A Simple Method for Modeling Open Cracked Beam

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ABSTRACT

A simple method is proposed to model the open cracked beam structures. In this method, crack is modeled as a beam element. Hence cracked beam can be assumed to be a beam with stepped cross sections, and problem of determining natural frequency and mode shape of cracked beam, can be solved as determining these characteristics for a beam with different lengths and cross sections. With this work, it is not necessary to model crack as lumped flexibility model in according to fracture mechanics and related sciences to obtain crack stiffness, and this spring model of crack can be used for further analysis.

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1. INTRODUCTION

The presence of a crack in structural members decreases their stiffness and natural frequencies. We can group different models of cracked structure in three basic categories [1]: equivalent reduction in local stiffness (smeared crack models), lumped flexibility models, and continuous cracked bar and beam models. A smeared crack model usually consists of a finite element model of the structure in which the damage is represented by an equivalent reduction in the stiffness of a particular element or group of the elements. Better representations of the effect of a crack over a region may be achieved by developing special finite elements, as reported by Haisty and Springer [2] and Gounaris and Dimaroginas [3]. Ibrahim [4] proposes an elasto-plastic finite element capable of accounting for the plastic deformation at the crack tip. Lumped flexibility models are based on substructuring concept. The undamaged portions of the structure are modeled using standard techniques such as FEM, component mode synthesis, or partial differential equations, and the crack is represented by lumped springs or by a compliance matrix, usually derived from the expression for the strain energy release rate or stress intensity factor. The first investigations are attributed to Kirmser [5]. They represented the effect of a notch by equivalent forces and moments at the location of the geometrical discontinuity. Dimarogonas [6] proposed the derivation of compliance constants from fracture mechanics and used it for vibration analysis. Christides and Barr initially developed models for transverse vibrations of a symmetric, double-edge cracked Euler-Bernoulli beam [7] and for the torsional vibrations of a cracked bar [8]. Both models rely on the characterization of the stress concentration due to the crack by means of a decay function. The mixed variational theorem used is an extension of the Hu-Washizu stationary principle[9], and it is now referred in the literature as the Hu-Washizu-Barr [10] principle. Christides and Barr’s model was improved and extended in following years by Shen ad Pierre [11-13], first by introducing an alternative estimation of the decay parameter from 2D finite element models and later by applying the same ideas to develop a model for beams with a single edge breathing crack. Chondros and Dimarogonas [14-17] used a similar variational approach, but they derived the so-called crack functions from energy considerations and fracture mechanics concepts.

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Studies on nonlinear behavior of cracked beam using perturbation models have been proposed, e.g., by Gudmundson [18], Tsyfanskii [19], Plakhienko and Yasinskii [20] and Ballo [21]. A review on the vibration analysis for a damage occurrence of a cantilever beam is given by Jassim et al. [22]. AL-Shudeifat used finite element modeling approach to study asymmetric cracked rotor [23]. Dynamic behavior of single-cracked beams considering the effects of axial stiffness, and shear deformations is carried out by Gomes and Almeida [24]. The damage is modeled using a rotational spring that simulates the crack based on fracture mechanics theory. Caddemi and Morassi investigate mathematical modeling and exact solutions of multi-cracked Euler-Bernoulli beams [25]. Dixit and Hodges, gave a general formulation termed the “Unified Framework”, which yielded nth-order expressions governing mode shapes and natural frequencies for damaged elastic structures such as rods, beams, plates and shells of any shape [26]. Rakideh et al. presented identification of crack location in beams with different boundary conditions based on neural network method [27].

