NON-LINEAR BEHAVIOR AND SHEAR STRENGTH OF
DIAGONALLY STIFFENED STEEL
PLATE SHEAR WALLS

F. Nateghi and E. Alavi*

Department of Structural Engineering, International Institute of Earthquake Engineering and Seismology
P.O. Box 19395-3913, Tehran, Iran
nateghi@iiees.ac.ir - e.alavi@iiees.ac.ir

*Corresponding Author

(Received: September 27, 2008 – Accepted in Revised Form: July 2, 2009)

Abstract In this study, non-linear behavior of diagonally stiffened steel plate shear walls as a seismic resisting system has been investigated, and theoretical formulas for estimating shear strength capacity of the system have been proposed. Several validated analytical finite element models of steel shear walls with various stiffener dimensions are generated to verify and compare the analytical and theoretical outcomes. Non-linear transient analysis under monotonic loading are carried out and the pushover curves of the models are obtained. It is observed that the diagonal stiffeners have been able to reduce the buckling effects of the infill steel plate, and they have increased the elastic shear buckling strength and the ultimate shear capacity of the system in comparison with the un-stiffened thin steel plate shear walls, and there are good agreements between the propounded theoretical method and the analytical results.

Keywords Steel Plate Shear Walls, Diagonal Stiffeners, Shear Strength, Non-Linear Analysis

1. INTRODUCTION

During the last three decades many researches have been carried out on steel plate shear walls, SPSWs, and consequently they have been classified as reliable seismic load resisting systems in the high risk zones. SPSWs have been used in structural design and retrofitting of existing buildings with different configurations and philosophies, stiffened and un-stiffened. The first approach utilizes heavily stiffened steel plate shear walls with horizontal and vertical stiffeners to ensure that the infill steel plate reaches its full plastic strength prior to the elastic out-of-plane buckling, and stiffening of the steel plate improves its strength and prevents tension field from developing in the plate. Takahashi, et al [1], studied the stiffened SPSWs with usual light and heavy stiffeners and their experimental results showed that this system has high capability of earthquake input-energy dissipation and stable hysteresis loop with spindle shape instead of S shape. It has also high lateral stiffness, which limits its elastic shear displacement. Sabouri-Ghomi, et al [2], however, believe that construction of the numerous horizontal and vertical stiffeners is very time consuming and causes high-fabrication cost.
The second approach is to use un-stiffened thin SPSWs, which relies on post-buckling strength of infill steel plate due to tension field action development in the steel plate after the elastic out-of-plane buckling and dissipation of seismic energy through the cyclic yielding of the infill in tension. Hence, nonlinear behavior exhibits at relatively small story drifts and the significant pinching in the hysteresis loops appears specially when the boundary elements are not relatively so strong, pinching phenomenon occurs due to reduction in stiffness and capacities of the infill steel plate upon load reversal until the tension field action can develop in the opposite direction, however, a well-designed un-stiffened SPSWs can reach ultimate wall capacity and sustains it through high-ductility demands. Many recent research programs have been performed on this system, Thorburn, et al [3], Timler, et al [4], Lubell [5], Driver, et al [6], Berman, et al [7], etc., and some simplified analytical strip models and provisions have been consequently suggested and implemented in the codes and standards such as CSA-2001 [8], AISC-341-05 [9] for analysis and design of the un-stiffened SPSWs. Other approaches such as using of low yield steel material by Nakashima, et al [10], slits in SPSWs by Toko Hitaka, et al [11], composite shear walls by Astaneh-Asl [12], etc. have been also studied for improving seismic behavior of SPSWs system.

This paper introduces diagonally stiffened steel plate shear wall as an alternative new type system, and intends to incorporate efficiencies of the stiffening approach into seismic behavior of un-stiffened thin steel plate shear wall with using minimum number of stiffeners by increasing elastic shear buckling stress limit of the steel plate and shear strength capacity of the shear wall, and by reducing overall buckling of the infill steel plate effects and consequently improving of the nonlinear behavior of the system.

Stiffening of a shear panel by diagonal stiffeners is not generally a new method, and this idea is previously elaborated in stiffening of the panel zones for instance in the rigid connections of gable frames. Moreover, Yonezawa, et al [13] studied plate girders with weds diagonally stiffened between vertical stiffeners, their experimental and theoretical investigations resulted good performance of the diagonal stiffeners in stiffening of the plate girders webs and concluded that the diagonal stiffeners allows tension field action to develop in the steel plate (despite to the heavily stiffened webs).

