Dynamic Behavior Analysis of a Planar Four-bar Linkage with Multiple Clearances Joint

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ABSTRACT

Clearances in the joints are inevitable in practice due to tolerances, and defects arising from design and manufacturing. In the presence of clearance at a joint, a mechanism gains some additional, uncontrollable degrees of freedom which are the source of error. Moreover, joints undergo wear and backlashes and so cannot be used in precision mechanisms. In this study, the dynamic behavior of a planar mechanism with revolute joints, in the presence of clearances is investigated. A continuous contact force model, based on the elastic Hertz theory together with a dissipative term, is used to evaluate the contact forces. Moreover, using a new contact model, the dynamic characteristics of planar mechanical system with multiple revolute joints in the presence of clearance are analyzed. Numerical results for four-bar linkage with one, two and three revolute clearance joints are presented and compared.

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1. INTRODUCTION

Parallel mechanisms have attracted the attention of many researchers. These mechanisms also have growing applications in robotics, machine tools, positioning systems, measurement devices, and so on [1-3]. It has been proved that such a closed-loop mechanism has great potential and advantages over the traditional opened-loop mechanism. The main advantage of parallel mechanisms is the high positioning accuracy due to their inherent rigidity [4]. But, due to joint clearance, a parallel mechanism’s end-effector exhibits position and orientation (or collectively referred to as pose) errors of various degrees [5].

Clearances in mechanisms are unavoidable due to wear, assembly and manufacturing tolerances. Performances of mechanisms in reality are deviated from the ideal mechanisms due to joint clearances [6]. In addition, joint clearance can lead to impulsive forces. These forces not only create increasing vibration amplitude, but also reduce system reliability, stability and life. Thus, joint clearance changes the dynamic response of a system. This justifies the deviations between the numerical predictions and the experimental measurements [6]. Therefore, these errors must be tightly controlled to ensure the desired performances. The problem of modeling joints with clearance in the context of multi body dynamics has been the subject of many studies during the last few decades [6-10]. Most of the reported work is devoted to the analysis of linkages [11-21], while the reported work on the synthesis of linkages is comparatively limited [22-24].

The main purpose of the present work is to study the dynamic behavior of planar four-bar linkage with rigid links in the presence of clearance in the multiple revolute joints. Different cases are considered with the purpose of performing a parametric study for quantifying the influence of the number of clearance joints on the dynamic responses of multi body systems with multiple clearance joints. In joints with clearances, if there is no lubricant, the journal moves freely inside the bearing boundaries until it contacts with the bearing. When the journal and the bearing are in contact, deformation takes place at the contact zone resulting in a contact force normal to the plane of collision.

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This force can be formulated by a non-linear continuous model proposed by Lankarani and Nikravesh [16]. The friction effects due to relative tangential velocity on the contact zone are also modeled according to the modified Coulomb friction law [25]. Therefore, the normal and tangential forces are introduced into the equations of motion of the system for the contact mode. A numerical simulation is included to illustrate the effects of clearances in several joints.

2. LITERATURE REVIEW

Dubowsky and Freudenstein [11, 12] formulated an impact pair model to predict the dynamic response of an elastic mechanical joint with clearance. Earles and Wu [13] introduced a model based on permanent contact condition. In their model, clearance is replaced by a mass-less virtual link that connects the journal center to the bearing center. Grant and Fawcett [14] analyzed the effects of lubrication, material properties and clearance size on the contact loss in a four-bar mechanism having joint clearance. Experimental results confirmed the validity of the proposed theoretical approach. Bengisu et al. [15] developed a separation parameter for a four bar mechanism based on a zero-clearance analysis. They also predicted contact-loss in a mechanism with multiple joint clearances. Using the general trend of the Hertz contact law, Lankarani and Nikravesh developed a contact force model in which a hysteretic damping function was incorporated with the intent to represent the energy dissipated during the impact [16]. Flores and Ambro'sio [17] used contact force approach to model and simulate the performance of a slider–crank mechanism with one clearance-joint. Also, Flores et al. [18] studied the effect of friction between the journal and the bearing, using a modified coulomb’s friction law. Schwab et al. [19] compared several models of contact forces. They analyzed and simulated a slider–crank mechanism with one clearance-joint of rigid or flexible connecting rod, for both dry and lubricated contact conditions. Ting et al. [20] studied the effects of the joint clearance on position and orientation deviation of mechanisms and robotic manipulators, using mass-less virtual link approach to model the clearances. Zhu and Ting [21] made the uncertainty analysis of planar and spatial robots with clearance joints. Feng et al. [22] developed an optimization method to control the inertia forces by re-distribution of masses of the moving links in planar mechanisms, in the presence of clearances at joints. Also, Erkaya and Uzmay [23] used a Genetic Algorithm Method to find the optimized link parameters for the path generation problem in the presences of clearance. Moreover, Zhang and Huang [24] made a robust tolerance design for function generation mechanisms.