The most used method for modeling vibration characteristics of cracked beam is the lumped flexibility model. In this model, the cracked section is considered as a torsional (rotational) spring. The stiffness of this spring is obtained from fracture mechanics, in which boundary conditions, location, opening (or length) of the crack is not important. Here a method is presented for modeling of crack, in which all of these parameters are considered in modeling of crack. In the proposed method, crack is modeled as a beam element. Hence a cracked beam can be assumed to be a beam with stepped cross sections, and problem of determining natural frequency and mode shape of cracked beam, can be solved as determining these characteristics for a beam with different cross sections. With this work, it is not necessary to model crack in according to fracture mechanics. With this modeling method, position, width and height of cracks are directly included in modeling. With putting aside fracture mechanics from analysis, modeling of crack will be simple, and analytical solution of crack problem will be possible.

2. MODELING OF CRACKED BEAM

Figure 1 (a) shows a beam of length \( L \) and thickness \( h \), containing an edge crack of depth \( c \) located at a distances \( x_1 \) and \( x_2 \) from the left end, and the middle point of the crack is \( (x_1 + x_2)/2 \) which is assigned as \( eL \), Young’s modulus of elasticity \( E \) and mass density \( \rho \) are assumed to be constant.

3. LUMPED MODELING OF CRACK

The lumped flexibility form for modeling cracked beam is simply shown in Figure 1 (b). In this simple model, the crack is modeled by massless spring with rotational (torsional) stiffness of \( K_c \). The effects of discontinuities in axial and transverse displacements are considered to be negligible compared with those of discontinuity in bending slope. The cracked beam is divided into two sub-beams connected by the rotational spring with stiffness of \( K_c \) at the cracked section whose bending stiffness can be given as:

\[
K_c = \frac{1}{G}
\]

where, \( G \) is the flexibility due to the crack and can be derived as[28]:

\[
\frac{(1-v^2)K_1^2}{E(a)} - \frac{M^2}{2} \frac{dG}{da}
\]

\( M \), is the bending moment at the cracked section; \( K_1 \) is the stress intensity factor (SIF) under mode I bending load; and \( E \) is Young’s modulus at the crack tip. The magnitude of SIF can be obtained from the data given in the literature [29] through Lagrange interpolation technique as:

\[
K_1 = \frac{6M \sqrt{\pi \eta}}{bh^2} F(\eta), \quad \eta = \frac{a}{h} (\eta \leq 0.7)
\]

where, \( \eta \leq 0.7 \) implies that crack depth ratio changes from 0.0 to 0.7, and:

\[
F(\eta) = 0.6384\eta^2 - 1.035\eta^3 + 3.720\eta^4 - 5.1773\eta^5 + 7.553\eta^6 - 7.332\eta^7 + 2.4909\eta^8
\]
The expression of $F(\eta)$ for other values of Young’s modulus ratios can also be obtained by using Lagrange interpolation formula. Substituting Equation (4) into Equation (3) and then into Equation (2) and integrating the results give:

$$G = \int_0^L 2\pi (1-\nu^2) \eta h^2 (\eta) \, d\eta$$

(5)

From Equations (5) and (1), the bending stiffness of the cracked section required for lumped flexibility modeling can be determined.

4. CRACKED EULER-BERNOUlli BEAM WITH LUMPED FLEXIBILITY MODELS

Consider an elastic cracked Euler-Bernoulli beam of length $L$, uniform cross-section area $A$ and moment of inertia $I$, with a crack at middle position of $eL$ as shown in Figure 1. Differential equations of motion and boundary conditions on two segments of the beam are as follows:

$$EIw'' + \rho A w = 0, \quad 0 < x < eL, \quad eL < x < L$$

(6)

$$EI\psi'' + \rho w = 0, \quad 0 < x < eL, \quad eL < x < L$$

(7)

where, $w(x,t)$ is the beam transverse deflection and $\rho$ mass density per unit length, $A$ cross section area of beam, $E$ Young’s modulus of elasticity and $I$ is area moment of Inertial. Also, at crack location:

$$w_{L} = w_{eL} = w_{t, eL}, \quad EIw''_{L} = 0$$

(8)