In this study, the derived formulas for stiffened plate girders by other researchers have been extended to SPSWs with diagonal stiffeners with considering differences between non-linear behavior of the plate girders and SPSWs and role of the surrounding steel frame, the results of theoretical and analytical studies on nonlinear behavior of diagonally stiffened SPSWs are presented and relevant formulas for estimating shear strength capacity of this system are also suggested. Several analytical finite element models of SPSWs, which are validated with experimental studies of other researchers and codes provisions, with various stiffener dimensions and slenderness of infill steel plate, are selected to verify and compare the analytical and theoretical outcomes.

2. SHEAR STRENGTH CAPACITY

The methodology to estimate the ultimate shear strength, \( V \), of diagonally stiffened infill steel plate with X-shaped stiffeners is based on summation of the shear strength contributions of separate parts of the wall, infill steel plate, stiffeners, peripheral frame, with the following assumptions:

1. Columns are rigid enough so that uniform tension field develops throughout the steel plate.
2. Steel plate is simply supported along its boundaries.
3. Principle of superposition can be applied.

Therefore, the ultimate shear strength is proposed as:

\[
V = V_{cr} + V_t + V_{st} + V_{sc} + V_f
\]

Where,

- \( V_{cr} \) Shear force, taken by the steel plate as shear buckling strength of diagonally stiffened plate.
- \( V_t \) Shear force, taken by the diagonal tension field action in the steel plate.
\( V_{st} \) Shear force, taken by the diagonal tensile stiffener.

\( V_{sc} \) Shear force, taken by the diagonal compressive stiffener.

\( V_f \) Shear force, taken by the frame if the beam-to-column connections are rigid.

The shear force, \( V_c \), is given by:

\[
V_c = \frac{\tau_c b t}{2} \tag{2}
\]

in which \( \tau_c \) is the shear buckling stress of diagonally stiffened plate and is obtained by:

\[
\tau_c = \frac{K \pi^2 D}{b^2 t} = \frac{K \pi^2 E}{12(1-\nu^2)} \left( \frac{t}{b} \right)^2 \leq \tau_{yw} = \frac{\sigma_{yw}}{\sqrt{3}} \tag{3}
\]

where \( D, E, b, t, \nu, \) and \( \sigma_{yw} \) are the flexural rigidity, Elastic Modulus, width and thickness, Poisson’s ratio, and yield stress of the steel plate respectively, \( K \) is elastic buckling coefficient of diagonally stiffened simply supported plate subjected to shear stress. With considering mathematical study and comparison of the results, it can be inferred that for X-shaped stiffened SPSW the proposed expressions by Yonezawa, et al \[13\] for compression stiffeners type as a function of aspect ratio can be used as:

\[
K = 11.9 + 10.1/\phi + 10.9/\phi^2 \tag{4}
\]

The magnitude of \( K \) depends upon aspect ratio \( \phi = \frac{d}{b} \), where \( d \) is the height of steel plate. For estimating the shear force \( V_f \), first a chronological and brief review of key researches on development of theories on modelling of the tension field action behavior and their differences in the plate girders and SPSWs, is presented as follows:

Wagner \[14\] used a complete and uniform tension field to determine the shear strength of a panel with rigid flanges and very thin web, and inferred that the shear buckling of a thin aluminium plate supported adequately on its edges does not constitute failure, this idea has been recently developed for modelling of thin SPSWs. Basler \[15\] carried out shear tests on plate girders with vertical stiffeners and developed a failure theory based upon a theoretical model with a diagonal band of tension yield. He ignored the effects and contribution of the flanges to shear strength of the plate girder. Rocky, et al \[16\] found that the rigidity of flanges has a strong influence upon the behavior of panel and when the flanges are very light the collapse mechanism approximates to that assumed by Basler, but if they are heavy the plastic hinges form in the four corners of the panel, and for intermediate flanges two numbers of the plastic hinges will be located at the flanges and the remained two ones at the corners of the panel.

