Application of Multi-objective Optimization for Optimization of Half-toroidal Continuously Variable Transmission

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ABSTRACT

Among different goals defined in vehicle design process, fuel consumption (FC) is one of the most important objectives, which significantly has taken into account lately, both by the customers and vehicle manufacturers. One of the significant parameters which impacts the vehicle FC is the efficiency of vehicle's power train. In this paper, a half-toroidal continuously variable transmission (CVT) is considered as the vehicle power train. Its efficiency is sensitive to its geometry, and variation of its geometry can result the vehicle FC reduction. On the other hand, geometry variation affects its weight and fatigue life, which are considered as major contributing factors in the power train design. This paper aims to optimize half-toroidal CVT in order to minimize its weight, FC of the vehicle equipped with it, and provide the desired fatigue life. After introducing half-toroidal CVT, the method of calculating the mentioned objective functions is presented. A specific importance weight for each objective is considered. These weights are functions of their related objectives. A single objective optimization is implemented for each objective, and their optimal values are obtained. Then, these objectives are optimized simultaneously using Global Criterion method.

KEYWORDS: Continuously Variable Transmission, Half-toroidal, Fuel Consumption, Fatigue Life, Multi-objective Optimization, Importance Weight, Global Criterion

NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_0$</td>
<td>The curvature radius of the input and output disks</td>
</tr>
<tr>
<td>$R_z$</td>
<td>The radius of the roller</td>
</tr>
<tr>
<td>$R_2$</td>
<td>The curvature radius of the roller in the contact area</td>
</tr>
<tr>
<td>$R_1, R_3$</td>
<td>The distances between the disks' rotation axis and the contact point with the roller</td>
</tr>
<tr>
<td>$L$</td>
<td>The lead of the loading cam</td>
</tr>
<tr>
<td>$S_p$</td>
<td>Slip coefficient</td>
</tr>
<tr>
<td>$F_n$</td>
<td>The normal force acting on the CVT elements</td>
</tr>
<tr>
<td>$m$</td>
<td>The vehicle mass</td>
</tr>
<tr>
<td>$A$</td>
<td>The vehicle frontal area</td>
</tr>
<tr>
<td>$P$</td>
<td>Power</td>
</tr>
<tr>
<td>$f_r$</td>
<td>The rolling resistance coefficient of the vehicle</td>
</tr>
<tr>
<td>$T$</td>
<td>Torque</td>
</tr>
<tr>
<td>$N$</td>
<td>The number of cycles</td>
</tr>
<tr>
<td>$S_c$</td>
<td>Allowable contact stress</td>
</tr>
<tr>
<td>$Z_N$</td>
<td>Load cycle factor</td>
</tr>
<tr>
<td>$\chi^*$</td>
<td>The optimum solution</td>
</tr>
<tr>
<td>$n$</td>
<td>The number of the disks</td>
</tr>
<tr>
<td>$n_d$</td>
<td>The speed ratio of the final drive</td>
</tr>
<tr>
<td>$a,b$</td>
<td>Semi-axis of the elliptical contact area</td>
</tr>
<tr>
<td>$C_D$</td>
<td>Aerodynamic drag coefficient of the vehicle</td>
</tr>
<tr>
<td>$R_d$</td>
<td>The radius of the wheels</td>
</tr>
<tr>
<td>$n_i$</td>
<td>The number of cycles at the stress level $\sigma_i$</td>
</tr>
<tr>
<td>$D$</td>
<td>The number of stress spectrum blocks to failure</td>
</tr>
<tr>
<td>$W_i$</td>
<td>The importance weight of the $i^{th}$ objective</td>
</tr>
</tbody>
</table>

Greek Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_{so}, \lambda_{cow}$</td>
<td>Spin momentum coefficients</td>
</tr>
<tr>
<td>$\theta$</td>
<td>The half cone angle in half-toroidal CVT</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>The roller tilt angle</td>
</tr>
<tr>
<td>$\mu_{so}, \mu_{cow}$</td>
<td>The effective friction coefficients in the contact areas</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Efficiency</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Stress</td>
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1. INTRODUCTION

Optimization of vehicles’ power train has been known as one of the methods for decreasing the vehicles’ fuel consumption (FC) [1, 2]. Variation of the power train’s geometry in order to increase its efficiency and consequently, to decrease the vehicle FC could have some impacts on its different parameters such as its weight and service life. Therefore, optimization of vehicles’ power train should be considered as a multi-objective optimization problem.

