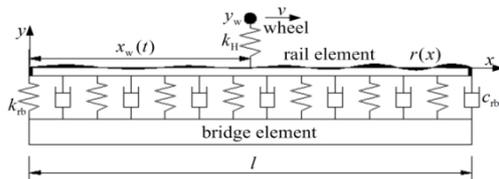


Figure 5. A moving wheel-rail-bridge interaction element.

### 3. MOTION EQUATIONS FOR TRAIN-RAIL-BRIDGE INTERACTION

Both the rail structure and the jump of wheel should be taken into account because of their importance for the dynamic interaction. As shown in Figure 4, in a typical

train-rail-bridge interaction system, the track structure, the train, the bridge and the embankments are modeled. Each vehicle is modeled as a mass-spring-damper system with 10 DOFs. The rail is modeled as a Bernoulli-Euler upper beam, and the bridge is modeled as a series of multi-span Bernoulli-Euler lower beams. It is assumed that if certain vehicles are not on the rail in question, they are supported on a rigid foundation. Let  $r(x)$  denote the initial rail irregularity.

#### 3. 1. A Moving Wheel-rail-bridge Interaction Element Considering Rail Irregularity and Jump

The three subsystems are connected by the wheel-rail interaction and the rail-bridge interaction. Both of the two interactions are considered in a typical wheel-rail-bridge element, as shown in Figure 5.

The wheel-rail interaction is of great importance, and a Hertz spring with stiffness  $k_H$  is introduced to simulate this interaction considering the transient jump. In the mathematical model, the spring coefficient is  $a_{in}k_H$ , and  $a_{in}$  ( $i=1, 2, \dots, N_v; n=1, 2, 3, 4$ ) denotes the wheel-rail contact coefficient.

The spring deformation  $y_{in}$  between the wheel and the rail can be written as:

$$y_{in} = y_w - y_r(x_w) - r(x_w) \quad (5)$$

where  $x_w$  is the local wheel coordinate;  $y_w$  is the wheel vertical displacement;  $r(x_w)$  denotes the rail irregularity;  $y_r(x_w)$  is the rail displacement.

The contact force  $F_{in}$  ( $i=1, 2, \dots, N_v; n=1, 2, 3, 4$ ) between the wheel and the rail can be written as:

$$F_{in} = -a_{in}k_H y_{in} \quad (6)$$

The motion of the bridge element is ruled by the following second-order equation:

$$[M_b^e]\{\ddot{q}_b^e\} + [C_b^e]\{\dot{q}_b^e\} + [K_b^e]\{q_b^e\} = -\int_0^l [N]^T f_{tb}(x, t) dx \quad (7)$$

where  $[M_b^e]$ ,  $[K_b^e]$  and  $[C_b^e]$  denote the element mass, stiffness and damping matrices, respectively;  $\{\ddot{q}_b^e\}$ ,  $\{\dot{q}_b^e\}$  and  $\{q_b^e\}$  denote the nodal acceleration, velocity and displacement, respectively.  $f_{tb}(x, t)$  is the distributed load from the rail, given by

$$f_{tb}(x, t) = -k_{rb}(y_r - y_b) - c_{rb}(\dot{y}_r - \dot{y}_b) \quad (8)$$

where  $k_{rb}$  and  $c_{rb}$  are the continuous stiffness and damping coefficients of the rail bed, respectively;  $y_r$  and  $y_b$  are the rail and bridge displacement, respectively. Substituting Equation (8) into Equation (7), one obtains:

$$[M_b^e]\{\ddot{q}_b^e\} + ([C_b^e] + [C_{be}^e])\{\dot{q}_b^e\} + ([K_b^e] + [K_{be}^e])\{q_b^e\} - [C_{rb}^e]\{\dot{q}_r^e\} - [K_{rb}^e]\{q_r^e\} = 0 \quad (9)$$

where  $[K_{be}^e] = [K_{rb}^e]$  and  $[C_{be}^e] = [C_{rb}^e]$  denote the coupling stiffness and damping matrices, respectively.

