IMPACT OF TRANSPORTATION SYSTEM ON TOTAL COST IN A TWO-ECHELON DUAL CHANNEL SUPPLY CHAIN

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Abstract The advent of e-commerce has prompted many manufacturers to redesign their traditional channel structure by engaging in direct sales. In this paper, we present a dual channel inventory model based on queuing theory in a manufacturer-retailer supply chain, consisting of a traditional retail channel and a direct channel which stocks are kept in both upper and lower echelon. The system receives stochastic demand from the both channel which each channel has an independent demand arrival rate. A lost-sales model which no backorder is allowed is supposed. The replenishment lead times are assumed independent exponential random variables for both warehouse and the retail store. Under the replenishment inventory policy, the inventory position is kept constant at a base-stock level. To analyze the chain performance, an objective function included holding, lost sales and transportation cost is defined. Simulated Annealing is used to find a good solution for inventory level in each echelon. At the end, we conduct a parametric analysis to study the effect of replenishment rate between warehouses on total cost and indicate how to decrease total cost by choosing a suitable transportation system.

Keywords Supply Chain Management, Direct Channel, Inventory, Transportation System, Simulated Annealing

1. INTRODUCTION

The advent of information technology plays a remarkable role in reshaping supply chain behavior in the recent years. Started during mid 1990’s, internet has become an important retail channel.
The commercial blossoming of the internet has introduced tremendous opportunities and has underscored the importance of effective supply management [1]. Recognizing the great potential of the Internet to reach customers, many brand name manufacturers, including Hewlett-Packard, IBM, Eastman Kodak, Nike, and Apple, have added direct channel operations [2]. Direct distribution enables companies to bring products to market faster. Now, companies are hugely benefiting from the early-to-market advantage and making a significant profit margin by eliminating the retailer and distributors margins. It seems doubtful the first sight that selling goods by both retailer and manufacturer make a profitable frame. Indeed, competition between manufacturer and retailer could lead to channel conflict [3], pricing policy for different channels [4] and distribution strategies.

However, considering the challenges posing in dual channels, examples of today's companies starting a direct channel when they already had a well-structured retail channel, such as IBM, Compaq, HP, and Sony, are the great evidence to support the issue that supply chain must react in order to meet consumer expectation instead of resisting on the traditional methods. Many retailers and manufacturers have already learnt that meeting consumer expectation is a valuable area for sharing margins. Frazier [5] stated that such a mixed channel would increase the product's penetration level and revenue on one hand, but would lead to decreased support from the channel partners. Dual-channel distribution may take many forms, one of which is when a manufacturer both sells through intermediaries and directly to consumers [6].

Chiang, et al [7] pointed out the most important economical reason for using of Multi-channel is reaching potential buyer segments that could not be reached by a single channel so that these channels can help to increase the market coverage. Also, they stated that the fundamental task in connection with the two-echelon inventory problem is to find the balance between the stock levels at the top and the bottom echelon. Although several studies have examined dual channel supply chains, the study of multi-channel supply chain in the internet-enabled versus retail channel has appeared recently in the literature. Rhee, et al [8] studied a hybrid channel design problem, assuming that there are two consumer segments: a price sensitive segment and a service sensitive segment. Chiang, et al [9] examined a price-competition game in a dual channel supply chain. Tsay, et al [2] provided an excellent review of recent work in the area and examined different ways to adjust the manufacturer–reseller relationship. Although their results show that a direct channel strategy makes the manufacturer more profitable by posing a viable threat to draw customers away from the retailer, their focus are on channel competition and coordination issues in the setting where the upstream echelon is at once a supplier to and a competitor of the downstream echelon. Also, their results depend on the assumption that customer’s acceptance of online channel is homogeneous.

Teimoury, et al [10] developed a dual channel model with separate lost sale cost for each channel, by extending Chiang, et al [7] model. In another excellent work, Boyaci [11] studied stocking decisions for both the manufacturer and retailer and assumes that all the prices are exogenous and demand is stochastic. In a similar setting, Cattani, et al [12] studied pricing strategies of both the manufacturer and the retailer. Dong, et al [13] developed the conditions under which the manufacturer and the retailer gain more or less from the adoption of efficient replenishment based on a game theoretic channel model of bilateral monopoly under the two channel relationship structures. Dumrongsiiri, et al [14] developed conditions under which the manufacture and the retailer share the market in equilibrium. Their results show that the differing in marginal costs and demand variability has a major influence on equilibrium prices. Xiao, et al [15] studied a model in that supply chain is coordinated and discuss how the cost disruption may affect the coordination mechanisms.

