

# A FUZZY MULTI OBJECTIVE PROGRAMMING MODEL FOR POWER GENERATION AND TRANSMISSION EXPANSION PLANNING PROBLEM

S.A. Torabi\*and M. Madadi

Department of Industrial Engineering, College of Engineering, University of Tehran  
P.O. Box 11155-4563, Tehran, Iran  
satorabi@ut.ac.ir - mmadadi@ut.ac.ir

\*Corresponding Author

(Received: May 10, 2009 – Accepted in Revised Form: November 5, 2009)

**Abstract** The increasing consumption of electricity over time forces different countries to establish new power plants and transmission lines. There are various crisp single-objective mathematical models in the literature for the long-term power generation and transmission expansion planning to help the decision makers to make more reasonable decisions. But, in practice, most of the parameters associated with the input data, such as forecasted demands for electricity consumption, economic and technical characteristics of new evolving generating and transmission technologies are imprecise (fuzzy) in nature. Moreover, making such strategic decisions require considering several objectives simultaneously and applying appropriate multi-objective programming approach to yield several compromise solutions. In this paper, we take into account these main characteristics of the problem in our proposed model. Also, the maintenance cost of generation units is modeled in such a way that handles its increasing nature over operating periods. Consequently, we propose a new fuzzy multi objective mixed integer linear programming model (FMOMILP) for integrated power generation and transmission expansion planning problem. After applying the effective strategies to convert the original FMOMILP into an equivalent single objective crisp one, through an illustrative example we show that the results obtained by proposed fuzzy model is more reliable than the crisp one. Finally, some concluding remarks are also provided.

**Keywords** Generation Expansion Planning (GEP), Transmission Expansion Planning (TEP), Fuzzy Multiple Objective Linear Programming

**چکیده** روند افزایشی، مصرف انرژی الکتریکی، کشورهای مختلف را مجبور به ساخت نیروگاهها و خطوط انتقال جدید کرده است. در ادبیات موضوع، مدل‌های ریاضی، قطعی، متعددی برای حل، مسئله برنامه‌ریزی توسعه تولید و انتقال انرژی الکتریکی، ارائه شده است. اما در عمل، بیشتر یارامتهای ورودی این مسئله غیرقطعی، هستند. در این مقاله، یک مدل برنامه ریزی عدد صحیح مختلط چند هدفه فازی جدید برای مسئله ارائه می‌شود. سپس، با به کارگیری استراتژی‌های کارا در تبدیل، مدل فازی اولیه به یک مدل قطعی، به کمک یک مثال عددی نشان داده می‌شود که مدل فازی علاوه بر ایجاد انعطاف پذیری در مدل، در مقایسه با مدل قطعی، نتایج قابل اعتمادتری را به دست می‌آورد.

## 1. INTRODUCTION

The electricity consumption in different countries is continuously increasing because of different factors like developing social life, increasing population and improving technologies. Therefore, it is vital to expand power supply to response to this increasing demand. Constructing new power plants and expanding the existing transmission lines are

the most widely employed method to satisfy this growing demand.

Generally speaking, there are three main processes (stages) in the electricity supply chain to deliver reliable electricity to customers, i.e., power generation, transmission and distribution. Of particular interest, this article deals with the two first processes, i.e., power generation and transmission expansion planning problems.

The Generation Expansion Planning (GEP) problem is defined as the problem of determining which, where, and when new generation units should be constructed over a long range planning horizon, to satisfy the expected energy demand. Moreover, the Transmission Expansion Planning (TEP) is actually a strategic expansion planning of transmission network, capable of satisfying demanded load with minimum total cost.

In GEP problem, different objectives are desired that often are conflicting, subject to some constraints such as power demand, plant availability, environmental and reliability constraints. Objectives typically include the minimization of the system's total cost, the maximization of the system's reliability, the minimization of environmental pollutant emission, and the minimization of imported fuels such as gas, oil, etc. Obviously, considering these objectives simultaneously make the GEP problem more difficult to solve.

Total system's cost can include construction costs, operational costs, maintenance costs, fuel costs, and emission costs. Minimizing the pollutant emissions is an objective which is considered recently in GEP literature following the sustainable development. According to the International Energy Agency in 2003, more than 50 % of the electricity produced in the world was generated from fossil-fuel sources. This results in high levels of pollutant emissions to the atmosphere. So, it is very important using those generation units with the renewable fuels as much as possible as an objective in the model to control these pollutant emissions. The major pollutants are sulfur oxides (SO<sub>x</sub>), nitrogen oxides (NO<sub>x</sub>), particulate matters (PM), and carbon dioxide (CO<sub>2</sub>). Furthermore, minimization of imported fuel for those countries importing some fossil fuels (like Iran) should be considered as another objective.

Obviously, making decisions regarding the power generation and transmission expansion planning in separate frameworks has some deficiencies say reaching to sub-optimality. For this, the recent trend is doing so in an integrated model. In an integrated power generation and transmission expansion planning (G and TEP) framework, the following five decisions must be made:

- What types of generation units should be added to the current system?

- How much generation capacities should be added?
- When (i.e., in which time periods) these capacities should be added?
- Where these new capacities should be located?
- Where the new required transmission lines should be installed?