Analogous to the bridge element, the equation of motion for the rail element can be written as:

$$[M_r^e]\{\ddot{q}_r^e\} + ([C_r^e] + [C_{re}^e])\{\dot{q}_r^e\} + ([K_r^e] + [K_{re}^e] + [K_{rw}^e])\{q_r^e\} - [C_{br}^e]\{\dot{q}_b^e\} - [K_{br}^e]\{q_b^e\} - a_{in}k_{H1}[N]_{x=x_w}^T y_w = \{P_{rw}^e\} \quad (10)$$

The wheel is supported by the contact force, so the equation of motion for the wheel can be written as:

$$m_w \ddot{y}_w + a_{in}k_{H1}y_w - a_{in}k_{H1}[N]_{x=x_w}\{q_r^e\} = a_{in}k_{H1}r(x_w) \quad (11)$$

where  $m_w$  is the mass of the wheelset. According to Equations (9), (10) and (11), the equations of motion for the moving wheel-rail-bridge interaction element can be assembled as:

$$\begin{bmatrix} M_b^e & 0 & 0 \\ 0 & M_r^e & 0 \\ 0 & 0 & m_w \end{bmatrix} \begin{Bmatrix} \ddot{q}_b^e \\ \ddot{q}_r^e \\ \ddot{y}_w \end{Bmatrix} + \begin{bmatrix} C_b^e + C_{be}^e & -C_{rb}^e & 0 \\ -C_{br}^e & C_r^e + C_{re}^e & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \dot{q}_b^e \\ \dot{q}_r^e \\ \dot{y}_w \end{Bmatrix} + \begin{bmatrix} K_b^e + K_{be}^e & -K_{rb}^e & 0 \\ -K_{br}^e & K_r^e + K_{re}^e + K_{rw}^e & -a_{in}k_{H1}[N]_{x=x_w}^T \\ 0 & -a_{in}k_{H1}[N]_{x=x_w} & a_{in}k_{H1} \end{bmatrix} \begin{Bmatrix} q_b^e \\ q_r^e \\ y_w \end{Bmatrix} = \begin{Bmatrix} 0 \\ P_{rw}^e \\ a_{in}k_{H1}r(x_w) \end{Bmatrix} \quad (12)$$

**3. 2. Solution Procedures for the Train-Rail-Bridge Interaction System**

From Equation (12), the equations of motion for the rail-bridge element without wheel can be obtained, and then the equations of motion for the wheel-rail-bridge interaction system can be assembled. Finally, according to the equations of motion for the classical vehicle model, the finite element equations of motion for the train-rail-bridge interaction system, shown in Figure 5, can be obtained [11]. It is interesting to note that only the time-dependent sub-matrices and sub-vectors need to be re-assembled at each time step. The flow chart of assembly and calculation in detail is shown in Figure 6. All the time-independent sub-matrices and sub-vectors are taken as the initial equations of motion for each time step.

**4. NUMERICAL EXAMPLE**

In this section, a six-span simply-supported box girder railway bridge traveled by a high-speed train consisting of five vehicles is analyzed to study the influences of random irregularities with different WIs on the dynamic responses. Without iteration, the system equations of motion will be solved by the Newmark- $\beta$  method [15] with time step  $\Delta t=0.001s$ .

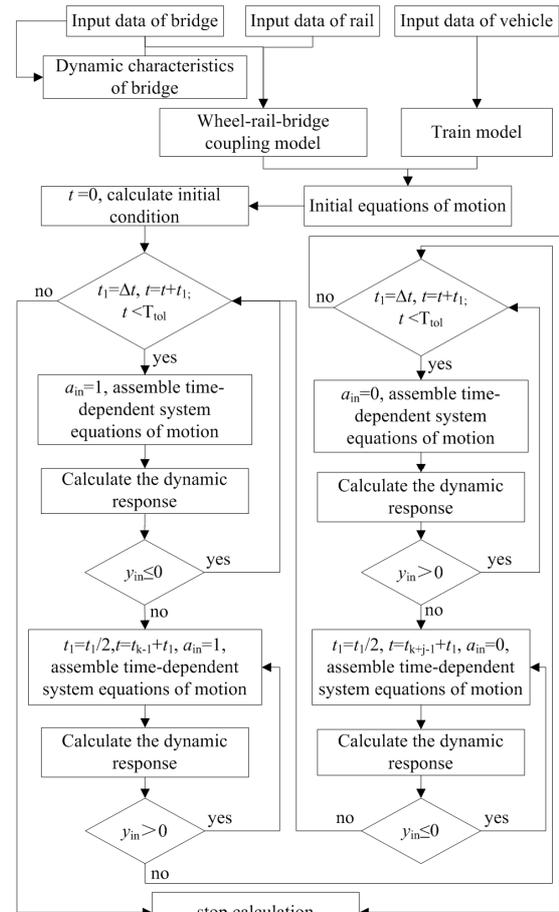


Figure 6. Flow chart of assembly and calculation.