In the inventory control models, the lost sale cost was considered the same in the both channel while this assumption is not adequate with the real world problems in virtue of the customer’s behaviors complexity. In fact, the customer's sensitivity of the internet-enabled channel is much more than the retail channel ones and the loyalty to a brand in virtual environment are lesser than the physical environment. So, it seems that the lost sale cost in direct channel is significant.

Also, the researchers developed inventory control models in a two echelon system to find the balance...
between the stock levels at the top and the bottom echelon without considering the effect of transporting cost. To show the impact of transporting cost on inventory levels, we consider a transportation cost between manufacture warehouse and retail store warehouse to indicate influence of transportation system on inventory levels and total cost. Furthermore, parametric analysis based on the model is conducted by varying the transportation systems with difference replenishment rates and translation costs to obtain generalized results.

The rest of the paper is organized as follows: Section 2 sets up the dual channel supply chain model. We analyze the performance of the two-echelon dual-channel system in Section 3. Section 4 is devoted to Simulated Annealing algorithm. Section 5 presents the numerical study. The computational results obtained by applying SA are discussed in Section 6. Conclusion and future researches are given in Section 7.

2. DUAL CHANNEL MODEL

Consider a two-echelon dual-channel supply system that consists of a manufacturer with a single warehouse at the top echelon and a retail store at the bottom echelon. The topology and product flows of the two-echelon dual-channel supply system are illustrated in Figure 1.

![Figure 1. Two-echelon dual channel inventory system.](image)

Furthermore, we have the following set of assumptions for the inventory model:
The product is available for customers at both retail store and the internet-based direct channel. The system receives stochastic demand from two customers segments: those who prefer the traditional retail store and those who prefer the direct channel which each segment has an independent demand arrival rate. In fact, the customers arrive at the retail store according to a Poisson process with constant intensity \( \lambda_r \), orders placed through the direct channel are in accordance with a Poisson process with rate \( \lambda_d \), and the total demand is elaborated as \( \lambda = \lambda_r + \lambda_d \). The demand of retail customers is met with the on-hand inventory from the bottom echelon while the demand in the internet-enabled channel is fulfilled through direct delivery with the on-hand inventory from the upper echelon.

Also, customers are lost when both retail store and the manufacturer warehouse are out of stock simultaneously. The replenishment lead times are assumed independent exponential random variables for both the warehouse and the retail store with mean \( (\mu_w)^{-1} \) and \( (\mu_r)^{-1} \) respectively. A one-for-one replenishment inventory policy is applied. We consider \( T \) as transportation cost per unit from upper echelon to bottom echelon. A customer served from back stock on-hand will trigger a replenishment order immediately by EDI (Electronic Data Interchange) therefore, the information lead time is assumed to be zero. Under this replenishment inventory policy, the inventory position is kept constant at a base-stock level. The base-stock levels at the warehouse and the retail store are denoted by \( S_w \) and \( S_r \) respectively.

We use the following notations in the state of equations and relations:
- \( \lambda \): Total demand.
- \( \lambda_d \): Customer arrival rate at the direct channel.
- \( \mu_w \): Manufacturer warehouse replenishment rate.
- \( \mu_r \): Retail store replenishment rate.
- \( S_w \): Base-stock level at the manufacturer warehouse.
- \( S_r \): Base-stock level at the retail store.
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On hand inventory at the manufacturer warehouse, $0 \leq x \leq S_w$.

On hand inventory at the retail store, $0 \leq y \leq S_r$.

$\pi_{xy}$: Steady-state probability.

$h_w$: Inventory holding cost per item per time unit at the manufacturer warehouse.

$C_{if}$: The average inventory holding cost.

$C_I$: The average lost sales cost.

$C_T$: The average transportation cost.

$L_{Sw}$: The probability of out of stock at the manufacturer warehouse.

$L_{Sr}$: The probability of out of stock at the retail store.

$L_{Sw,Sr}$: The probability of out of stock at the both channels.

$T$: Transportation cost per unit from upper echelon to bottom echelon.

Total cost

2.1. The Markov Model

In this article we extend the Markov model which Chiang, et al [7] used in their article, by adjusting it based on our assumptions. The corresponding Markov model can be constructed with the state space $(x,y)$. There are four events which lead to change the state: a customer arrives at the retail store, an order is placed through the direct channel, a replenishment order arrives at the manufacturer warehouse and a replenishment order arrives at the retail store. The transition diagram of the proposed model is as shown in Figure 2 to verify the balance equations and better understand of the system.

