

# SIMULTANEOUS DUE DATE ASSIGNMENT AND LOT SIZING WITH UNCERTAIN FLOW TIMES

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**Abstract** Due date assignment for customer orders has been studied in various production environments using different modeling approaches. In this paper the researchers consider a production system in which the orders of several customers are produced in a single batch because of the economy of scale. If a batch is completed before receiving customer orders, inventory carrying cost is incurred but if it is completed after receiving customer orders, shortage cost is incurred and finally if the order is delivered after its due date, tardiness cost is charged. The problem is to decide on batch size, due date of batch (imposed on supply process) and lead time (to be quoted to the next coming customers) so that relevant costs are minimized. The objective function is to minimize total cost of supply, inventory carrying, shortage and tardiness. Production flow times are probabilistic which follow a probability distribution. The proposed model is solved using real-coded genetic algorithms and numerical results are presented. This work was motivated by a heavy equipment production system which has some products with relatively low demand rates, high prices and long supply lead times.

**Keywords** Due Date Assignment, Lot-Sizing, Lead Time Quotation, Real-Coded Genetic Algorithms

**چکیده** در مورد تعیین موعد تحویل سفارش مشتریان در محیط های تولیدی مختلف و با استفاده از روش های مدل سازی متفاوتی، تحقیق شده است. در این مقاله، یک سیستم تولیدی را در نظر می گیریم که به دلیل صرفه جویی ناشی از مقیاس، سفارش چندین مشتری در یک بچ، تولید می شوند. اگر تولید بچ، قبل از دریافت سفارش مشتریان پایان یابد، هزینه نگهداری موجودی وجود می آید اما اگر بعد از دریافت سفارش مشتریان تکمیل شود، هزینه کمبود خواهیم داشت و در نهایت اگر سفارشی بعد از موعد تعیین شده، به مشتری تحویل داده شود، هزینه تأخیر ایجاد می گردد. مسئله عبارت است از تصمیم گیری در مورد اندازه بچ، موعد تحویل بچ (اعمال شده بر فرایند تأمین) و مدت تحویل (که به مشتریان اعلام می گردد)، به نحوی که مجموع هزینه ها حداقل گردد. تابع هدف، حداقل سازی کل هزینه های تأمین، نگهداری موجودی، کمبود و تأخیر است. زمان های جریان تولید، احتمالی بوده و از یک توزیع احتمالی پیروی می کنند. مدل ارائه شده، با استفاده از الگوریتم ژنتیک با کدینگ حقیقی، حل شده و نتایج عددی ارائه شده اند. این تحقیق بر اساس یک سیستم تولید تجهیزات سنگین انجام گرفت که تعدادی محصول با تقاضای نسبتاً کم، قیمت بالا و مدت تأمین طولانی، تولید می نماید.

## 1. INTRODUCTION

In make-to-order production systems, it is crucial to have short lead times and a good due date performance. With the latter it is meant that orders are delivered as close as possible to their due dates. Researches have shown that price, quality and response time are the most important criteria in

contractors selection. Hence trying to reduce lead times and on time delivery is essential for competition.

In quoting lead times to the customers a trade-off has to be made between the length of lead times and reliability of the lead time. Promising a short lead time might lead to an impossible task for the order realization function with regard to delivering

the order close to the order due date. On the other hand long lead times make it easy for the order realization function to obtain a good due date performance but these lead times are, in general, not acceptable for the customers. Long lead times not only dissatisfy customers but also increase WIP and total inventories. Many authors have studied this trade-off between the length of lead times quoted to the customer and the delivery reliability. In due date assignment two kinds of costs exist:

- Too long lead time costs (lost sales, lower prices, customer dissatisfaction,...)
- Too short lead time costs (Tardiness penalties, more production costs, quality problems, customer dissatisfaction,...)

Due dates must be set so as to minimize total costs. In other words, due dates must be attractive for customers and profitable for manufacturers.