Sirikum, et al [1] formulate GEP problem for thermal power plants as a mixed integer nonlinear programming (MINLP) model. Their model incorporates the reliability constraints, environmental requirements and location constraints to determine an optimal electricity generation plan that minimizes the expected sum of discounted investment costs and variable costs which comprises of fuel costs, operating costs, and environmental costs of each new generating unit. They propose a genetic algorithm (GA) to solve their model heuristically.

In another study [2] they propose a mixed integer nonlinear model, which is more comprehensive than the first one, to evaluate the most economical investment plan for additional thermal power generating units with emission controls and apply a hybrid GA-Benders' decomposition (GA-BD) method for solving this complex problem. Park, et al [3] propose a hybrid approach combining a refined GA with the tunnel-based dynamic programming (DP), a method employed in the Wien Automatic System Planning Package (WASP) to solve GEP problem. The main advantage of this approach lies in the GA's capability to find the global optimum and the tunnel-based DP's high performance to get a local optimum. Fukuyama, et al [4] applied a parallel genetic algorithm (PGA) to solve generation expansion planning. They use an effective coding scheme and selection method in their proposed approach.

Because of the technical and economic differences of the additional power plants, Sevilgen et al [5] use economic methodologies to determine the best technology for the additional capacity. They use annual leveled cost method for this purpose, and the technology giving the minimum value for the additional load range is chosen. They also consider the changing rates of economic parameters such as interest rate, construction escalation, fuel escalation, maintenance escalation and discount factor which can affect the annual leveled cost

considerably and change the economic range of the plants in their study.

Antunes et al [6] present a multiple objective mixed integer linear programming model allowing the consideration of modular expansion capacity values of supply-side options. They consider demand-side management (DSM) as an option in the planning process and integrated resource planning (IRP) as its objective in their model. Non-dominated solutions to their MOMILP model are generated by means of an interactive algorithm based on a reference direction approach. Kannan, et al [7] apply particle swarm optimization (PSO) to solve the GEP problem. They propose a new technique called VMP which can be used in all PSO algorithms and it is found that the VMP procedure reduces the execution time, and the chances of yielding infeasible solution while increasing the efficiency. Finally, Chen, et al [8] propose a decision tool for utilities to perform the optimal generation expansion planning in a deregulated electricity market. Combining the immune algorithm (IA) and Tabu search (TS), a refined immune algorithm (RIA) is developed to solve this problem. By considering the various load types (peak load, middle load, basic load) and independent power producers (IPPs) competition, a generation expansion planning model is established under the operational constraints, reliability constraints and CO<sub>2</sub> constraints.

In the above-mentioned works, the location of generating units and cost of transmission lines are usually neglected. These approaches are appropriate for such situation in which the transmission network is strongly adequate and it is not necessary to expand it, but in the real situations especially for developing country where the transmission networks are often weak, this assumption cannot be valid. Therefore, developing the integrated generation and transmission expansion planning models are of particular interest. In this respect, Ceciliano Meza, et al [9] present a multi objective model minimizing total costs, environmental impact, imported fuel and fuel price risks simultaneously and decide about the location of the planned generation units taking into account the cost of transmission in a multi period planning horizon. Their solution approach is based on analytical hierarchy process (AHP).

GEP and especially G and TEP problems are typically large size problems which make the

solution process cumbersome. To alleviate this difficulty, Liu, et al [10] propose two different methods in order to reduce the number of variables in these models. One of these methods is on the basis of network topology and power system physical structure and characteristics, and to further reduce the number of variables and constraints, some operation state variables are replaced with the investment variables.

To the best of our knowledge, there are few integrated models in the literature developing generation and transmission expansion plans simultaneously. Furthermore, almost all of previous research works done in this area, assume that all input data can be determined precisely in spite of inherent fuzziness in the most of these parameters such as forecasted demands for electricity consumption as well as economic and technical characteristics of new generating and transmission technologies.

However, the literature review reveals that there is no research work applying the fuzzy mathematical modeling approach to take care of inherent imprecision in most of the input parameters of such integrated problems. Therefore, in this paper we propose an integrated power generation and transmission expansion planning model which is formulated as a fuzzy multi objective mixed integer linear programming model (FMOMILP). Moreover, in the proposed formulation, the maintenance cost is modeled in a new different way. In fact, it is assumed to be independent of the amount of generation and increases exponentially over time during the planning horizon.

The rest of this paper is organized as follows. In Section 2, we first demonstrate the proposed fuzzy MOMILP and then describe the process of converting the original fuzzy programming model into an equivalent auxiliary crisp one as well as its solution approach. Section 3 provides an illustrative example and the corresponding results. Finally, the Section 4 is devoted to concluding remarks.

## 2. PROBLEM FORMULATION

Most of the previous GEP problems have been modeled as single-objective programming models considering just the total cost objective function.