Among different modern power trains, continuously variable transmission (CVT) is one of the common power trains. Unlike conventional automatic power train, CVT changes the speed ratio between engine and wheels continuously. Therefore, the dependence between the engine’s revolution and the vehicle speed is lower than the case of employing conventional transmissions. In case of conventional transmissions, the engine operates in the domain in which its brake specific fuel consumption (bsfc) is minimal. However, the most efficient point is not achievable through use of this power train. On the contrary, this point is reachable in case of using CVT because of the lower dependence between the engine and the wheels [3, 4]. The efficiency of CVT is highly sensitive to its geometry. Therefore, its efficiency can be improved by proper design of its geometry.

These power trains have been optimized in different studies. Huibo et al. [5] optimized toroidal CVT in order to minimize its cavity radius and maximize its efficiency in the whole range of its speed ratio. In this optimization, the constraints were the oil film thickness and its contact strength. The optimization was carried out using sequential quadratic programming (SQP). Akbarzadeh et al. [6] optimized geometry of half toroidal CVT using Genetic Algorithm (GA). The objective function was sum of functions such as the normal force and efficiency. Delkhosh et al. [7] optimized half toroidal CVT in order to maximize its efficiency and found a geometry which maximized CVT efficiency in whole range of its operating condition. They [8] optimized full toroidal CVT in order to increase its efficiency and decrease its weight using Parameter Space Investigation (PSI) method. In the same line, they [3] embedded a fixed ratio (FR) mechanism between full toroidal CVT and the vehicle final drive and optimized CVT and FR with the aim of minimizing the vehicle FC in NEDC driving cycle. Also, they [9] optimized half and full-toroidal CVT in ECE driving cycle. The impact of vehicle weight on the optimal CVTs was also investigated by them.

As discussed above, variation of CVT geometry changes its efficiency, service life and weight. Therefore, its optimization should be carried out considering these parameters as the objective functions. Although the study on this field seems to be necessary, there has been almost no research in this area up to now. In this paper, half toroidal CVT is introduced and the method of calculating its efficiency, fatigue life and weight and also FC of the vehicle equipped with this power train is described. Finally, by considering aforementioned parameters as objective functions and use of Global Criterion Method as a multi-objective optimization method, the CVT geometry is optimized.

2. CONTINUOUSLY VARIABLE TRANSMISSION

Continuously variable transmission (CVT) is a kind of power transmission, which creates a continuous range of the speed ratio. Although the efficiency of CVT is lower than the conventional power trains, owing to some beneficial aspects of these power trains, the usage of them has been considering a lot for years. One of the major preferences of these power trains is their ability to change the speed ratio continuously which causes the engine rpm to vary almost independent of the vehicle speed. Thus, engine can operate in its fuel-optimal rpm, in which its FC is minimal. Unlike the conventional transmissions, CVT’s efficiency is severely dependent on its geometry and also operating condition, which includes the input torque, speed and speed ratio. Toroidal CVT is a major kind of these power trains, which is divided into half-toroidal and full-toroidal ones. In these power trains, their input power is transmitted via rolling and sliding of the roller on the disks. An oil film in the contact area prevents metal to metal contact. As demonstrated in [10], half-toroidal CVT has higher efficiency compared to the full-toroidal one. A simple geometry of a half-toroidal traction drive which includes a roller, input disk and output one is shown in Figure 1. According to this figure, varying the values of $R_1$ and $R_2$ change and consequently the transmission speed ratio is altered.

As discussed, there are three functions which play a major role in the costs of using half-toroidal CVT as the vehicle power train. These functions are discussed in the following.