The total longitudinal length of the rail is 240 m, and the parameter values adopted from the literature [11] are used for the vehicle and the rail. The Hertz spring stiffness  $k_{H1}=2 \times 1.4 \times 10^9$  N/m [13].

**4. 1. Influence of Rail Random Irregularities with Wavelengths in Interval (0.1~150 m)**

The influences of the five random irregularities generated in section 2 on the dynamic responses of the interaction system are analyzed. The maximum dynamic responses at two high speeds of 250 and 300 km/h are listed in Table 1, where the Smooth represents the smooth rail. For the bridge mid-span displacement, the responses considering irregularities are consistent with those of the Smooth, indicating that all the irregularities have less influence. With regard to the bridge mid-span acceleration, the responses of the sub-interval (0.1~30 m) are nearly the same as those of the WI (0.1~150 m) and much greater than those of the Smooth (about 4.6 times greater). On the vertical acceleration of the central car body, all the irregularities can have significant influences, indicating that the irregularity with any WI should be controlled to improve the riding comfort.

TABLE 1. Maximum dynamic responses at two high speeds (km/h)

WI (m)	Bridge displacement (mm)		Bridge acceleration ( $m/s^2$ )		Car body acceleration ( $m/s^2$ )		Contact force (N)	
	250	300	250	300	250	300	250	300
0.1~150	1.02	1.11	0.868	1.284	0.201	0.233	261921	281470
0.1~30	1.01	1.10	0.867	1.281	0.180	0.183	260274	279084
30~60	1.01	1.09	0.157	0.230	0.101	0.113	142997	144781
60~90	1.02	1.09	0.155	0.225	0.110	0.111	140395	141280
90~150	1.01	1.08	0.153	0.226	0.060	0.078	139622	140344
Smooth	1.01	1.08	0.154	0.225	0.018	0.021	138823	140261

Meanwhile, the sub-interval (0.1~30 m) is more sensitive than the other three sub-intervals. While for the contact force, the sub-interval (0.1~30 m) has great influences that should be the sensitive WI. The first natural frequency of the bridge is 4.61 Hz, thus the resonance speed in theory [4] is about 390 km/h. The time histories of some dynamic responses near the resonance speed are shown in Figures 7 and 8. As shown in Figure 7, the response of the bridge increases as there are more vehicles passing the bridge, indicating the occurrence of resonance. The vertical displacement of the rail relative to bridge at the mid-span is shown in Figure 8. The relative displacement is very small due to the strong constraint effect of the rail bed; the periodic excitation leads to the periodic forced vibration of rail.

#### 4. 2. Influence of Rail Random Irregularities with Wavelengths in Interval (0.1~30 m)

As deduced from the analysis of Section 4.1, the dynamic response is mainly contributed by the irregularity with WI (0.1~30 m), which will be taken as the large WI for a more precise analysis. With the generation method for random irregularity developed in Section 2, the irregularity samples of sub-intervals (0.1~1 m), (1~5 m), (5~12 m), (12~20 m) and (20~30 m) are generated, and the influences of them are compared with those of the large WI to study the sensitive WI.

Considering the six rail random irregularities, the maximum dynamic responses of the train-rail-bridge interaction at different train speeds ranging from 200 to 400 km/h have been shown in Figures 9-12. As shown in Figure 9, the results of the sub-interval (1~5 m) are much greater than those of the smooth rail and are nearly the same as those of the large WI (0.1~30 m), indicating that the bridge acceleration is more sensitive to the sub-interval (1~5 m). As shown in Figure 10, the responses of the sub-interval (0.1~1 m) are much greater than those of the smooth rail (e.g., about 8.5 times greater at a speed of 350km/h). The sub-interval (1~5 m) has some influence, while the influences of the other three sub-intervals are very small. So the sensitive WI should be (0.1~1 m) for the rail acceleration. The vertical acceleration at the centroid of the central car body is shown in Figure 11.