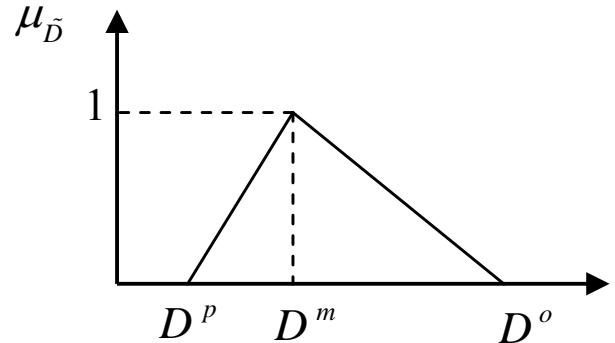
However, with the aid of multi-objective approaches, decision makers may grasp the conflicting nature and the trade-offs among the different objectives in order to generate some satisfactory compromise solutions for the G and TEP problem helping to reach a final preferred decision.

Here, the following three objectives have been considered in the model formulation:

- Minimizing the total cost of the system including construction, maintenance, operation, and transmission costs;
- Minimizing pollutant emissions since the electricity expansion planning policies and strategies have been currently giving emphasis not only on the fuel, operation, and investment costs, but also on the environmental issues.
- Maximizing of system reliability (in terms of cost of energy not served as a penalty for unreliability).

There is some trade-off between these objectives. For instance, if new generation units are added, the capital cost would be high, but because of having more capacity in the system, the cost of energy not served would be low, as the system would be more reliable.

The proposed model considers the transmission network in order to take into account the geographical impact of the new generation units, obtaining better results in the electricity supply chain. Furthermore, since the most of input data (parameters) including demand, reserve margin, transmission loss and generation unit capacity, construction cost, fuel cost, maintenance cost, transmission cost, forced outage rate, outage cost, amount of emissioned pollutant cannot be determined precisely, so that we model them by appropriate fuzzy numbers with triangular possibility distributions. As such, for each fuzzy parameter, we consider three prominent values i.e. the pessimistic, optimistic, and the most likely values based on considering both available objective data and subjective data quoted by the field experts. As an example, Figure 1 depicts the triangular possibility distribution of imprecise demand denoted by  $D = (D^p, D^m, D^o)$  where  $D^p$ ,  $D^m$ ,  $D^o$  denote the respective pessimistic, the most likely and the optimistic values, respectively.



**Figure 1.** The triangular possibility distribution of fuzzy demand  $\tilde{D}$

Following notations are used in the problem formulation:

## 2.1. Parameters

$CC_{klt}$	Construction cost of one generation unit of type k at location l in period t.
$MC_{klt}$	Maintenance cost of one generation unit of type k at location l in period t.
$FC_{klt}$	Fuel and operational cost of one generation unit of type k at location l in period t.
$TC_{ll't}$	Cost of adding new transmission line from location l to location $l'$ .
$E_{ko}$	Rate of pollutant type o emissioned by one generation unit of type k.
$OC_{klt}$	Outage cost of one generation unit of type k at location l in period t.
$FOR_k$	Forced outage rate of one generation of type k.
$r$	Discount rate.
$d$	A coefficient handling increasing nature of maintenance cost (which is estimated according to generating unit type).
$\tilde{\ell}$	Transmission loss percent.
$R$	Reserve margin.
$D_{lt}$	Demand of location l in period t.
$G_k$	Capacity of one generation unit of type k
$T$	Number of periods in the planning horizon
$A$	Set of transmission lines between different locations.

## 2.2. Decision Variables

$x_{klt}$	Cumulative number of generating units of type k which is constructed at location l by period t
$X_{klt}$	Number of generation unit of type k at location l in period t
$Y_{klt}$	Amount of power generated by unit of type k at location l until period t
$z_{ll't}$	The additional transmission capacity (MW) installed in line $(l,l')$ at period t
$iz_{ll't}$	Amount of power transmitted through line $(l,l')$ at period t
$cz_{ll't}$	The maximum transmission capacity of line $(l,l')$ in period t.

The proposed FMOMILP model can be stated as follows:

$$\begin{aligned} \text{Min } Z_1 = & \sum_k \sum_l \sum_t (1+r)^{-1} [\bar{C}_{klt} \cdot (x_{klt} - x_{kl,t-1}) \\ & + (1+d)^{T-(t-1)} \cdot \bar{MC}_{klt} \cdot X_{klt} + \bar{FC}_{klt} \cdot Y_{klt}] \\ & + \sum_{(l,l') \in A} \sum_t (1+r)^{-1} \cdot \bar{TC}_{ll't} \cdot z_{ll't} \end{aligned} \quad (1)$$

$$\text{Min } Z_2 = \sum_k \sum_l \sum_t \sum_o \bar{E}_{ko} \cdot Y_{klt} \quad (2)$$

$$\text{Min } Z_3 = \sum_k \sum_l \sum_t \bar{\Theta} C_{klt} \cdot \bar{FOR}_k \cdot Y_{klt} \quad (3)$$