2. 1. Fuel Consumption

One of the most important parameters which impact the vehicle FC is the power train efficiency. The more the efficiency of the power train is, the fewer FC will be. In order to estimate FC of the vehicle equipped with half-toroidal CVT, calculation of CVT’s efficiency is crucial.

There are several models proposed to calculate half-toroidal CVT efficiency. These model’s inputs are CVT geometry, input torque and speed, and its speed ratio. In this paper, a model introduced in [10] and [7] is employed to determine its efficiency. In this model, the contact between the roller and the disks has been
simulated using Hertz contact rule. In addition, the viscosity of the oil has been presented as a function of its temperature and pressure. Finally, the efficiency of CVT in this model is:

\[
\eta = \frac{P_m}{P_{in}} = \mu + \gamma \sin(\theta + \gamma) (1 - Sp)
\]  

(1)

In order to calculate the vehicle FC, experimental data of the engine fuel consumption is needed. Figure 2 depicts the brake specific fuel consumption (bsfc) contours of the considered vehicle’s engine. These data are obtained from the engine manufacturer. The required characteristics of the vehicle for calculating its FC are shown in TABLE 1. In this study, the vehicle motion is considered in ECE driving cycle. In order to calculate the vehicle FC, a method proposed in [3] is used, where the method has been validated comparing the experimental model.

2.2 Stress and Fatigue Life Calculation

The second function impacts the cost of using half-toroidal CVT is its fatigue life. Higher fatigue life leads to lower costs of CVT maintenance. Therefore, it is necessary to determine the fatigue life of CVT during driving cycle. Fatigue life of CVT totally depends on the contact stress between its elements. The contact stress on the roller and disks in the contact points is generally a high value and can rise up to 3GPa [10]. The stress in the contact area is a function of the normal force exerted on it (F_N), the contact points’ modulus of elasticity and Poisson’s ratio. These parameters specify the semi-axis of elliptical contact area, as well. This stress can be achieved using [11]:

\[
\sigma = \frac{1.5F_N}{\pi ab}
\]  

(2)

where the normal force is calculated by:

\[
F_N = \frac{2\pi T_m}{L \cos \gamma}
\]  

(3)

Obviously, the roller and the disks experience a repetitive stress which changes due to diverse operating conditions. Accordingly, calculating their fatigue life seems to be essential. In the driving cycle, in order to allow the engine to operate in the fuel-optimal rpm, CVT speed ratio changes frequently, and the different points of the disks experience various values of the contact stress, when CVT speed ratio changes by tilting the roller and varying \( \gamma \). On the other hand, during CVT operation, any contact point of the roller experiences contact stress. Therefore, the roller contact points may be more critical than the disks’ contact points in terms of the contact stress exerted on them. Consequently, in order to determine CVT fatigue life, the fatigue life of its roller is studied.

During any rotation of the roller about its axis, a specific point on the roller experiences two various nonzero stresses during contact with the input and output disks. Therefore, the stress in the considered point varies according to Figure 3. In this figure, \( \sigma_{in} \) and \( \sigma_{out} \) are the stress values on the roller during contact with the input and output disks, respectively.

TABLE 1. The characteristics of the considered vehicle

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_r )</td>
<td>0.013</td>
<td>( m ) (kg)</td>
<td>1100</td>
</tr>
<tr>
<td>( C_D )</td>
<td>0.355</td>
<td>( R_i ) (m)</td>
<td>0.279</td>
</tr>
<tr>
<td>( A ) (m(^2))</td>
<td>2.09</td>
<td>( n_d )</td>
<td>3.895</td>
</tr>
</tbody>
</table>

Figure 1. A simple geometry of half-toroidal traction drive [7]