Following dimensionless parameter are defined:

$$\xi = \frac{x}{L}, \quad \tau = \frac{EI}{\rho A e^2}, \quad \eta_i = \frac{w_i}{L}, \quad (i=1,2)$$

(9)

By substituting dimensionless parameter from Equation (9), in Equations (6) and (7), assuming harmonic solution as $\eta_i(\tau, \xi) = A_i e^{\lambda_i \xi}$, the related characteristic equation will be obtained with eigenvalues of $\lambda_i^2 = \pm \pi^2 = \pm \beta_i^2$, $\beta_i (i = 1,2)$. Then mode shape of beam has following form [30]:

$X_i(\xi) = C_{i, 1} \cos \beta_i \xi + C_{i, 2} \sin \beta_i \xi$

(10)

Values of $\beta_i$ and the ratios of coefficients of mode shapes of $C_i/C_1$ can be obtained from applying boundary and compatibility conditions at crack location.

5. CRACKED TIMOSHENKO BEAM WITH LUMPED FLEXIBILITY MODELS

Differential equations of motion and boundary conditions for transverse deflection $w$ and slope of $\psi$ for beam in according to Timoshenko beam theory is as follows [30]:

For $0 < x < eL$ and $eL < x < L$:

$$\rho A w'' - kAG(w'' - \psi'' = 0)$$

and boundary conditions are:

$$EI\psi''|_{x=0} = 0, \quad kAG(w'' - \psi''|_{x=L})$$

(11)

(12)

and at crack location:

$$w(t, eL) = w(t, eL), \quad \psi''|_{x=0} = \frac{K_c}{EI} w''|_{x=0}$$

(13)

where, $w(x,t)$ is transverse deflection, and $\psi(x,t)$ is the slope of the deflection curve, $G$ indicates shear modulus of elasticity. $k$ represents the shear correction factor, which is assumed to be 5/6. Following dimensionless quantities are introduced:

$$\xi = \frac{x}{L}, \quad \eta = \frac{EI}{\rho A e^2}, \quad \tau = \frac{\theta \tau}{\eta}, \quad s = \theta \tau, \quad \tau = \frac{EI}{\rho A e^2},$$

(14)

With assuming a harmonic solution in following form $\eta_i(\tau, \xi) = X_i(\xi) e^{\lambda_i \xi}$ and $\psi_i(\tau, \xi) = \Psi_i(\xi) e^{\lambda_i \xi}$ and obtaining $X_i(\xi)$ in terms of $X_i(\xi)$, we have [30]:

$$\frac{d^2}{dx^2} + \pi^2 \left(1 + \frac{1}{\tau s} \right) \frac{d^2}{dx^2} + \pi^2 \left(1 + \frac{1}{\tau s} \right) X_i = 0$$

(15)

in the form of $X_i = A_i e^{\lambda \xi}$, the characteristic equation for determining of mode shape eigenvalue of $\lambda$, and mode shape will be obtained as:

$$\lambda_{i, 2} = \frac{\pi^2 \left(1 + \frac{1}{\tau s} \right)}{4 \tau s}$$

(16)

$$X_i(\xi) = C_{i, 1} \cos \lambda_i \xi + C_{i, 2} \sin \lambda_i \xi + C_{i, 3} \cos \lambda_i \xi + C_{i, 4} \sin \lambda_i \xi$$

(17)

Accordingly the boundary conditions are:
The equation of motion for Euler-Bernoulli beam is given by:

\[ X_i(\varepsilon) - X_2(\varepsilon) + X_3(\varepsilon) - X_4(\varepsilon) - \frac{1}{K_{cb}} \Psi_i(\varepsilon) = 0 \]  

(18)

\[ \Psi_2(\varepsilon) - \Psi_3(\varepsilon) = 0 \]

\[ X_2(\varepsilon) - X_3(\varepsilon) - \Psi_2(\varepsilon) + \Psi_3(\varepsilon) = 0 \]

where, \( K_{cb} \) is given by following equation:

\[ K_{cb} = \frac{L}{\cos \theta_i} \]  