Subject to:

$$\begin{aligned} & \sum_k Y_{klt} + (1-\tilde{\ell})[\sum_{(l,l') \in A} iz_{llt} - \\ & \sum_{(l,l') \in A} iz_{llt}] \geq \bar{D}_{lt} (1+\bar{R}), \quad \forall l, t \end{aligned} \quad (4)$$

$$Y_{klt} \leq \hat{Y}_{kl} + (X_{klt} - X_{kl0}) \bar{G}_k, \quad \forall k, l, t \quad (5)$$

$$Y_{klt} \leq \hat{Y}_{kl}, \quad \forall k, l \quad (6)$$

$$X_{klt} = X_{kl,t-1} + (x_{klt} - x_{kl,t-1}), \quad \forall k, l, t \quad (7)$$

$$X_{kl0} = \hat{X}; \quad x_{klt} \leq \hat{x}; \quad \forall k, l, t \quad (8)$$

$$x_{klt} \geq x_{kl,t-1}; \quad \forall k, l, t \quad (9)$$

$$\sum_k Y_{klt} \leq iz_{ll't}; \quad \forall (l, l') \in A, t \quad (10)$$

$$cz_{ll't} \leq c\hat{z}_{ll't}; \quad \forall (l, l') \in A, t \quad (11)$$

$$iz_{ll't} \leq cz_{ll't}; \quad z_{ll't} \leq \hat{z}_{ll't}; \quad \forall (l, l') \in A, t \quad (12)$$

$$cz_{ll't} = cz_{ll',t-1} + z_{ll't}; \quad \forall (l, l') \in A, t \quad (13)$$

$$X_{klt}, x_{klt} = \text{integer}; \quad z_{llt}, iz_{llt}, cz_{llt} \geq 0; \quad \forall k, l, t \quad (14)$$

In the proposed model, the three minimization objectives pertaining to the total cost of the system, pollutant emission and system's reliability are considered simultaneously:

Constraint (4) states that the demand plus reserve margin at each location, per each unit time should be satisfied by generating units of respective location and net transmitted power to that location.

Constraint (5) limits the amount of power generated by each type of unit at each location in each period by the initial generation capacity of this unit type plus the maximum capacity of added units.  $\hat{Y}_{kl}$  denotes the maximum capacity of generating units of type k at location l in the beginning of planning horizon.

Constraints (6) limit the amount of power which can be generated in the beginning of planning horizon by its maximum values.

Constraint (7) states that the number of power generating units of type k at location l in period t is determined by the corresponding number in period t-1 plus the number of units which are added at period t.

Constraints (8) force the initial number of generating unit of type k and the maximum number of unit of type k at location l that can be added during the planning horizon.

Constraint (9) states that the cumulative number of constructed unit of type k at location l in period t is greater than or equal to corresponding number at period t-1.

Constraint (10) limits the amount of power which can be transmitted from location l to other locations with total amount of power generated in location l.

Moreover, some policies exist that limit the transmission lines in each period, which are stated by constraint (11).

Constraint (12) limits amount of power which is transmitted and added in each period to its maximum capacity.

Constraint (13) states that the maximum power which can be transmitted in period t is determined by its corresponding value in period t-1 plus the amount which is added during the period t.

### 3. EQUIVALENT AUXILIARY CRISP MODEL

We apply an extended version of a well-known approach proposed by Lai, et al [13] to transform the initial FMOMILP model into an auxiliary crisp multi-objective mixed integer linear programming model. For doing so, the conversion process is divided into two parts: objectives defuzzification and constraints defuzzification.

**3.1. Objectives Defuzzification** There are several approaches to cope with an objective function with imprecise parameters, among them we apply the effective method proposed by Lai, et al [13] which has been considerably applied in the literature, for example see [11,12].

According to this method, the three objectives of our model are converted into the nine equivalent objectives. In this approach, for each minimization type objective, we try to minimize the most likely value of associated fuzzy parameter, maximize the left tolerance (the most likely value minus pessimistic value) and minimize the right tolerance (optimistic value minus the most likely value). The equivalent crisp objectives are as follows where the objective 1,2 and 3 are converted to set of objectives 15-17, 18-20 and 21-23, respectively:

$$\begin{aligned} \text{Min } Z_{11} = & \sum_k \sum_l \sum_t (1+r)^{-1} [\text{CC}_{klt,\beta}^m \cdot (x_{klt} - x_{kl,t-1}) \\ & + (1+d)^{T-(t-1)} \cdot \text{MC}_{klt,\beta}^m \cdot X_{klt} + \text{FC}_{klt,\beta}^m \cdot Y_{klt}] + \\ & \sum_{(l,l') \in A} \sum_t (1+r)^{-1} \cdot \text{TC}_{ll't,\beta}^m \cdot Z_{ll't} \end{aligned} \quad (15)$$

$$\begin{aligned} \text{Max } Z_{12} = & \sum_k \sum_l \sum_t (1+r)^{-1} [(\text{CC}_{klt,\beta}^m - \text{CC}_{klt,\beta}^p) \\ & \cdot (x_{klt} - x_{kl,t-1}) + (1+d)^{T-(t-1)} \cdot (\text{MC}_{klt,\beta}^m - \text{MC}_{klt,\beta}^p) \cdot X_{klt} \\ & + (\text{FC}_{klt,\beta}^m - \text{FC}_{klt,\beta}^p) \cdot Y_{klt}] + \\ & \sum_{(l,l') \in A} \sum_t (1+r)^{-1} \cdot (\text{TC}_{ll't,\beta}^m - \text{TC}_{ll't,\beta}^p) \cdot Z_{ll't} \end{aligned} \quad (16)$$