3. MODELING OF REVOLUTE JOINTS WITH CLEARANCE

Figure 1(a) shows the typical configuration of a revolute joint with clearance. The joint elements are the bearing and journal, withdiameters R_b and R_j, respectively. The radial clearance, c, is the difference between their radii. It is noteworthy that the existence of clearance in revolute joints introduces two extra degrees of freedom, that is, the horizontal and vertical displacements of the center of the journal and consequently, the journal and bearing can freely move relative to each other. They experience three different modes of relative motion; namely, contact mode, free flight mode and impact mode, as illustrated in Figure 1(b). In the contact mode, the journal and the bearing are in permanent contact and only a sliding or rolling relative motion is assumed. Clearly, this mode is terminated when the journal loses the contact with the bearing, i.e., the free flight mode is started. If this is the case, the journal can move freely inside the bearing boundaries; i.e., there is no contact between the journal and the bearing. Therefore, no reaction force is developed at the joint.

Finally, in the impact mode, which occurs at the termination of the free-flight mode, there is an impact force between the journal and bearing. This causes a discontinuity in the kinematic and dynamic characteristics of the system. At the termination of the impact mode, the journal can enter either in free flight or in contact modes. It is noteworthy that in the contact mode, there is the normal contact force F_N and the tangential contact force F_T between the journal and bearing as illustrated in Figure 2(b), while, in the case of no contact between the journal and the bearing, the contact forces vanishes [1, 2], i.e.,

\[
F = F_N + F_T,
\]

\[
\begin{align*}
F &= 0 \text{ if } \delta < 0 \\
F &= 0 \text{ if } \delta \geq 0
\end{align*}
\]
where $\delta$ is the relative penetration depth as depicted in Figure 2(a) and is given as:

$$\delta = e - c$$  \hspace{1cm} (2)

in which $e$ is the absolute eccentricity defined as:

$$e = \sqrt{\Delta X^2 + \Delta Y^2}$$  \hspace{1cm} (3)

where $\Delta X$ and $\Delta Y$ represent, respectively the horizontal and vertical displacements of the journal with respect to the bearing.

Moreover, the magnitude of the joint force can be obtained from the Hertzian contact theory under the assumption that the dimension of the contact region between the journal and bearing is much smaller than the radius of the bearing. The contact force model proposed by Lankarani and Nikravesh [10] that accounts for both the elastic and damping effects is employed in the present work. It is noteworthy that the damping effect is associated with the energy dissipated during the impact together with the dissipative effect associated with the Coulomb friction on the contact surface [23]. Thus, the normal reaction force at the revolute joint in the presence of clearance can be expressed by [6, 16]:

$$F_N = K\delta^* + D\delta$$  \hspace{1cm} (4)

where $K$ is the generalized stiffness parameter, depending on the geometry of the contacting surfaces and their physical properties, while $D$ is the hysteresis damping coefficient. Therefore, the first term in the right hand side of Equation (4) represents the elastic force based on the Hertz contact law while the second term accounts for the damping force due to energy dissipation. Also, $n$ is a constant depending on the material properties of the contact surface. For the contacting bodies, $K$ is given as [6]:

$$K = \frac{4}{3[\sigma_i + \sigma_j]} \left[ \frac{K_i R_i}{R_i + R_j} + \frac{K_j R_j}{R_i + R_j} \right]$$  \hspace{1cm} (5)