(19)

### 6. CRACKED EULER-BERNOUlli BEAM WITH CRACKS AS STEPPED CHANGE IN BEAM’S CROSS SECTION

As pointed out in the introduction section, here crack is modeled as a beam element with different cross sections other than intact section of beam. Hence a cracked beam as shown in Figure 1 can be modeled as three interconnecting beams, with different areas, moment of inertias, heights and lengths. Then intact and cracked segments of beam has area moment of inertia of

\[ I_1 = \frac{1}{12} bh_i^3 \quad \text{and} \quad I_c = \frac{1}{12} (h - h_i)^3 \]

respectively, where \( h \) is the thickness of the beam and \( h_i \) is the depth of the crack. The length of cracked beam segment is equal to its opening, i.e. \( x_2 - x_1 \). The equation of motion for Euler-Bernoulli cracked beam is similar to Equations (6) and (7), and related mode shape solution is similar to Equation (10). Then for a beam according to Figure 1 and with one crack, we have (from left end of beam):

\[ d^4X_i \quad d\varepsilon^4 \quad \theta_i^4 \quad \theta_i = \frac{m_i l}{E I} \]  

(20)

\[ \Psi_i(\varepsilon) = C_{i1} \cosh \lambda_{i1} x_i + C_{i2} \sinh \lambda_{i1} x_i 
+ C_{i3} \cos \lambda_{i2} x_i + C_{i4} \sin \lambda_{i2} x_i \quad \text{for} \quad \lambda_{i1} \leq \lambda_{i2} \]

For cracked section \( h_i = h - h_c \). Since the value of \( \omega \) (natural frequency) is high, for simplicity in numerical calculation it is better to select \( \theta_i \) as unknown. The ratio of \( \theta_i / \theta_1 \) can be written as:

\[ \theta_i = \frac{\theta_1}{\sqrt{h_i}} = \frac{\theta_1}{\sqrt{h}} \]  

(21)

and boundary and compatibility conditions at two ends of beam and location of cracks are:

\[ X_2(\varepsilon) = X_3(\varepsilon), \quad X_2(\varepsilon) = X_3(\varepsilon) \quad \text{at} \quad x = x_1, x_2 \]

\[ E_1X_2(\varepsilon) = E_2X_2(\varepsilon), \quad E_1X_3(\varepsilon) = E_2X_3(\varepsilon) \quad \text{at} \quad x = x_2, x_3 \]

(22)

\[ l_2X_2(\varepsilon) = l_3X_2(\varepsilon), l_2X_3(\varepsilon) = l_3X_3(\varepsilon) \]

### 7. CRACKED TIMOSHENKO BEAM WITH CRACKS AS STEPPED CHANGE IN BEAM’S CROSS SECTION

Following the same procedure as described for Euler-Bernoulli beam, and with considering the crack as a beam with length equal to its opening and thickness equal to the thickness of the intact section minus depth of beam, following modeling of cracked beam in according to Timoshenko beam theory is given. The equation of motion for Timoshenko beam is given by Equations (11) and (12), and related mode shape solution is also given by Equation (17). Then for a beam in accordance to Figure 1 and with one crack, we have:

\[ (i = 1, 2, 3, 0 \leq \xi < \xi_1, \xi_1 < \xi < \xi_2, \xi_2 < \xi \leq 1) \]

\[ X_1(\varepsilon) = \frac{C_{i1} \cosh \lambda_{i1} x_i + C_{i2} \sinh \lambda_{i1} x_i}{\sinh \lambda_{i1} x_i + \cosh \lambda_{i2} x_i} + C_{i3} \cos \lambda_{i2} x_i + C_{i4} \sin \lambda_{i2} x_i \]

\[ \Psi_1(\varepsilon) = \left\{ \begin{array}{ll} \frac{\bar{a}^2}{\eta_i} + \lambda_{i1} & C_{i1} \sinh \lambda_{i1} x_i + C_{i2} \cosh \lambda_{i1} x_i \\
\frac{\bar{a}^2}{\eta_i} - \lambda_{i2} & C_{i3} \sin \lambda_{i2} x_i - C_{i4} \cos \lambda_{i2} x_i \end{array} \right\} \]