$$\begin{aligned} \text{Min } Z_{13} = & \sum_k \sum_l \sum_t (1+r)^{-1} [(\text{CC}_{klt,\beta}^0 - \text{CC}_{klt,\beta}^m) \\ & \cdot (x_{klt} - x_{kl,t-1}) + (1+d)^{T-(t-1)} \cdot (\text{MC}_{klt,\beta}^0 - \text{MC}_{klt,\beta}^m) \cdot X_{klt} \\ & + (\text{FC}_{klt,\beta}^0 - \text{FC}_{klt,\beta}^m) \cdot Y_{klt}] + \\ & \sum_{(l,l') \in A} \sum_t (1+r)^{-1} \cdot (\text{TC}_{ll't,\beta}^0 - \text{TC}_{ll't,\beta}^m) \cdot Z_{ll't} \end{aligned} \quad (17)$$

$$\text{Min } Z_{21} = \sum_k \sum_l \sum_t \sum_o E_{ko,\beta}^m \cdot Y_{klt} \quad (18)$$

$$\text{Max } Z_{22} = \sum_k \sum_l \sum_t \sum_o (E_{ko,\beta}^m - E_{ko,\beta}^p) \cdot Y_{klt} \quad (19)$$

$$\text{Min } Z_{23} = \sum_k \sum_l \sum_t \sum_o (E_{ko,\beta}^o - E_{ko,\beta}^m) \cdot Y_{klt} \quad (20)$$

$$\text{Min } Z_{31} = \sum_k \sum_l \sum_t OC_{klt,\beta}^m \cdot FOR_{k,\beta}^m \cdot Y_{klt} \quad (21)$$

$$\begin{aligned} \text{Max } Z_{32} = & \sum_k \sum_l \sum_t (OC_{klt,\beta}^m - OC_{klt,\beta}^p) \cdot \\ & (FOR_{k,\beta}^m - FOR_{k,\beta}^p) \cdot Y_{klt} \end{aligned} \quad (22)$$

$$\begin{aligned} \text{Min } Z_{33} = & \sum_k \sum_l \sum_t (OC_{klt,\beta}^o - OC_{klt,\beta}^m) \cdot \\ & (FOR_{k,\beta}^o - FOR_{k,\beta}^m) \cdot Y_{klt} \end{aligned} \quad (23)$$

In these equivalent crisp objectives,  $\beta$  denotes the minimum acceptable possibility level of occurrence for the corresponding imprecise/fuzzy data and should be determined by the decision maker.

**3.2. Constraint Defuzzification** Regarding the fuzzy constraints (4) which have imprecise parameters both in the left-hand side and right-hand side, we use the fuzzy ranking concept proposed by Lai, et al [13] and replace each imprecise constraint with three equivalent auxiliary inequality constraints, i.e., constraints 24-26. Moreover, to resolve the imprecise right-hand sides of constraints (5), the weighted average method [11-13] is used for converting the  $G_k$  parameter into its equivalent crisp number. Therefore, the equivalent constraint to constraints (5) are constraints (27) where the corresponding weights are considered as 1/6, 4/6, 1/6 for pessimistic, the most likely and optimistic values, respectively.

$$\begin{aligned} \sum_k Y_{klt} + (1-\ell_{\beta}^p) [\sum_{(l,l') \in A} iz_{l'lt} - \sum_{(l,l') \in A} iz_{ll't}] \\ \geq D_{lt,\beta}^p (1+R_{\beta}^p); \quad \forall l, t \end{aligned} \quad (24)$$

$$\sum_k Y_{klt} + (1 - \ell_\beta^m) [\sum_{(l,l') \in A} iZ_{l'lt} - \sum_{(l,l') \in A} iZ_{ll't}] \geq D_{lt,\beta}^m (1 + R_\beta^m); \forall l, t \quad (25)$$

$$\sum_k Y_{klt} + (1 - \ell_\beta^o) [\sum_{(l,l') \in A} iZ_{l'lt} - \sum_{(l,l') \in A} iZ_{ll't}] \geq D_{lt,\beta}^o (1 + R_\beta^o); \forall l, t \quad (26)$$

$$Y_{klt} \leq \hat{Y}_{kl} + (X_{klt} - X_{kl0}) (\frac{G_{k,\beta}^p + 4G_{k,\beta}^m + G_{k,\beta}^o}{6}); \forall k, l, t \quad (27)$$

**3.3. Solution Approach** There are several methods for solving multi-objective linear programming (MOLP) models, especially fuzzy programming approaches such as Lai and Hwang (LH) and more recently Torabi, et al (TH) methods [11,12]. In this paper, we apply the TH method, to solve the resulting auxiliary crisp model. This approach always yields efficient compromise solution. The steps of this solution method can be summarized as follows:

**Step 1.** Convert the original fuzzy objectives into their equivalent crisp objectives as shown through equations 15 up to 23.

**Step 2.** Given the minimum acceptable possibility level for imprecise parameters,  $\beta$ , convert the fuzzy constraints into the corresponding crisp ones, and formulate the resulting auxiliary crisp MOMILP model involving objectives 15-23 as well as constraints 6-14 and 24-27.