Let $\pi_{xy}$ be the steady-state probability that $x$ items is on-hand at the manufacturer warehouse and $y$ items is on-hand at the retail store. The flow balance equations that require for all states input and output rates to each state are equal are given by:

![Transition Diagram](image-url)
\[
\pi_{xy} [\lambda_r \times (A) + \lambda_y \times (B) + (S_w - x) \times \mu_w + \\
[(S_r - y) \times \mu_r] \times (C)] = \pi_{(x,y+1)} [\lambda_r \times (C)] \\
+ \pi_{(x+1,y)} \times [\lambda_y \times (D)] + \pi_{(x-1,y)} \times \\
[\mu_w \times (S_w - x + 1) \times (B)] + \pi_{(x+1,y-1)} \times \\
[\mu_r \times (S_r - y + 1) \times (E) \text{ for } x=0,1,...,S_w \text{ and } y=0,1,...,S_r] \\
\]

(1)

The left hand side of (1) represents the average transitions from state \((x,y)\). The first two terms in the bracket specify the transitions due to receiving demand. Specifically, \(A\) and \(B\) state whether the retail store and the manufacturer warehouse are out of stock or not respectively. The last two terms in the bracket are the rates at which in-transit replenishment order arrive at the manufacturer warehouse and the retail store respectively. Also the right hand side of (1) represents the average transitions into state \((x,y)\).

\[A = \begin{cases} 1 & \text{if } y > 0 \\ 0 & \text{otherwise,} \end{cases} \]

(2)

\[B = \begin{cases} 1 & \text{if } x > 0 \\ 0 & \text{otherwise,} \end{cases} \]

(3)

\[C = \begin{cases} 1 & \text{if } y < S_r \\ 0 & \text{otherwise,} \end{cases} \]

(4)

\[D = \begin{cases} 1 & \text{if } x < S_w \\ 0 & \text{otherwise,} \end{cases} \]

(5)

\[E = \begin{cases} 1 & \text{if } 0 < y \leq S_w \text{ and } 0 \leq x < S_r \\ 0 & \text{otherwise,} \end{cases} \]

(6)

The left hand side of (1) represents the average transitions from state \((x,y)\). The first two terms in the bracket specify the transitions due to receiving demand. Specifically, \(A\) and \(B\) state whether the retail store and the manufacturer warehouse are out of stock or not respectively. The last two terms in the bracket are the rates at which in-transit replenishment order arrive at the manufacturer warehouse and the retail store respectively. Also the right hand side of (1) represents the average transitions into state \((x,y)\). The first two terms denote the transitions due to satisfying demand by inventory on-hand at the retail store and the manufacturer warehouse, correspondingly. The last two terms indicate the transitions due to receiving in-transit replenishment orders from the manufacturer and the retail store, respectively.

\(C\) and \(D\) state whether the stock in retail store is lower than \(S_r\) and the stock in the manufacturer warehouse is lower than \(S_w\), respectively. Besides, \(E\) states whether the stock in retail store is bigger than zero and smaller than or equal to \(S_r\) and stock in the manufacturer warehouse is bigger than or equal to zero and smaller than \(S_w\), respectively. To find the steady-state probabilities, we can solve the corresponding system of linear equations which contain the balance equations given in (1) and the normalizing constraint:

\[
\sum_{w}^{S_w} \sum_{r}^{S_r} \pi_{xy} = 1 \\
x = 0 \text{ and } y = 0
\]

(7)

### 3. ANALYSIS OF DUAL CHANNEL MODEL

In this section, we analyze the performance of the two-echelon dual-channel system. Three different operational cost factors are considered: the average inventory holding cost, average lost sales cost and transportation cost.

#### 3.1. Average Inventory Holding Cost

Given the steady-state probabilities, the average inventories for the manufacturer warehouse and the retail store can be modeled in finite horizon, respectively as

\[
I_w = \sum_{x=1}^{S_w} x \times \pi_{xy} \\
I_r = \sum_{x=0}^{S_r} y \times \pi_{xy}
\]

(8)

(9)

Let \(h_w\) and \(h_r\) be the inventory holding cost incurred by the firm per item unit at the manufacturer warehouse and the retail store, respectively. Then, the average inventory holding cost, \(C_{HI}\), is represented by

\[
C_{HI} = h_w \times I_w + h_r \times I_r
\]

(10)

The first item of \(C_{HI}\) states the inventory holding cost from the manufacturer warehouse and the second item specifies the inventory holding cost from the retail store.