(23)

Boundary and compatibility conditions for Timoshenko beam are:

\[ EPP_1[\delta \Psi_1] = 0, \quad EPP_2[\delta \Psi_2] = 0 \]

\[ kAG(X_1 - \Psi_1) \delta X_1 = 0, \quad kAG(X_2 - \Psi_2) \delta X_2 = 0 \]

\[ X_1(\xi_1) = X_2(\xi_1), \quad \Psi_1(\xi_1) = \Psi_2(\xi_1) \]

\[ E_1P_1[\delta \Psi_1] = E_2P_2[\delta \Psi_2] \]

\[ kAG(X_2 - \Psi_2) \delta X_2 = 0 \]

(24)

\[ X_2(\xi_2) = X_3(\xi_2), \quad \Psi_2(\xi_2) = \Psi_3(\xi_2) \]

\[ E_2P_2[\delta \Psi_2] = E_3P_3[\delta \Psi_3] \]

It should be noted that formulation of the intact beam is similar to cracked beam, with putting aside the compatibility boundary conditions of cracked beam [30].

### 8. RESULTS AND DISCUSSIONS

Now after presenting the proposed crack modeling method and describing lumped flexibility modeling of crack, the results obtained from these two methods and comparison between them are given for Euler-Bernoulli and Timoshenko beam theories. Steel-high strength
As expected with increasing depth of crack, natural frequencies of cracked beam are reduced, and with increasing or decreasing the rigidity of boundary conditions, natural frequencies are changed accordingly.

### TABLE 2. Three first natural frequencies for CF beam, for crack location $\eta = 0.5$, crack opening $d_c / L = 0.002$ and various crack depths.

<table>
<thead>
<tr>
<th>$\eta / h$</th>
<th>Intact beam [30]</th>
<th>Presented method</th>
<th>Lumped modeling [31]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EB</td>
<td>TB</td>
<td>EB</td>
</tr>
<tr>
<td>0.1</td>
<td>3.516</td>
<td>3.51</td>
<td>3.5</td>
</tr>
<tr>
<td>0.2</td>
<td>22.03</td>
<td>21.95</td>
<td>22.0</td>
</tr>
<tr>
<td>0.3</td>
<td>61.69</td>
<td>61.19</td>
<td>61.69</td>
</tr>
<tr>
<td>0.4</td>
<td>61.69</td>
<td>61.19</td>
<td>61.69</td>
</tr>
</tbody>
</table>

### TABLE 3. Three first natural frequencies for CS beam, for crack location $\eta = 0.5$, crack opening $d_c / L = 0.002$ and various crack depth.

<table>
<thead>
<tr>
<th>$\eta / h$</th>
<th>Intact beam [30]</th>
<th>Presented method</th>
<th>Lumped modeling [31]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EB</td>
<td>TB</td>
<td>EB</td>
</tr>
<tr>
<td>0.1</td>
<td>15.4</td>
<td>15.3</td>
<td>15.4</td>
</tr>
<tr>
<td>0.2</td>
<td>15.4</td>
<td>15.3</td>
<td>15.4</td>
</tr>
<tr>
<td>0.3</td>
<td>15.4</td>
<td>15.3</td>
<td>15.4</td>
</tr>
<tr>
<td>0.4</td>
<td>15.4</td>
<td>15.3</td>
<td>15.4</td>
</tr>
</tbody>
</table>

As expected with increasing depth of crack, natural frequencies of cracked beam are reduced, and with increasing or decreasing the rigidity of boundary conditions, natural frequencies are changed accordingly.
In Table 4, the effect of varying crack opening on three first natural frequencies of CC beam for crack location of $\eta=0.5$ and depth of $h/h=0.1$ is shown. From this table, it can be seen that for lower frequencies, natural frequencies predicted in according to lumped flexibility model of Euler-Bernoulli beam is greater than stepped modeling. While in higher models, lumped flexibility model gives higher prediction of natural frequency in comparison to present modeling of crack.