**Step 3.** Determine the positive ideal solution (PIS) and negative ideal solution (NIS) for each objective function by applying a similar method presented in [11,12].

**Step 4.** Specify a linear membership function for each objective function as follows. For the objectives which should be minimized, their corresponding linear membership functions are as follows (see Figure 2):

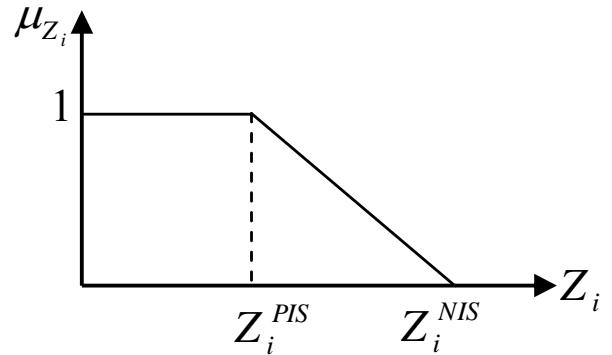
$$\mu_i(v) = \begin{cases} 1 & \text{if } Z_i \leq Z_i^{PIS} \\ \frac{Z_i^{NIS} - Z_i}{Z_i^{NIS} - Z_i^{PIS}} & \text{if } Z_i^{PIS} \leq Z_i \leq Z_i^{NIS} \\ 0 & \text{if } Z_i \geq Z_i^{NIS} \end{cases}$$

Furthermore, for the maximization objectives we would have (see Figure 3):

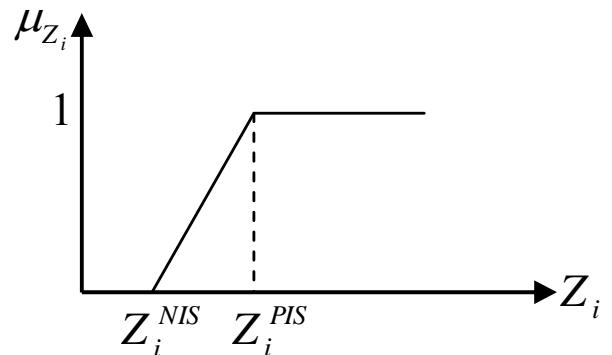
$$\mu_i(v) = \begin{cases} 0 & \text{if } Z_i \leq Z_i^{NIS} \\ \frac{Z_i - Z_i^{NIS}}{Z_i^{PIS} - Z_i^{NIS}} & \text{if } Z_i^{NIS} \leq Z_i \leq Z_i^{PIS} \\ 1 & \text{if } Z_i \geq Z_i^{PIS} \end{cases}$$

**Step 5.** Convert the auxiliary MOMILP model into an equivalent single-objective MILP one using the following new auxiliary crisp formulation proposed by Torabi, et al [11]:

$$\max \lambda(v) = \gamma \cdot \lambda_0 + (1 - \gamma) \cdot \sum_h \theta_h \mu_h(v)$$



**Figure 2.** Linear membership function of a minimization objective.



**Figure 3.** Linear membership function of a maximization objective.

s.t.

$$\mu_h(v) \geq \lambda_0; \forall h=1, \dots, 9$$

$$v \in F(v) \text{ and } \lambda_0, \gamma \in [0,1].$$

Where  $\theta_h$  and  $\gamma$  indicate the relative importance of the  $h$ -th objective function and the coefficient of compensation, respectively, and  $F(v)$  denotes the solution space of initial constraints in auxiliary crisp model. Noteworthy, the  $\theta_h$  parameters are determined by the decision maker based on her/his preferences such that  $\sum_h \theta_h = 1$ . Also  $\gamma$  controls the minimum satisfaction level of objectives as well as the compromise degree among the objectives, implicitly [11].

**Step 6.** Given the coefficient of compensation  $\gamma$  and relative importance of the fuzzy goals ( $\theta$ ), solve the auxiliary crisp model by a MIP solver (like CPLEX or LINGO). If the decision maker is satisfied with this current efficient compromise solution, stop. Otherwise, provide another efficient solution by changing the value of some controllable parameters like  $\beta$  and  $\gamma$  [11].

#### 4. ILLUSTRATIVE EXAMPLE

The proposed model has several novelty over the existing models in the literature (e.g., considered objectives and fuzziness in some critical parameters), so it cannot be compared with the existing models in the literature. Therefore, in this section an illustrative example is presented to show the practicality of the proposed model. We consider a problem with two locations, three types of generation units, and a three-periods planning horizon. It is assumed that country is divided into two parts: north and south. The considered generation unit types have denoted by X, Y and Z. The pollutant types of these generation units are: CO<sub>2</sub> and SO<sub>2</sub>. There are some policies such that in north area no type of generating unit must be constructed at the first period of planning horizon and in south area Z unit type cannot be constructed

at the first period. The maximum of transmission is 1000 MW at first and at each period of planning periods, 300 MW can be added to existing transmission lines.