Figure 2. bsfc data of the considered vehicle’s engine

Figure 3. A schematic of stress variation for a specific point on the roller during rotation about its axis
The value of cyclic contact stress in a specific point of the roller can be calculated knowing the input torque and the speed ratio (see Equation (2)). The times of acting a definite stress on the specific contact point for a defined driving cycle can be obtained, by studying the roller in time deviation ($\Delta t$) which is small enough. Knowing the value of the roller rotational speed during drive cycle, the times of facing the considered contact point with input or output disk can be determined. Therefore, times of applying contact stress on the considered contact point is obtained. The allowable contact stress ($S_c$) at $10^7$ cycles can be chosen according to ANSI/AGMA 2001-D04 [12]. In order determine its value in other cycle numbers, the load cycle factor ($Z_0$) is needed. One of the conventional materials used in toroidal CVTs is case-hardened steel with HRC=60 hardness [13]. In this case, $S_c$ is almost 1.9GPa and $Z_0=1.249N^{0.0138}$[12]. Thus, the number of cycles to failure for specific stress level is achieved by:

$$N_i = \left( \frac{1.2495 S_c}{\sigma_i} \right)^{1/0.0138}$$

In this paper, Miner’s rule is used to determine the expected fatigue life of the roller under cyclic stresses. The expected life of the roller is attained by:

$$D = \frac{1}{\sum \frac{n_i}{N_i}}$$

The fatigue life of CVT can be shown in terms of the number of driving cycles or the distance (in kilometers) can be travelled using CVT.

2.3. Weight Calculation The third function which affects the cost of using CVT is its weight. Higher weight of CVT increases the overall weight of the vehicle and therefore, increases the vehicle FC. The power train weight is more important in case of CVT transmissions. These power trains are commonly heavier than the conventional automatic transmissions [14].

In order to determine half-toroidal CVT weight, a simplification is carried out. In this investigation, in order to calculate the weight of a CVT power train, we calculate the sum of its primary elements including input and output disks and the rollers. The reason behind this simplification is that the other elements have smaller weight compared to these elements. Furthermore, the optimization parameters presented in the following sections don’t alter the ignored elements’ weight. Thus, these elements don’t affect the optimization results. CVT weight is calculated as a function of its density and geometrical parameters including $R_o$, $R_{zh}$, $e$, $\theta$ and $n$. Since the equation which presents its weight is a large equation, it is not presented here.

3. MULTI-OBJECTIVE OPTIMIZATION METHOD

Engineering design often deals with multiple, possibly conflicting, objective functions or design criteria. On the other hand, we can say all design and engineering activities are fundamentally multi-objective in nature because of the existence of inherent tensions between some objectives which most of the time are conflicting. For example, one may want to maximize the performance of a system while minimizing its cost. Such design problems are the subject of multi-objective optimization [15]. There is general consensus that multi-objective optimization methods can be broadly decomposed into two categories: Scalarization approaches and Pareto approaches. In the first group of methods, the multi-objective problem is solved by translating it back to a single objective, scalar problem. The Pareto methods, on the other hand, keep the elements of the objective vector separate throughout the optimization process and typically use the concept of dominance to distinguish between inferior and non-inferior solutions. However, the end goal of all these methods is the same to provide designers and decision makers with a set of ‘optimal’ alternatives to choose from [16]. In the research reported here, scalarization method is used. A simple method of this group is to define a function which is sum of the weighted functions where the weight of each objective is determined with regard to its importance compared to other objective functions. However, as demonstrated in [17] and [18], it fails to provide acceptable results especially in non-convex optimization spaces. Moreover, the solution points’ distribution is highly sensitive to the importance weights of the objective functions [19]. In the present study, we use “Global Criterion Method” as one of the most common general scalarization methods. This method is introduced briefly as follows:

3.1. Weighed Global Criterion Method Global criterion method is one of the scalarization methods for multi-objective optimization in which all objective functions are combined to form a single function. Although a global criterion may be a mathematical function with no correlation to preferences, a weighted global criterion is a type of utility function in which method parameters are used to model preferences. This method was firstly introduced by Rao [20]. In this method, the optimum solution (X) is found by minimizing a preselected global criterion, F(X), such as the sum of the squares of the relative deviations of the individual objective functions from the feasible ideal solutions. Thus, X is found by: Minimizing

$$F(X) = \left( \sum_{i=1}^{n} w_i \left( \frac{f_i - f_i^*}{f_i^*} \right) \right)^{1/2}$$

subject to $g_j(X) \leq 0, \ j=1, 2...$
m; where p is a constant value (an usual value of p is 2) and $\sum w_i = 1$. For a fixed value of $p$, a single optimal point is achieved while Pareto optimal set is attainable by continuous variation of $p$ [21].