Following these studies, Thorburn, et al \[3\] developed an analytical method based on the Wagner’s work to evaluate the shear resistance of thin un-stiffened SPSWs and introduced the strip model to represent the shear panel as a series of inclined tensile strips. Timler, et al \[4\] modified formula for angle of strips inclination with the column \( \alpha \), which is proposed by Canadian Steel Design Standard, CAN/CSA-S16-01 \[17\], Equation 8.

As a consequence, it is here presumed that the surrounding frame members of diagonally stiffened SPSWs have also enough strength and stiffness, whereat plastic hinges form at the panel corners and none plastic hinges occurs along with members lengths due to tension field action development into the infill plate, and failure mechanisms are like plate girders with heavy flanges and un-stiffened thin SPSWs, hence, by using the inclination \( \theta \) of tension field with respect to the horizontal axis, value of diagonal tensile stress, \( \sigma_t \), from Von Mises yield criterion and work of Sabouri-Ghomi, et al \[2\] is obtained as:

\[
3\tau_c^2 + 3\tau_c^2\sigma_t \sin 2\theta + \sigma_t^2 - \sigma_{yw}^2 = 0 \tag{5}
\]

\[
\sigma_t = -\frac{3}{2}\tau_c \sin 2\theta + \sqrt{\sigma_{yw}^2 + \left(\frac{9}{4} \sin^2 2\theta - 3\right)\tau_c^2} \tag{6}
\]

\[
\theta = 90 - \alpha \tag{7}
\]

Where inclination \( \alpha \) is:

\[
\alpha = \tan^{-1} \left( \frac{1 + \frac{t L}{2 A_c}}{1 + \frac{t h}{A_p + \frac{h^3}{360 L}} \left( \frac{1}{A_p} + \frac{1}{360 L} \right)} \right) \tag{8}
\]
in which $A_c$, $A_b$, $I_c$, $L$, and $h_s$ are the cross-sectional areas of column, beam, moment inertia of the column, span length and the height of story respectively, Figure 1.

Then the shear force $V_t$ can be determined from vertical component of the tension field force as:

$$V_t = \frac{1}{2} t b \sin 2\theta$$

(9)

The shear forces, $V_{st}$ and $V_{sc}$ are expressed as:

$$V_{st} = A_s \sigma_{st} \cos \theta_d$$

(10)

$$V_{sc} = A_s \sigma_{sc} \cos \theta_d$$

(11)

Where $A_s$ is the cross section area of tensile or compressive diagonal stiffener, and $\theta_d$ is the inclination of the diagonal stiffeners with respect to the horizontal axis, Figure 1. Expression $\sigma_{st}$ is the tensile axial stress, and $\sigma_{sc}$ denotes the compressive axial stress in the diagonal stiffeners. They are determined based on Hook’s law (normal stress $\sigma_t$ derives perpendicular to the field tensioned stress, $\sigma_t$, direction due to constraining the plate at the stiffeners contact line), and Mohr’s circle analysis in order to stress transformation, Timoshenko [18], by superposing the shear buckling and tension field stresses effects and using $\sigma_t$ and $\theta$ by means of Equations 6 and 7. Hence, the tension stress in the diagonal stiffeners can be obtained as:

$$\sigma_{st} = \sigma_t [1 - (1 + \nu) \sin^2(\theta_d - \theta)] + [(1 + \nu) \tau_{cr} \sin 20\theta_d] \leq \sigma_{ys}$$

(12)

and the compression stress in the diagonal stiffeners can be resulted as:

$$\sigma_{sc} = -\sigma_t [1 - (1 + \nu) \sin^2(\theta_d + \theta)] + [(1 + \nu) \tau_{cr} \sin 20\theta_d] \leq \sigma_{crs}$$

(13)

where $\sigma_{ys}$ is the yield stress and $\sigma_{crs}$ is the compressive buckling stress of diagonal stiffeners, the value of $\sigma_{crs}$ can be obtained and used for the diagonal stiffeners of SPSWs from the following formula given by Basler-Thürlimann, and proposed by Yonezawa, et al [13] for diagonal stiffeners of plate girders as:

$$\sigma_{crs} = \sigma_{ys} \left(1 - 0.53(\lambda - 0.45)^{1.36}\right) \text{ for } 0.45 \leq \lambda < \sqrt{2} ;$$

(14)

$$\sigma_{crs} = \sigma_{ys} \text{ for } \lambda < 0.45 ;$$

(15)