The majority of published papers about due date assignment assume a make-to-order production system in which each customer order is quoted and processed separately. The researcher's contribution is formulating and solving due date assignment in a make-to-demand production system in which the order of several customers are aggregated and produced in a single batch.

The rest of this paper is organized as follows. In Section 2, some of the relevant literature is summarized while Section 3 presents this paper's analytical model. In Section 3, the specific problem is described and modeled. In Section 4 real-coded genetic algorithms are applied to solve the sample problems and analyze the results.

## 2. LITERATURE REVIEW

In this paper a problem is considered which is simultaneously related to due date assignment and lot sizing literature. Hence the principal research streams in these distinct areas are reviewed. Sections 2.1, 2.2 are about due date assignment and lot sizing respectively.

**2.1. Due Date Assignment Literature** There are many published papers about due date

assignment problem. Here three principal research streams are recognized:

- Studies that combine due date assignment with orders sequencing and scheduling problems [13,15,22,36]. In this line of research, production systems may include single machines, parallel machines, job shop, flow shop or continuous.
- Studies that use a simulation approach to estimate order lead time based on some independent variables [12].
- Studies that use flow time approach for due date assignment. In this approach, each order has a flow time which usually has a probability distribution function. [23,28]

Key factors that differentiate between the above streams are:

- Amount and precision of information considered about workshop schedule and new arriving orders: When due date assignment is mixed with scheduling, very detailed information about workshop load (jobs already in system) is considered. But when flow time approach is used, information about workshop load is considered with some approximations. Some empirical methods use non-stochastic estimates like total work content (TWK), total work on the critical path (TWKCP), job processing time plus a slack (SLK), the number of operations (NOP), processing time and current queue length (JIQ), job processing time, waiting time and number of jobs in the system (JIS).
- Decision variables : may include optimal due date, product price [16], order acceptance ...
- Objective function: may be minimizing costs (including tardiness and earliness cost, long quoted lead time cost) or maximizing profit (when decision has an effect on sales revenue)

The flow time approach is followed in this paper so papers that are in this line of research are concentrated on.

Bertrand and Ooijen [28] examine lead time quotation for coming orders. Each order has some revenue which is a descending linear function of quoted lead time. Order's flow time has a specific

distribution function. Late deliveries incur two kinds of tardiness costs (independent of tardiness time and proportional to the amount of tardiness). The objective function is maximizing profit which is revenue minus lead time related costs. Optimal lead time is obtained by derivation. Finally they study the model performance using a simulation model. Ooijen and Bertrand [23] follow the approach of Bertrand and Ooijen [28]. In this paper they try to use new order information and work loads are taken into account. They define Internal and external due dates. Internal due dates are used for determining the priorities on the shop floor and external due dates are quoted to the customer. Internal due date is a function of arrival time of order, number of operations of order, processing time of operations, number of orders in the shop at the arrival time of order, number of machines in the shop and steady state utilization. At the end of the paper they use simulation to verify model performance.

Song, et al [11] investigates the problem of assigning product due dates for complex multistage assemblies. Manufacturing and assembly processing times are stochastic. In order to analyze multistage assemblies their product structures are decomposed into two-stage subsystems. A two-stage assembly system has some components that are processed in parallel and then assembled. In assigning product due dates it is assumed that there is a production plan of start times which is being followed. An operation begins at its planned start time, unless it is delayed. It would then start immediately after the completion of previous activities, or in the case of assemblies, when all components are available. The completion time of a component and first stage is obtained based on these assumptions. They use three different forms of earliness and tardiness cost functions (absolute deviation, squared deviation and linear combination of positive and negative deviations).