#### 3.2. Average Lost Sale Cost

Recall that when a stock-out occurs in either channel result in lost sales. Moreover, customers are lost when both retail store and the manufacturer warehouse are out...
of stock simultaneously. The probabilities that a stock-out occurs only at the manufacturer warehouse and only at the retail store, respectively, depends upon the steady-state probabilities in the following way:

\[ L_{Sw} = \sum_{y=1}^{y} \pi_{0y} \]  

(11)

\[ L_{Sr} = \sum_{x=1}^{x} \pi_{x0} \]  

(12)

\[ L_{Sw,r} = \pi_{00} \]  

(13)

Which \( L_{Sw} \), \( Sr \) denote the probability that both channel are simultaneously out of stock. Now, assume that the opportunity cost of losing a customer in the direct channel and the retail store channel are \( l_d \) and \( l_r \) per customer, respectively. Then, we can specify the total average lost sales cost (\( CL \)) as:

\[ C_L = l_d \times \lambda_d \times \sum_{y=1}^{y} \pi_{0y} + l_r \times \lambda_r \times \sum_{x=1}^{x} \pi_{x0} + (l_d \times \lambda_d + l_r \times \lambda_r) \times \pi_{00} \]  

(14)

3.3. Average Transportation Costs Each product that being sold in retail store has transportation cost (\( T \)) to transport from plant’s warehouse to retail store warehouse. Therefore system has transportation costs as follow:

\[ C_T = T \times \lambda_r \times \sum_{x=0}^{x} \sum_{y=1}^{y} \pi_{xy} + T \times (S_r) \]  

(15)

The second part of (\( C_T \)) calculates transportation costs for base-stock in retail store warehouse.

3.4. Total Cost To evaluate the performance of the two-echelon dual-channel inventory system, the sum of the inventory holding cost, lost sales cost and transportation cost are used. Therefore, the total cost is elaborated as \( C_T = C_H + C_L + C_T \). Note that the only decision variables are the base-stock level, \( S_w \) and \( S_r \). So, we have total cost as a function of \( S_w \) and \( S_r \), which is:

\[
Te(S_w, S_r) = h_w \times \sum_{x=1}^{x} y \times \pi_{xy} + \\
h_r \times \sum_{y=1}^{y} \sum_{x=1}^{x} \pi_{xy} + l_d \times \lambda_d \times \sum_{y=1}^{y} \pi_{0y} + \\
l_r \times \lambda_r \times \sum_{x=1}^{x} \pi_{x0} + (l_d \lambda_d + l_r \lambda_r) \times \pi_{00} + \\
T \times \lambda_r \times \sum_{x=0}^{x} \sum_{y=1}^{y} \pi_{xy} + T \times (S_r)
\]  

(16)

We intend to find the base-stock levels that minimize \( TC = (S_w, S_r) \).

4. SIMULATED ANNEALING

Metropolis, et al [16] proposed an algorithm to simulate the evolution of a solid in a heat bath until it reached its thermal equilibrium. The Monte Carlo method was used to simulate the process, which started from a certain thermodynamic state of the system, defined by a certain energy and temperature. Then, the state was slightly perturbed. If the change in energy produced by this perturbation was negative, the new configuration was accepted. If it was positive, it was accepted with a probability given by \( e^{-\Delta E/kT} \), where \( k \) is the so-called Boltzmann constant, which is a constant of nature that relates temperature to energy [17]. This process is repeated until a frozen state is achieved [18,19].

Thirty years after the publication of Metropolis’ approach, Kirkpatrick, et al [20] and Corny [21] independently pointed out the analogy between this “annealing” process and combinatorial optimization. These researchers indicated several important analogies: a system state is analogous to a solution of the optimization problem; the free energy of the system (to be minimized) corresponds to the cost of the objective function to be optimized; the slight perturbation imposed on the system to change it to another state corresponds to a movement into a neighboring position (with respect to the local search state); the cooling schedule corresponds to the control mechanism adopted by the search algorithm; and the frozen state of the system corresponds to the final solution.
generated by the search algorithm (using a population size of one). These important analogies led to the development of an algorithm called “Simulated Annealing”.

The SA has two inside and outside loops. The insideloop controls the achievement to equilibrium in the current temperature and outside loop controls the rate of temperature decrease Figure 3. Its parameters are as follows:

- **EL (Epoch Length)**: number of accepted solutions in each temperature (criteria for exit from Inside loop).
- **R**: maximum number of the temperature transfer (stopping criteria or criteria for outside loop).
- **n**: counter for number of accepted solutions in each temperature.
- **r**: counter for number of temperature transfer.
- **T0**: initial temperature.
- **α**: rate of the current temperature decrease.
- **X0**: a feasible solution.
- **F(X)**: the value of objective function for X.

At first, SA starts by choosing an initial solution from feasible space \((S_c \geq 0, S_r \geq 0)\) that \(S_c\) and \(S_r\) are integer. To select a neighborhood SA randomly chooses one of the points around the existent point. Note that there are upper most 8 points in the neighborhood of a point in feasible space.