Where,

$$\lambda = \left(\frac{b_s}{t_s}\right) \sqrt{12(1 - \nu^2)(\sigma_{ys}/E)(\pi^2 k_S)} ;$$

(17)

$$k_S = \left(\frac{b_s}{t_s}\right)^2 + 0.425$$

(18)

in which $b_s$, $t_s$ and $l$ are width, thickness and effective length of the diagonal stiffeners respectively. Besides, upon the analytical results and to preclude the local buckling of the stiffeners it is recommended that the diagonal stiffeners should have the width-thickness ratio $(b_s/t_s)$ limitation as per AISC-360-05 [17] expression for

**Figure 1.** SPSW with the diagonal stiffeners and relevant parameters under shear load.
the transverse stiffeners as given by:

\[ b_s / t_s \leq 0.56 \frac{E}{\sigma_{ys}} \]  

(19)

The shear force of the surrounding frame depends upon the rigidity of beam-to-column connections, and as an evident if the connections were assumed theoretically to be the simple type then \( V_f = 0 \), and for rigid type \( V_f \) can be estimated as shear capacity of a collapse mechanism from forming rigid plastic hinges in the boundary members [7]. Besides, Pushover analysis can be performed for finding the governing failure mechanism of rigid frames. Results of studies on the stiffened shear walls show that in a single story with rigid beam-to-column connections the plastic hinges will most probably form at the top and the bottom of the columns instead of the beams, and \( V_f \) can be estimated as given in Equation 20, Nateghi, et al [19].

\[ V_f = 4M_{pc} / h_s \]  

(20)

where

\[ M_{pc} = \text{Column Plastic Moment} \]

Eventually, Substitution of Equations 2, 9-11 and 20 into Equation 1 in terms of the defined expressions results Equation 21, which represents the ultimate shear strength of the single story, diagonally stiffened SPSW with rigid beam-to-column connections and X-shaped stiffeners, as follows:

\[ V = \tau_{cr} bt + \frac{1}{2} \sigma_{ts} bt \sin 2\theta + A_s (\sigma_{st} + \sigma_{sc}) \cos \theta_d + \frac{4M_{pc}}{h_s} \]  

(21)

3. ANALYTICAL STUDY

Several numerical models were generated from one story stiffened and un-stiffened SPSWs to assess ability of the presented theoretical approach to evaluate the shear load capacity of diagonally stiffened SPSWs, and to describe their behavior during seismic loads. All modelling were conducted using general-purpose nonlinear finite element program, ANSYS, which is properly suited for the solution of highly nonlinear engineering problems like SPSWs.

3.1. Numerical Methods Verification

The numerical models are validated with available experimental data in the literature, and those models with not having the experimental data are validated with the codes results expectations for their un-stiffened or commonly stiffened models. The kinematic hardening plasticity model has been invoked with multi-linear kinematic hardening material model for mild steel materials; and the elastic properties are assumed isotropic. If the loading on the structure is considered perfectly in-plane, the buckling will not analytically develop unless the out-of-plane deflections are applied to initiate the buckling, hence, to prevail to this problem in the analysis of SPSWs several methods are suggested by researchers in the literature that some of them are presented as follows:

Xue, et al [20] used initial imperfections obtained from superposition of several shear buckling modes of the infill plates in finite element analysis of a 12-story SPSW. Driver, et al [6] incorporated initial imperfections in the finite element analysis of their test specimen based on the first buckling mode of the infill plate. Behbahani, et al [21] tested a large-scale 3-story SPSW specimen, the specimen showed high initial stiffness, excellent ductility and energy absorption, stable hysteresis loops, however, characteristic pinching of hysteresis loops was observed in the inelastic range. They also developed a finite element model based on the nonlinear dynamic explicit formulation for numerical study of the specimen and a kinematic hardening model to simulate the Bauschinger effect, they applied an initial imperfections corresponding to the buckling mode of the shear wall with maximum amplitude of 10 mm in the analysis.

Consequently, two analysis methods are used in this study. In the first method a preliminary eigenvalue buckling analysis of the structure has been performed to predict the buckling mode shapes, then appropriate perturbations have been
applied to simulate the possible buckling response, and small displacement transient non-linear analysis has been done; this method is used for preliminary analysis.