Here, $\sigma_i$ and $\sigma_j$ are defined as:

$$\sigma_i = \frac{1 - v_j^2}{E_i}, \quad k = i, j$$  \hspace{1cm} (6)

where $v_j$ and $E_j$ are the Poisson's ratio and the Young's modulus of each body, respectively. Moreover, the hysteresis damping coefficient is given by [6]:

$$D = \chi \delta^*$$  \hspace{1cm} (7)

in which the hysteresis factor $\chi$ is defined as:

$$\chi = \frac{3K(l - c_i^2)}{4\delta_i}$$  \hspace{1cm} (8)

where $\delta_i$ and $c_i$ are the initial impact velocity and the restitution coefficient, respectively. Substituting the values of $K$ and $D$ from Equations (5) and (7) into Equation (4) yields [6]:

$$F_N = K\delta^* \left[ 1 + \frac{3(l - c_i^2)}{4\delta_i} \right]$$  \hspace{1cm} (9)

In addition, for tangential contact force $F_T$, Ambrósio presented a modification for the Coulomb's friction law expressed as follows [25]:

$$F_T = -c_f c_i F_N \frac{v_T}{v_i}$$  \hspace{1cm} (10)

where $v_T$ is the relative tangential velocity; $c_f$ is the friction coefficient and the dynamic correction coefficient $c_d$ is given as:

$$c_d = \begin{cases} 0 & |v_i| < v_0 \\ \frac{|v_i| - v_0}{v_i - v_0} & v_0 \leq |v_i| \leq v_i \\ 1 & |v_i| > v_i \end{cases}$$  \hspace{1cm} (11)

in which $v_0$ and $v_i$ are given bounds for the tangential velocity. Finally, in the contact mode, $Q_v$ and $\psi$ are obtained from the following equation, while in the non-contact mode $Q_v$ is zero [6]:

$$Q_v = K_n \delta^* \left[ 1 + \frac{3(l - c_i^2)}{4\delta_i} \right], \quad \psi = \alpha + \varphi$$  \hspace{1cm} (12)

in which:

$$K_n = \sqrt{1 + c_i^2 + c_f^2} K$$  \hspace{1cm} (13)

where $\omega_j$ and $\omega_B$ are the journal and the bearing angular velocities, respectively.

![Figure 2. (a) Revolute joint with clearance (b) Joint contact forces](image)

### Table 1. Geometric and inertial properties of the four-bar mechanism

<table>
<thead>
<tr>
<th>Link Type</th>
<th>Length (m)</th>
<th>Mass (kg)</th>
<th>Moment of inertia (kg.m$^2$)</th>
<th>u (m)</th>
<th>v (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>crank</td>
<td>0.03</td>
<td>0.3</td>
<td>1.82 $10^{-6}$</td>
<td>0.015</td>
<td>0</td>
</tr>
<tr>
<td>coupler</td>
<td>0.18</td>
<td>0.0737</td>
<td>2.02 $10^{-4}$</td>
<td>0.09</td>
<td>0</td>
</tr>
<tr>
<td>follower</td>
<td>0.12</td>
<td>0.049</td>
<td>6.19 $10^{-5}$</td>
<td>0.06</td>
<td>0</td>
</tr>
<tr>
<td>Bearing radius (mm)</td>
<td>Clearance size (mm)</td>
<td>Restitution coefficient</td>
<td>Young's modulus (GPa)</td>
<td>Poisson's ratio</td>
<td>Friction coefficient</td>
</tr>
<tr>
<td>---------------------</td>
<td>---------------------</td>
<td>------------------------</td>
<td>-----------------------</td>
<td>----------------</td>
<td>---------------------</td>
</tr>
<tr>
<td>10</td>
<td>1</td>
<td>0.9</td>
<td>207</td>
<td>0.3</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2. The clearance geometric and material properties

5. CASE STUDY

In this section, the four-bar linkage is used to demonstrate the effect of clearance joint on the dynamics of the mechanism. The kinematic model of the mechanism with three clearance joints is depicted in Figure 3. The links are assumed to be rigid. This mechanism will be analyzed here, in several cases; i.e., "a". One clearance joint, (only joint A is with clearance and the other joints are ideal), "b". Two clearance joints, (joints A and B are with clearances and joints A₀ and B₀ are ideal), "c". Three clearance joints, (all the joints have clearances, except joint A₀ that is ideal).