### 5. numerical study

A company considered that implements a two-echelon dual-channel supply system consisting of a manufacturer with a single warehouse at the top echelon and a retail store at the bottom echelon as shown in Figure 1. Also, the product is available in two channels, the traditional retail store and the direct channel. To purchase their goods, customers can refer to both the retail store and the online system. The base characteristic of the company shown in Table 1.

### 6. computational results

This section presents the results obtained by the Simulated Annealing (SA). All computational results described in this section are produced by a Matlab code. For solving the examples with Simulated Annealing (SA), we set the SA parameters as follows: EL=50, T0=10, α=0.98. Also, Algorithm is stopped when temperature becomes less than 0.1.

Run of SA is shown in Figure 4. SA reaches to the solution (7,6) with total cost (1417.7).

#### 6.1. parametric analysis

In this section, we conduct a parametric analysis to study the effect of transportation system on stock levels and total cost. We assume there are four transportation systems that replenishment rate and transportation cost for each system shown in Table 2. As it can be seen increasing replenishment rate between warehouses, increases the transportation cost per unit from upper echelon to bottom echelon.

The plots in Figure 5 indicate that, in general, the base-stock level at the manufacturer warehouse shows negligible changes as \(\mu_r\) increases, while on the other hand, the base-stock level at the retail store decreases in \(\mu_r\). This trend is expected because by increasing replenishment rate between warehouses the goods are replenished at lower

---

**Figure 3.** Pseudo-code of SA [18].
TABLE 1. The Base Characteristic of the Company.

<table>
<thead>
<tr>
<th>Base Parametric Values for Sport Shoes Company</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>System-related Parameters:</td>
<td></td>
</tr>
<tr>
<td>Manufacturer warehouse replenishment rate</td>
<td>( \mu_w = 20 )</td>
</tr>
<tr>
<td>Retail store replenishment rate</td>
<td>( \mu_r = 20 )</td>
</tr>
<tr>
<td>Customer-related Parameters:</td>
<td></td>
</tr>
<tr>
<td>Customer arrival rate at the direct channel</td>
<td>( \lambda_d = 15 )</td>
</tr>
<tr>
<td>Customer arrival rate at the retail store</td>
<td>( \lambda_r = 30 )</td>
</tr>
<tr>
<td>Cost-related Parameters:</td>
<td></td>
</tr>
<tr>
<td>Opportunity cost of losing a customer at the direct channel</td>
<td>( l_d = 1000$ )</td>
</tr>
<tr>
<td>Opportunity cost of losing a customer at the retail store</td>
<td>( l_r = 600$ )</td>
</tr>
<tr>
<td>Inventory holding cost per item per time unit at the warehouse</td>
<td>( h_w = 75$/year )</td>
</tr>
<tr>
<td>Inventory holding cost per item per time unit at the retail store</td>
<td>( h_r = 80$/year )</td>
</tr>
<tr>
<td>Transportation cost per unit from upper echelon to bottom echelon.</td>
<td>( T = 15$ )</td>
</tr>
</tbody>
</table>

![Figure 4. Run of SA.](image)

TABLE 2. The Base Characteristic of the Transportation Systems.

<table>
<thead>
<tr>
<th>Transportation System</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \mu_r )</td>
<td>5</td>
<td>10</td>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>( T($) )</td>
<td>8</td>
<td>10</td>
<td>15</td>
<td>20</td>
</tr>
</tbody>
</table>
time; hence, keeping a lot of inventory in retail store isn’t needed.

The plots in Figure 6 indicate that use of transportation system B with replenishment rate of 10 and transportation cost of 10 $/unit has the lowest total cost for company.

7. CONCLUSION

We study a two-echelon dual channel supply chain in which inventories are kept in both upper and lower echelon. In this chain products are sold in two channels and consumers choose a channel to buy the good accordingly. We develop a model to capture the major features of such supply chains. Our objective is to use the model to understand how transportation system for transporting the goods from upper echelon to bottom echelon influences the inventory levels and total cost. With this model we can easily choose the best tradeoff between replenishment rate of warehouses and transportation cost per unit from upper echelon to bottom echelon. It means that the model can determine the best inventory levels and transportation system, simultaneously. However, due to the complex nature of the problem, the model imposes potentially limiting assumptions. For example, customer demands are modeled as a Poisson process. The results from our model may change if different demand characteristics are applied. The framework of analysis in this paper is the one for one replenishment inventory; while considering transported size of inventory as a decision variable is another potential area to extend this research.

8. REFERENCES