In the second method, finite element geometrically nonlinear analysis by means of large deflection transient analysis has been executed; therefore the local buckling and the post-buckling effects of steel elements have been incorporated into the results. This method usually needs very trial and error, and used after the first method.

In both methods, the implicit solution procedure based on Newmark’s algorithm is utilized, and 4-node plastic shell with six degrees of freedom at each node, shell 181, is employed for 3D-modelling of the shear walls, and appropriate time-stepping by the trial and error is used to overcome to the convergence problem. The analytical results are validated by comparing them with the available experimental results in the literature, for that mean SPSW2 specimen of Lubell, et al [5,22] work is selected and modelled. Figure 2 represents the experimental model of SPSW2. Steel materials of the boundary elements (S 75 x 8) and the infill steel plate (1.5 mm) have been different in this specimen. Figure 3 shows the stress-strain curves of the steel materials obtained from the coupon tests, and used in the numerical analysis. The load-displacement curves from non-linear finite element modelling, FEM, analysis and the experimental results are compared in Figure 4. This figure also contains other experimental results of samples SPSW1 and SPSW4 from Lubell’s works, shown in dash-lines, which are not reviewed here. It can be inferred that the analytical model have been successful to estimate the actual shear capacity of the system with good approximate precision (less than 5% deviation).

The other nonlinear results such as horizontal displacements, the stresses based on Von Mises yield criterion and out-of-plane deformation are presented in Figure 5. It is observed that the infill steel plate is completely yielded and the columns have reached near the failure limits at their connections to the beams.

3.2. Numerical Analysis Results

After validation of the analytical methods, the specimen SPSW2 has been stiffened by two-sided diagonal stiffeners with various sizes and push-over analysis has been executed and ultimate shear capacities of the new stiffened systems are evaluated. Figure 6 and Figure 7 show FEM results of the diagonally stiffened SPSW2. It can be observed that the infill plate is yielded, and with reduction of the width to thickness ratio of the diagonal stiffeners, their local buckling has been reduced. Furthermore, the diagonal stiffeners have been able to deduct the buckling lengths of the inclined strips in comparison with the un-stiffened model. The force-displacement curves are drawn in Figure 8. This figure also shows that the drifts of all systems have reached near 4%, which is a good range for the ductile systems. The shear strengths of the stiffened SPSWs have become greater, and their stiffness has been nearly from 15% to 30% more than the un-stiffened steel shear wall results. The increments rates relate also to the diagonal stiffeners dimensions. For further investigation a full-scale sample, SPSW(s) 3m × 3m, is arbitrarily selected and its analytical results are presented hereinafter.

Beam-to-column connections of this specimen is also assumed to be rigid and the boundary elements are such designed to meet the requirements of steel plate shear walls and AISC 341-05 [9] provisions as seismically compact sections. The columns flanges and the webs are PL. 300 mm × 18 mm and PL. 300 mm × 12 mm, respectively. The beams flanges and the webs are PL. 250 mm × 12 mm and PL. 200 mm × 18 mm, respectively. The continuity plates, the base plates and the relevant stiffeners are made of PL. 18 mm (thick). The infill plate is PL. 2700 mm × 2700 mm × 3 mm, and all plate connections to each other are

![Figure 2 SPSW2 experimental model, lubell-97.](image-url)
assumed to be continuous at shell element contact surfaces nodes, none weld is modelled. The steel materials are assumed mild steel similar to ST-37 type with the yield stress limit of 240 MPa, and the ultimate stress limit of 360 MPa.

Some of the un-stiffened and diagonally stiffened analytical results are presented in Figure 9 and Figure 10. With reference to Figures 9a,b it can be derived that the diagonal stiffeners have had effective role on reduction of the buckling lengths of the strips near to the half, which this results increase of the elastic shear buckling strength of the infill plate. Furthermore, as reflected in Figure 10 the stiffeners have decreased the concentrated stresses at the edges and distributed them as well.

The pushover curves of the rigid frame only and
the un-stiffened and stiffened samples up to 3.3% drift (100 mm) are shown in Figure 11, it shows that as the previous model the shear strengths of the stiffened models are increased in comparison with the un-stiffened model and all models have had ductile performance in the non-linear ranges.