The required parameters (input data) of the model are represented through Table 1 to 4. Other required data are as follows:

Discount rate ( $r$ ) = 10 %, reserve margin ( $\tilde{R}$ ) = (5, 10, 15%), transmission loss ( $\tilde{\ell}$ ) = (10, 15, 20%), Transmission cost ( $TC_{II'}$ ) = (400, 500, 550), ( $l, l'$ ) =  $\in A$  and  $d = 0.2$  for all types of generation units. Figure 4 shows for example, the membership function of capacity for Y generation units and its corresponding interval on minimum acceptable level ( $\beta = 0.5$ ). In other words, we have  $[G_{Y,\beta}^p, G_{Y,\beta}^m, G_{Y,\beta}^o] = [520, 560, 620]$  for  $\beta = 0.5$ .

**4.1. Computational Results** In this section, the proposed model is compared with the crisp model using the data withdrawn from the above example. The aim of this comparison is to demonstrate the usefulness and appropriateness of proposed fuzzy model operating in an uncertain decision-making environment. For the crisp model, we use the most likely values of corresponding fuzzy parameters.

According to the TH method [11], the parameters ( $\gamma, \theta$  and  $\beta$ ) are determined by the decision maker (DM). If the current solution does not satisfy DM, then problem will be solved for new values of these parameters.

In this respect, the decision maker provided the relative importance of objectives linguistically as:  $\theta_1 \geq \theta_2 = \theta_3, \theta_4 \geq \theta_5 = \theta_6$  and  $\theta_7 \geq \theta_8 = \theta_9$  for fuzzy model which means the more importance of  $\theta_1, \theta_4$  and  $\theta_7$  over others. Based on this relationship, the corresponding weight vector is set as:  $\theta = (1/6, 1/12, 1/12, 1/6, 1/12, 1/12, 1/6, 1/12, 1/12)$ . Notably, although the weight vector  $\theta$  can be more precisely determined using well-known MCDA techniques like Analytic Hierarchy Process (AHP) method, but we are not care about this issue and easily set it after an interaction with the decision maker. We also set the value of  $\gamma$  equal to 0.5. Recalling the above preference relationship among the values of  $\theta_h$ , the reason for selecting  $\gamma = 0.5$  is that getting somewhat unbalanced compromise solutions with higher satisfaction degree for  $\theta_1, \theta_4$  and  $\theta_7$  is of particular interest.

**TABLE 1. Cost and Capacity Data for the Different Type of Generation Units.**

Type	Parameters	$\tilde{C}C_{klt}$ (\$)	$\tilde{M}C_{klt}$ (\$)	$\tilde{F}C_{klt}$ (\$/MW)	$\tilde{F}OR_k$ (%)	$\tilde{G}_k$	Number of Candidate ( $\hat{x}$ )	
							$L_1$	$L_2$
X		(3000000,4000000, 500000)	(740,780,800)	(7,10,12)	(0.05,0.07,0.1)	(480,560,680)	1	2
Y		(750000,830000, 900000)	(890,930,960)	(4,5,7)	(0.09,0.1,0.12)	(250,350,410)	3	2
Z		(850000,900000, 300000)	(490,510,540)	(0.5,1,1.5)	(0.01,0.03,0.04)	(100,200,350)	2	3

**TABLE 2. Outage Cost ( $\tilde{O}C_{klt}$ ) - (\$/MW).**

Location \ Type	X	Y	Z
North	(3,5,6)	(6,8,9)	(1.5,2,3)
South	(5.5,6,7)	(8,9,9.5)	(2,3,5)

**TABLE 3. Forecasted Demand ( $\tilde{D}_{lt}$ )-(MW).**

Planning Period \ Location	1	2	3
North	(21272,21288,21294)	(21860,21886,21934)	(22509,22575,22664)
South	(19578,19595,19610)	(20202,20235,20245)	(21042,21234,21295)

**TABLE 4. Pollutant Omission Rate ( $\tilde{E}_{ko}$ ).**

Pollutant Emission \ Type	$CO_2$	$SO_2$
X	(0.3,0.35,0.39)	(0.54,0.59,0.63)
Y	(0.75,0.79,0.85)	(0.2,0.24,0.31)
Z	(0,0,0)	(0,0,0)

Now, for different values of  $\beta$  (i.e., 0.3, 0.5, 0.7, 0.9 and 1), the corresponding auxiliary models have been solved and the obtained results have been shown in Table 5. Moreover, for the crisp model we set  $\theta = (0.5, 0.25, 0.25)$  considering the relative importance of objectives linguistically quoted by the DM as  $\theta_1 \geq \theta_2 = \theta_3$ . The respective solution has also been shown in the last row of the Table 5.

From the results shown in Table 5, it can be observed that the best found solution by the fuzzy model outperforms the crisp one in all performance indicators (i.e.,  $\lambda(v)$ ,  $Z_1$ ,  $Z_2$  and  $Z_3$ ).