3.2. Optimization

In this section, the optimization goals introduced earlier are optimized simultaneously, employing the mentioned optimization method. In this paper, the importance weights in the optimization are dependent on objective functions as follows:

The importance weight of fatigue life function decreases with the growth of this function. It is on account of the fact that very large fatigue life of the power train is not necessary. On the contrary, by rising of the CVT weight, its importance weight will increase. The reason behind is that very massive power train for the considered vehicle cannot be feasible. The variations of the above weight factors versus their objective functions are shown in Figure 4 and Figure 5. As shown in Figure 4, the weight coefficient of the CVT fatigue life for the service life less than 40,000 km is equal to 1/3. This parameter decreases by increasing the service life, and for service life more than 120,000 km, it is equal to zero. This data is derived from the presented data of the vehicle manufacturer \(^1\) [22]. According to Figure 5, the importance weight factor of the CVT weight criteria remains 1/10 whenever the CVT weight values are less than 10 kg. Furthermore, as depicted in the diagram, this parameter shows an increase with the growth of CVT weight and reaches the maximum point at 1. Similar to the fatigue life data, these data is also derived from the published data of the vehicle manufacturer [22]. As stated by manufacturer, the power train weight must be less than 50 kg. For more information, the weight of the manual transmission currently utilized as the vehicle power train is 25 kg. Since the sum of the weight parameters must be equal to 1, the weight coefficient of the vehicle’s FC for any value of CVT fatigue life and weight can be calculated.

In order to accomplish the optimization procedure, the particle swarm optimization (PSO) method is employed. This method is illustrated in [7]. The reason of selecting PSO as the optimization method is its ability to optimize complex and non-differentiable functions, and its simplicity [22]. As described before, the optimization problem has two constraints: the first constraint is that the power train weight must be less than 50 kg, and the second constraint is on the maximum stress exerted on the disks and the roller during the identified driving cycle. Regarding the considered material for these elements, the maximal contact stress which CVT elements can endure is 2.37 GPa.

\(^1\) Saipa corporation

While dealing with the solutions which violate the constraints, the penalty functions that is used will be the exterior penalty function [23]. The optimization parameters are those parameters which determine half-toroidal CVT geometry. Their ranges are shown in Table 2. The optimization problem is formulated as follows: Minimize $f_i(X)$; where:

\[
\begin{align*}
    f_1(X) &= \text{vehicle FC} \\
    f_2(X) &= 1/\text{Fatigue life} \\
    f_3(X) &= \text{CVT weight}
\end{align*}
\]

Subject to

- Maximum contact stress $\leq 2.37$ GPa
- CVT weight $\leq 50$ kg

Since we want to increase fatigue life of CVT, its objective function is considered $1/\text{Fatigue life}$. Minimization of this function means that the fatigue life is maximized.
The optimization parameters and their range [7]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_e$ (mm)</td>
<td>[20-80]</td>
<td>$\epsilon$</td>
<td>[0.1-0.9]</td>
</tr>
<tr>
<td>$\frac{R_\infty}{R_e}$</td>
<td>[0.3-0.9]</td>
<td>$\theta$ (deg)</td>
<td>[50-80]</td>
</tr>
</tbody>
</table>