The effect of crack opening on natural frequency of Euler-Bernoulli’s beam in according to these two mentioned methods of crack modeling will be more clear, with Tables 5. and 6. In these figures, three first natural frequencies of beams with different boundary conditions with one crack at location of $\eta=0.5$, depth of $h/h=0.5$ and different amounts of crack openings are shown. As expected, in all cases with increasing the crack opening, the amounts of natural frequencies are reduced. Now the effects of varying crack location on natural frequencies of beam are investigated. Table 7 shows the effect of crack location on natural frequency of beam. As seen from these tables, the differences between these two presented methods with varying the location of crack is more apparent in this case, i.e. change in location of crack. The amounts of differences are dependent to boundary conditions, and the amount of proximity of crack to the nodal point of related intact mode shape. In Figure 2, mode shapes and their slopes for three first natural frequencies of cracked CC beam obtained from stepped modeling of crack are shown. As seen from this figure, mode shapes and their slopes are continuous in cracked section of beam, while there are qualitative change in the shape of slope of mode shapes in cracked section of beam, i.e. $\ddot{x}^2/\ddot{w}^2$ and $\ddot{x}^3/\ddot{w}^3$ are discontinuous in the cracked domain.

**Table 4.** Three first natural frequencies for clamped-clamped beam (CC) for different crack openings, $\eta = 0.5$ and $h/h=0.1$.

<table>
<thead>
<tr>
<th>$d/L$</th>
<th>Intact beam [30]</th>
<th>Lumped modeling [31]</th>
<th>Present method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EB</td>
<td>TB</td>
<td>EB</td>
</tr>
<tr>
<td>0.001</td>
<td>22.3</td>
<td>22.2</td>
<td>22.3</td>
</tr>
<tr>
<td>120.9</td>
<td>118.8</td>
<td>120.5</td>
<td>118.5</td>
</tr>
</tbody>
</table>
| 0.01 | 61.6 | 61.0 | 61.6 | 61.0 | 61.6 | 61.0 | 0.120 | 118 | 120 | 118 | 120 | 118 | 120 | 118 | 120 | 118

**Table 5.** Three first natural frequencies for CF beam with different crack openings, $\eta = 0.5$ and $h/h=0.5$.

<table>
<thead>
<tr>
<th>$d/L$</th>
<th>Intact beam [30]</th>
<th>Present method</th>
<th>Lumped method [31]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EB</td>
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<td>EB</td>
</tr>
<tr>
<td>0.001</td>
<td>3.51</td>
<td>3.51</td>
<td>3.5</td>
</tr>
<tr>
<td>61.6</td>
<td>61.1</td>
<td>61.6</td>
<td>61.1</td>
</tr>
<tr>
<td>3.51</td>
<td>3.51</td>
<td>3.5</td>
<td>3.5</td>
</tr>
<tr>
<td>0.003</td>
<td>22.0</td>
<td>21.9</td>
<td>21.6</td>
</tr>
</tbody>
</table>