It is noteworthy that each clearance, add two degree-of-freedoms (DOF) to the mechanism. Therefore, the 1-dof four bar linkage in the ideal case has 3-DOF, 5-DOF and 7-DOF, for the cases "a", "b" and "c", with the followings generalized coordinates, respectively.

\[ q_i = [\theta_2, \theta_3, \theta_4] \]
\[ q_i = [\theta_2, x_{G2}, Y_{G2}, \theta_3, \theta_4] \]
\[ q_i = [\theta_2, x_{G3}, Y_{G3}, \theta_3, \theta_4, x_{G4}, Y_{G4}] \]

where \( \theta_i \) (i=2,3,4) are the angles of the links relative to the horizontal axis; \( x_{Gi} \) and \( Y_{Gi} \) (i=3,4) are the coordinates of the mass centers of the connecting rod and the follower in the fixed coordinate system.

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The geometric and inertial properties of the mechanism are listed in Table 1; while the bearing radius, the magnitude of the clearance and material properties associated with the journal and the bearing are listed in Table 2. The clearances and bearings radii are the same for all the joints, while the length of \( A_0B_0 \) is 0.2m.

For the numerical solution of the ordinary differential equations (ODE) dynamic equations, we used Matlab software. Matlab offers some functions, namely ode45, ode15s, ode113, ... to solve ODE. Each of these solvers has an order of accuracy and they are suitable for some types of problems (stiff or non-stiff). Using these solvers, one can change the error tolerance (RelTol, AbsTol, etc.) for the accuracy of the solution. By decreasing the RelTol and AbsTol, accuracy of the solution can be increased. Also, the "InitialStep" must be selected. This property specifies the size of the first step size of the solver. Due to the high input revolution velocity of the input, with the default value of the initial step of ODE Integrator, the journal may penetrate into the bearing. Hence, it is better to restrict this value. In addition, one can specify bounds on the sizes of subsequent time steps, as well. In the present study, "ode15s" solver, which is more suitable for problems with stringent error tolerances or for solving computationally intensive problems, has been used.
Furthermore, for the tolerances we used $1e-20$ for 'AbsTol', $1e-12$ for 'RelTol' and $1e-10$ for 'InitialStep'. Finally, we include a dynamic simulation for the mechanism in these three cases wherein the crank is rotated with a constant angular velocity of 5000 rpm, while initially the journal and the bearing centers are coincident. The values of clearance vector in joint A, for the three cases, are plotted in Figure 4.

![One clearance joint](image1)
![Two clearance joints](image2)
![Three clearance joints](image3)

**Figure 4.** Values of clearance vectors in joint A

![Two clearance joints](image4)
![Three clearance joints](image5)

**Figure 5.** Contact force in the clearance joint A

It has been revealed that for one clearance joint, the value is a smooth function of the time, while this is not true for two other cases. Therefore, one can expect higher contact forces in these cases. The values of the contact forces in joint A are depicted in Figure 5 and compared with the case without clearance.

The plots clearly show that in the mechanism with joint clearance, the contact forces experience some sharp changes for cases "b" and "c". Further, the value of the contact force increases with increasing the number of clearance joints. It is noteworthy that the maximum value of the contact force in joint A, for cases "a", "b" and "c" is 1.4, 3.7 and 5.2-folds compared to the case without clearance. Therefore, one should expect much worse dynamic performance for the mechanisms with more clearance joint.

The value of the required input torque is depicted in Figure 6 which indicates that the required input torque experiences sudden changes.

![One clearance joint](image6)

**Figure 6.** Contact force in the clearance joint A

Additionally, the value of angular acceleration of the coupler and the follower are depicted in Figures 7 and 8, respectively. The plots for case "b" and "c" show sharp changes, while for case "a" the plots are smooth. It is noteworthy that higher values of the accelerations lead to higher inertia forces which act as the source of shocks and vibrations.

All the plots shown in Figures 4 - 8 clearly reveal the undesirable effect of the increase of the number of clearance joints on the dynamic performance of the four-bar linkage.
6. CONCLUSIONS

The planar four-bar mechanism has been used in this work as a demonstrative application example. Numerical results for the four-bar mechanism with several revolute clearance joints have been presented and discussed. Using a nonlinear continuous contact
force model, the normal contact forces in the revolute joints with clearance have been formulated. While the tangential friction forces have been modeled using a modified Coulomb friction coefficient. From numerical simulation, it can be concluded that the system response substantially changes when the number of clearance joints increase. These changes are mainly due to higher values for the contact forces and links accelerations that cause shocks and vibrations in the mechanism.

7. REFERENCES


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