The optimal solution for each objective, taken alone

<table>
<thead>
<tr>
<th>Objective</th>
<th>Vehicle FC (L/100km)</th>
<th>(Fatigue Life)$^{-1}$</th>
<th>CVT Weight (kg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vehicle FC</td>
<td>9.54</td>
<td>6.22×10$^{-3}$</td>
<td>6</td>
</tr>
<tr>
<td>(Fatigue Life)$^{-1}$</td>
<td>10</td>
<td>5.28×10$^{-19}$</td>
<td>47.82</td>
</tr>
<tr>
<td>CVT weight</td>
<td>10.2</td>
<td>7.3×10$^{-8}$</td>
<td>2.76</td>
</tr>
</tbody>
</table>

3.3 Optimization Results

The global criterion method was used and the results were achieved. As discussed above, the first stage of this method is to optimize each of the considered objectives by itself, subject to the mentioned constraints. The optimization results of this stage are shown in Table 3. Each row shows the optimization results of one of the objectives. At this row, added to the optimal value of the considered objective, the values of other objectives for this case are shown.

As shown in this table, for the case of optimizing the vehicle FC, fatigue life of CVT indicates a low value (160 kilometers) in terms of the distance which can be travelled by the CVT. For the case of optimizing the fatigue life, the vehicle FC will be 5% higher than its optimum value while CVT’s weight reaches a large value and approaches to the upper bound of the allowable range. Moreover, at the same time, the fatigue life is 1.88×10$^{18}$km. According to Figure 4, this value is very larger than the required value. In the case of optimizing the CVT weight, the vehicle FC shows an increase and reaches the amount that is 7% higher than its optimum value, meanwhile the CVT’s fatigue life will be much more greater than its required value.

As a result, we can notice that through the process of optimization of each objective, other objectives are away from their optimal points. Moreover, in some cases, the optimized objective reaches the value which is beyond the required level. Therefore, multi-objective optimization with respect to the importance weight factors seems to be necessary.

As the next step, the multi-objective optimization method is implemented and the optimal values of the objective functions are achieved. This objective is

$$F(X) = \left( \sum_{i=1}^{n} \left( \frac{f_i^* - f_i(X)}{f_i^*} \right)^2 \right)^{1/2}.$$ 

This value added to the optimal geometries is shown in Table 4. In addition, the values of the objectives are shown in this table. According to this table, CVT fatigue life in terms of the travelled distance is about 465,000 kilometers. Therefore, regarding Figure 4, its importance weight is zero. This is because the proposed CVT meets the demanded service life totally, and therefore, this objective hasn’t affected the optimization objective function. Comparing the results shown in Table 4 with Table 3, it is concluded that the objectives achieved from multi-objective optimization are not the optimal values, but are acceptable ones. As shown in Table 3, the vehicle FC is only 1.5% larger than its optimum value. In this case, the optimal CVT’s weight is 28.67 kg which comparing to a normal manual power transmission (25 kg) is acceptable. Meanwhile in that case, the fatigue life of CVT is larger than the required value.

4. CONCLUSION

This paper aimed to optimize half-toroidal CVT. For this aim, we tried to minimize its weight and the vehicle FC, simultaneously while remaining in the optimal area of CVT service life. In the first place, the models for calculating objective functions were presented. These objectives were FC of the vehicle equipped with CVT power train, CVT fatigue life and its weight. Then, an importance weight was considered for each objective which denoted its importance compared to the other objectives. These weight factors were considered as a function of their objective functions and were changed with the variation of their objectives, regarding the vehicle desires. Next, the optimal solution for each objective function was taken alone, and their optimal points were obtained. Finally, a multi-objective optimization problem was solved, using global criterion method. It was found that this optimization gives a geometry which provides acceptable FC, fatigue life and weight.
5. REFERENCES


Application of Multi-objective Optimization for Optimization of Half-toroidal Continuously Variable Transmission

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In this paper, a multi-objective optimization approach is proposed for the design of half-toroidal Continuously Variable Transmission (CVT) systems. The main objectives are to optimize the fuel consumption and fatigue life of the transmission while minimizing the weight and cost. The optimization problem is formulated using the global criterion approach, which balances the trade-offs between the objectives. The results show significant improvements in the performance of the CVT design compared to the baseline configuration. DOI: 10.5829/idosi.ije.2014.27.09c.15