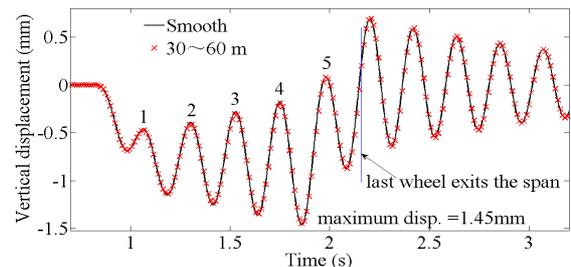


Figure 7. Bridge mid-span vertical displacement.

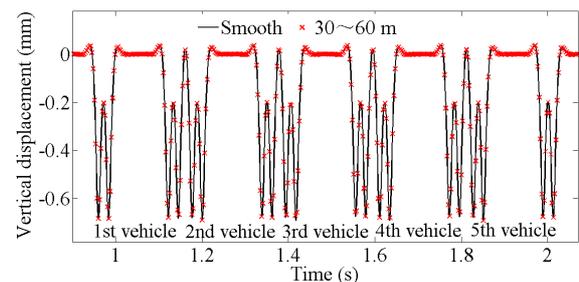


Figure 8. Vertical displacement of the rail relative to bridge.

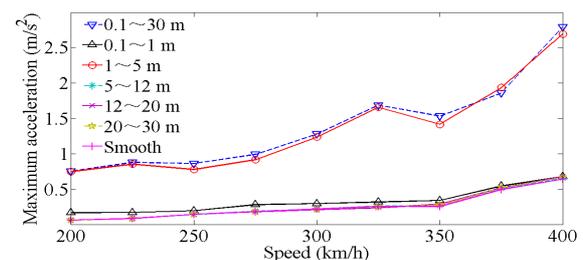


Figure 9. Maximum bridge mid-span acceleration.

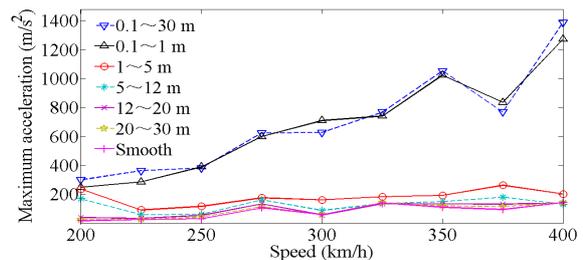


Figure 10. Maximum rail acceleration at the mid-span.

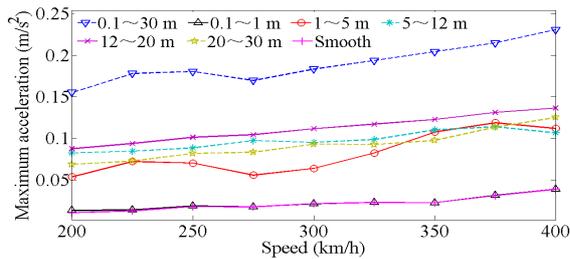


Figure 11. Maximum car body acceleration.

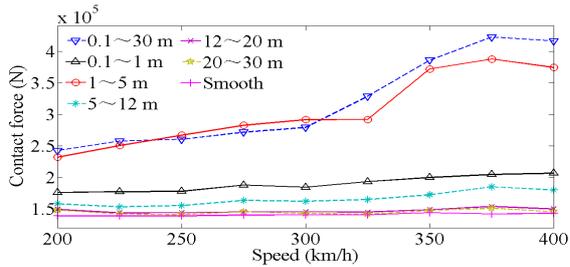


Figure 12. Maximum contact force.

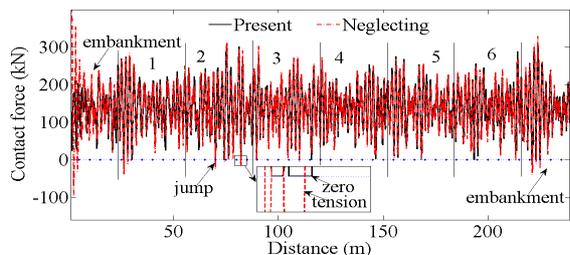


Figure 13. Contact force at speed  $v=325$  km/h.