## 5. CONCLUSION

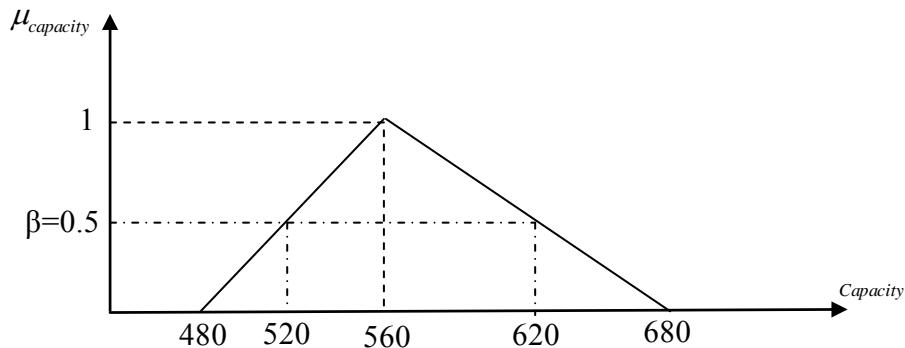
Most of the parameters involved in generation and transmission expansion planning problems cannot be precisely determined in real world. This paper proposes a fuzzy decision model to cope with this

uncertainty in the G and TEP context. Besides, the proposed fuzzy model offers flexibility for getting different compromise solutions based on DM preferences and reaching to more accurate and reliable solutions as shown by our numerical example.

There are various future research directions. Among them, due to computational complexity of the proposed model, especially in real size instances, developing an efficient meta-heuristic is of particular interest.

## 6. ACKNOWLEDGEMENT

This study was supported by the University of Tehran under the research grant No. 8109920/1/05. The authors are grateful for this financial support. We are also grateful to the anonymous reviewers for their valuable comments and constructive criticism.



**Figure 4.** The membership function of  $\tilde{G}_k$  and its interval for  $\beta = 0.5$ .

**TABLE 5. Results of TH Method for Different Value of  $\beta$ .**

$\beta$	$\lambda(v)$	$Z_1$	$Z_2$	$Z_3$
0.3	0.52	7870962	115615	75151
0.5	0.50	6268411	114851	74596
0.7	0.50	5536919	114275	74103
0.9	0.48	4973283	113376	73227
1	0.46	3827858	110394	70374
Crisp Solution	0.47	5014153	127136	73869

## 7. REFERENCES

1. Sirikum, J. and Techanitisawad, A., "Power generation Expansion Planning With Emission Control: A Nonlinear Model and a GA-Based Heuristic Approach", *International Journal of Energy Research, Wiley Inter Science*, (2005), 81–99.
2. Sirikum, J., Techanitisawad, A. and Kachitvichyanukul, V., "A New Efficient GA-Benders' Decomposition Method for Power Generation Expansion Planning with Emission Controls", *IEEE Transactions on Power Systems*, Vol. 22, No. 3, (2007), 1092-1100.
3. Park, J.B. and Won, J.R., "A Hybrid Genetic Algorithm/Dynamic Programming Approach to Optimal Long-Term Generation Expansion Planning", *Electrical Power and Energy Systems*, Vol. 20, No. 4, (1998), 295-303.
4. Fukuyama, Y. and Chiang, H.D., "A Parallel Genetic Algorithm for Generation Expansion Planning", *IEEE Transaction on Power Systems*, Vol. 11, No. 2, (1996), 955-961.
5. Sevilgen, S.H., Erdem, H.H., Cetin, B., Akkaya, A.V., and Dagdas, A., "Effect of Economic Parameters on Power Generation Expansion Planning", *Energy Conversion and Management*, Vol. 46, (2005), 1780–1789.
6. Antunes, C.H., Martins, A.G. and Brito, I.S., "A Multiple Objective Mixed Integer Linear Programming Model for Power Generation Expansion Planning", *Journal of Energy*, Vol. 29, (2004), 913-627.
7. Kannan, S., Slochanal, S.M.R., Subbaraj, P. and Padhy, N.P., "Application of Particle Swarm Optimization + Technique and Its Variants to Generation Expansion Planning Problem", *Electric Power Systems Research*, Vol. 70, (2004), 203–210.
8. Chen, S.L., Zhan, T.S. and Tsay, M.T., "Generation Expansion Planning of the Utility with Refined Immune Algorithm", *Electric Power Systems Research*, Vol. 76, (2005), 251–258.
9. Ceciliano Meza, J.L., Yildirim, M.B. and Masud, A.S.M., "A Model for the Multi Period Multi Objective Power Generation Expansion Problem", *IEEE Transactions on Power Systems*, Vol. 22, No. 2, (2007), ???-???
10. Liu, G., Sasaki, H. and Yorino, N., "Application of Network Topology to Long Range Composite Expansion Planning of Generation and Transmission Lines", *Electric Power Systems Research*, Vol. 57, (2000), 157-162.
11. Torabi, S.A. and Hassini, E., "An Interactive Possibilistic Programming Approach for Multiple Objective Supply Chain Master Planning", *Fuzzy Sets and Systems*, Vol. 159, (2008), 193-214.
12. Torabi, S.A. and Hassini, E., "Multi-Site Production Planning Integrating Procurement and Distribution Plans in Multi-Echelon Supply Chains: An Interactive Fuzzy Goal Programming Approach", *International Journal of Production Research*, Vol. 47, No. 19, (2007), 5475-5499.
13. Lai, Y.J. and Hwang, C.L., "Fuzzy Multi Objective Decision Making, Methods and Applications", Springer, Berlin, Germany, (1994).