**Figure 5.** SPSW2 FEM models (a) von-mises stresses (Pa.) and (b) out-of-plane deflection (meter).
4. COMPARISON OF THE RESULTS

Comparison between the theoretical and the analytical outcomes has been made in Table 1 for both SPSW2 and SPSW(s) models. It can be inferred that there are good agreements between the theoretical and the analytical results. Besides, variations of some parameters and their effects on...
the final results can be investigated thereof, for instance, it can be extracted that $V_{cr}$ has been increased more than 3 times in the diagonally stiffened SPSWs in comparison with the unstiffened models, and their ultimate shear strengths have been increased about 15% to 40% or even more, depending also to the stiffeners dimensions.
Figure 8. FEM Force-displacement curves for frame only, un-stiffened and stiffened SPSW2 with various diagonal stiffeners.

Figure 9. Out-of-plane deflection (meter) for (a) un-stiffened SPSW (s) and (b) diagonally stiffened SPSW (s) with 2 × 2 PL 100 × 12 mm.

5. CONCLUSIONS

The following summarized results can be pointed out from the theoretical and analytical study on diagonally stiffened steel shear walls:

1. The elastic buckling shear capacities of the infill steel plates in the diagonally stiffened SPSWs have been increased more than 3.0 times in comparison with the un-stiffened SPSWs.
2. Two-sided diagonal stiffeners have increased ultimate shear strength of SPSWs.
3. The push-over curves showed that the diagonally stiffened shear walls also had appropriate ductility.
4. With reference to the given out-of-plane
displacements contours, it can be observed that the diagonal stiffeners have affected on the buckling effective lengths of the infill steel plate, whereat the buckling lengths of the inclined strips have been reduced up to their half in comparison with the unstiffened SPSWs.

5. By reduction of the width to thickness ratio of diagonal stiffeners, their contributions in bearing of the loads have been slightly increased and the local buckling phenomenon in them has been reduced.

6. The presented theoretical formulas have been able to estimate the ultimate shear capacities of the diagonally stiffened SPSWs.

Figure 10. Von-mises stresses (Pa.) for (a) un-stiffened SPSW(s) and dia. stiffened SPSW(s) with (b) 2 × 2 PL 100 × 5 mm, (c) 2 × 2 PL 100 × 10 mm and (d) 2 × 2 PL 100 × 12 mm.
Figure 11. FEM force-displacement curves for frame only, un-stiffened and stiffened SPSW(s) with various diagonal stiffeners.

### TABLE 1. Comparison between Theoretical and Analytical Results for Ultimate Shear Loads.

<table>
<thead>
<tr>
<th>Model</th>
<th>Diagonal Stiffeners</th>
<th>Theoretical Values</th>
<th>Analytical Values</th>
<th>K/LA (0/VA)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ts mm</td>
<td>bs/ts</td>
<td>$\tau_{cr}^{(2)}$ MPa</td>
<td>$\alpha$ Deg.</td>
</tr>
<tr>
<td>0$^{(1)}$</td>
<td>-</td>
<td>6.3</td>
<td>37</td>
<td>310.9</td>
</tr>
<tr>
<td>SPSW 2 (Lubell-97)</td>
<td>2</td>
<td>14.75</td>
<td>20.6</td>
<td>37</td>
</tr>
<tr>
<td>SPSW 2 (Lubell-97)</td>
<td>4</td>
<td>7.375</td>
<td>20.6</td>
<td>37</td>
</tr>
<tr>
<td>SPSW 2 (Lubell-97)</td>
<td>6</td>
<td>4.92</td>
<td>20.6</td>
<td>37</td>
</tr>
<tr>
<td>0$^{(1)}$</td>
<td>-</td>
<td>2.2</td>
<td>40</td>
<td>236.7</td>
</tr>
<tr>
<td>SPSW (6)</td>
<td>5</td>
<td>20</td>
<td>7.7</td>
<td>40</td>
</tr>
<tr>
<td>SPSW (6)</td>
<td>10</td>
<td>10</td>
<td>7.7</td>
<td>40</td>
</tr>
<tr>
<td>SPSW (6)</td>
<td>12</td>
<td>8.33</td>
<td>7.7</td>
<td>40</td>
</tr>
</tbody>
</table>

(1) Un-stiffened

(2) For un-stiffened SPSW \( K = \frac{5.35}{4/\phi^2}, \phi \geq 1 \) have been taken in calculation of $\tau_{cr}$ in the Equation 3, [17].
6. REFERENCES