Obviously, the contribution of the large WI (0.1~30 m) is larger than that of any other sub-interval. Therefore the large WI (0.1~30 m) should be taken directly to estimate the most unfavorable response. The response of the sub-interval (0.1~1 m) gets a well consistent with that of the smooth rail. As shown in Figure 12, the sensitive WI should be (1~5 m), which is the same for the bridge acceleration. For the speed of 325 km/h, the contact forces of the large WI (0.1~30 m) considering the present model and neglecting the transient jump are both shown in Figure 13. The contact force neglecting the jump includes incorrect tensions; while the minimum contact force of the present model decreases to zero at certain times, meaning the transient jump of wheel. These results also indicate that the transient jump of wheel should be taken into consideration for the derailment risk assessment.

## 5. CONCLUSION

A generation method for the rail random irregularity with arbitrary WI has been developed, and with a moving wheel-rail-bridge interaction element derived,

the motion equations for the train-rail-bridge system have been established. According to the sub-interval principle, the sensitive WI of each response has been studied. From this study, the following conclusions can be drawn:

- (1) Excellent agreement can be observed not only between the PSDs in theory and those of the generated irregularities, but also between the irregularity with the large WI and the summation of all the irregularities with the sub-interval wavelengths, indicating that the generation method are reasonable and effective.
- (2) Resonance can occur near the resonance speed, which can result in dramatic amplification of the response. The transient jump of wheel can be simulated by the present model, and should be considered for the derailment risk assessment.
- (3) The bridge acceleration is more sensitive to the irregularity with WI (1~5 m). The rail acceleration is mainly contributed by the irregularity with WI (0.1~1 m).
- (4) For the car body acceleration, the influence of the irregularity with WI (0.1~1 m) is insignificant, while all the irregularities with wavelengths in interval (1~150 m) can have significant influences. The irregularity with WI (1~5 m) mainly contributes to the contact force.
- (5) The irregularity with wavelength shorter than 5m should be mainly controlled in the design and maintenance of the high-speed railway, and in order to improve the riding comfort, the irregularity with any wavelength should be controlled.

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### PAPER INFO

چکیده

#### Paper history:

Received 27 December 2013

Accepted in revised form 13 March 2015

#### Keywords:

Train-Rail-Bridge Interaction  
Rail Random Irregularity  
Wavelength Interval  
Sensitive Wavelength  
Dynamic Response  
High-Speed Railway Bridge

در این تحقیق، روش تولیدی برای بی‌نظمی تصادفی راه آهن با فاصله طول موج دلخواه (WI) توسعه یافته است، و دقت و بهره‌وری آن نشان داده شده است. سپس المان متحرکی از برهمکنش چرخ-راه آهن-پل حاصل گردید تا معادلات المان محدود حرکت برای سیستم تعاملی قطار-راه آهن-پل ایجاد کند، و فلوچارت مونتاژ و محاسبه برای معادلات سیستم داده می‌شود. با توجه به اصل زیر فاصله، اثرات بی‌نظمی با WI بزرگ و زیر فواصل روی پاسخهای دینامیک با یک مثال عددی تجزیه و تحلیل می‌شود و WI حساس هر پاسخ مورد بحث قرار می‌گیرد. نتایج نشان می‌دهد که شتاب پل و نیروی تماس هر دو حساسیت بیشتری نسبت به بی‌نظمی با WI (۵ ~ ۱ متر) دارند. بی‌نظمی با WI (۱ ~ ۰.۱ متر) تاثیر کمتری روی شتاب بدنه خودرو دارد اما به طور عمده منجر به شتاب راه آهن می‌شود. با این حال، همه بی‌نظمی‌ها با طول موج در فاصله (۱۵۰ ~ ۱ متر) می‌تواند تاثیر قابل توجهی روی شتاب بدنه خودرو داشته باشد. در همین حال، پرش گذرا از چرخ می‌تواند شبیه سازی شود و باید برای ارزیابی ریسک از خط خارج شدن ترن در نظر گرفته شود.

doi: 10.5829/idosi.ije.2015.28.04a.04