**Table 6.** Three first natural frequencies for CS beam with different crack openings, $\eta = 0.5$ and $h/h=0.5$.

<table>
<thead>
<tr>
<th>$d/L$</th>
<th>Intact beam [30]</th>
<th>Present method</th>
<th>Lumped Method [31]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>EB</td>
<td>TB</td>
<td>EB</td>
</tr>
<tr>
<td>15.4</td>
<td>15.3</td>
<td>15.3</td>
<td>15.3</td>
</tr>
<tr>
<td>104.2</td>
<td>102.8</td>
<td>103.6</td>
<td>102.3</td>
</tr>
<tr>
<td>15.4</td>
<td>15.3</td>
<td>15.2</td>
<td>15.2</td>
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<td>15.4</td>
<td>15.3</td>
<td>15.2</td>
<td>15.1</td>
</tr>
<tr>
<td>104</td>
<td>102</td>
<td>102</td>
<td>101</td>
</tr>
</tbody>
</table>

Finally some results are presented to show the validity of the presented method with experimental or other available results given in literature. For this purpose a model with geometric and mechanical properties of $E=207$ Gpa, $\rho=7860$ kg/m$^3$, $\nu=0.3$, $b=12.7$ mm, $h=12.7$ mm, $L=400$ mm, $\eta=0.3$, and $d/L=5$ are selected [32]. Experimental results for these parameters are given elsewhere [32]. The obtained results are shown in Table 8. As seen from this table, a good agreement between the results of the proposed method and the experimental model exists, and this shows the validity of the presented modeling of crack. The proposed modeling approach can be used to investigate the beam with different crack shapes, such as circular, elliptical and V shape cracks. Figure 3 shows a beam with V shape crack; there is no analytical solution for these problems, so to obtain the natural frequencies the Galerkin method should be used. Table 9 shows three first natural frequencies of Euler-Bernoulli cantilever cracked beam with V shape crack shown in Figure 3. For modeling crack, stepped beam
modeling presented is used and beam is modeled in
according to Euler-Bernoulli theory. Galerkin method is
used for obtaining natural frequencies.

**TABLE 7.** Three first natural frequencies for CC, with
different locations of crack, crack opening \( d_c/L = 0.002 \) and
depth of \( h_c/h = 0.5 \).

<table>
<thead>
<tr>
<th>Crack location</th>
<th>Intact beam ([30])</th>
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<th>Lumped modeling ([31])</th>
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<tr>
<td></td>
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<tr>
<td>0.1</td>
<td>22.3</td>
<td>22.2</td>
<td>22.2</td>
</tr>
<tr>
<td>0.2</td>
<td>61.6</td>
<td>61.0</td>
<td>61.5</td>
</tr>
<tr>
<td>0.4</td>
<td>61.6</td>
<td>61.0</td>
<td>61.4</td>
</tr>
</tbody>
</table>

**TABLE 9.** Three first natural frequencies for CF, with
different crack openings and depth and crack location \( \eta = 0.5 \).

<table>
<thead>
<tr>
<th>( \frac{h_c}{h} )</th>
<th>Intact beam</th>
<th>Different crack openings</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cracked beam</td>
<td></td>
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<td>3.5158 3.5159</td>
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<td>22.0344</td>
<td>22.0294 22.0319</td>
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<tr>
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<td>61.6972</td>
<td>61.6972</td>
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As clear from obtained results, natural frequencies
decrease with increasing crack depth and crack opening,
and their values are less than intact beam.

9. CONCLUSION

There are different methods for modeling of cracks in
the literature. In the current work a simple method is
presented for modeling of crack. This method is based
on modeling crack as a stepped change in cross section
of beam. With this modeling approach, crack will be a
beam element, with specific depth, area, area moment
of inertia, and length, and similar to intact beam, its
solution will be obtained analytically. Based on the
proposed idea, beams with different number of cracks
and boundary conditions are simulated and results are
compared with well-known lumped flexibility modeling
of crack, in which crack is modeled as massless
rotational spring. The obtained results show good
conformity of the stepped modeling of crack with
results obtained from lumped flexibility model. The
presented modeling approach is very simple, can simply
be extended to structures with different shapes of cracks such as V or circular crack shapes that do not have analytical solutions.

10. REFERENCES

A Simple Method for Modeling Open Cracked Beam
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