Saibal and Jewkes [39] study due date assignment when both demand rate and product price are lead time sensitive. Orders arrive according to a Poisson process and processing times are exponentially distributed. The firm's objective is to maximize profits by selecting an optimal guaranteed delivery time taking into account that (i) reducing delivery time will require investment, and (ii) the firm must be able to satisfy a pre-specified service level. Mean demand rate is

modeled as a decreasing linear function of price and delivery time while price itself is a decreasing linear function of the guaranteed delivery time. Hence demand rate is a function of delivery time. They assume that the investment function takes a linear form and later discuss the extension to non-linear investment functions. The firm has a unit operating cost, initially assumed to be constant (i.e., economies of scale do not exist).

Veral [12] uses a simulation model for estimating the order flow time which is the base of due date assignment. An average allowance for operation  $i$  at machine  $j$  is calculated using the processing time of operation  $i$  at machine  $j$  and average utilization of machine  $j$ . The flow allowance for a job is calculated as the sum of all its operations.

**2.2. Lot Sizing Literature** The concept of batching has been examined in various families of problems. Two major families of problems are lot sizing and lot scheduling problems. For a review of lot sizing problems the reader is referred to [52]. Lot sizing problems are production planning problems with setups between production lots. Because of these setups, it is often too costly to produce a given product in every period. On the other hand, generating fewer setups by producing large quantities to satisfy future demands, results in high inventory holding costs. Thus, the objective is to determine the periods where production should take place, and the quantities to be produced, in order to satisfy demand while minimizing production, setup and inventory holding costs. Other costs might also be considered. Examples are backorder cost, changeover cost, etc. Several models have been proposed for lot sizing problems. One of the ancestors of these models is the Economic Order Quantity (EOQ) model, which is a continuous time model with an infinite time horizon. It considers a single item and imposes no capacity restrictions. Later, the EOQ model was extended to consider multiple items and capacity limits. Research on continuous models is still carried out.

Unlike the above mentioned continuous models, the discrete lot sizing models assume that the planning horizon is finite and divided into discrete periods for which demand is given and may vary between periods. Wagner and Whitin's single item problem is a seminal work in this area.

The classification of lot sizing problems is based on several criteria: number of machines, number of production stages (levels), capacity constraints and their nature (fixed or variable), length of production periods, etc. Based on production stages, single-level and multi-level problems can be considered. While if the focus is on period length, big time bucket problems and small time bucket problems can be considered. Small time bucket problems have short production periods, which normally are of the order of a few hours. Big time bucket problems consist of longer time periods, normally of the order of a few days or weeks. The latter can be seen as a time aggregation of the former. This leads to the consideration of hierarchical planning problems.

Lot sizing problem may be considered capacitated or uncapacitated. The latter can be seen as a lot sizing problem with a single (or aggregate) product, in which the production capacity is assumed to be high enough to never be binding in an optimal solution.

For a review of lot scheduling problems the reader is referred to [7,9]. In lot scheduling problems jobs may be batched if they share the same setup on a machine or a machine can process several jobs simultaneously. In these problems, doing jobs in batches has fewer costs than doing them separately.

One situation where benefits may result from batching occurs when machines require setups if they are to process jobs that have differing characteristics. The setup may reflect the need to change a tool or to clean the machine. In a family scheduling model, jobs are partitioned into families according to their similarity, so that no setup is required for a job if it belongs to the same family of the previously processed job. However, a setup time is required at the start of the schedule and on each occasion when the machine switches from processing jobs in one family to jobs in another family. In this model, a batch is a maximal set of jobs that are scheduled contiguously on a machine and share a setup. Large batches have the advantage of high machine utilization because the number of setups is small. On the other hand, processing a large batch may delay the processing of an important job belonging to a different family.

In lot scheduling, problems involving the

scheduling of a single machine, of  $m$  parallel machines, and of  $m$ -machine flow shops, job shops and open shops may be considered. There is a set of jobs to be processed. The processing time of jobs, release dates, due dates and the weights of jobs is known.

In scheduling problems it is often assumed that due dates are determined before and some of them in which due date is a part of decision variables, the problem and costs structure is different from due date assignment problem [see References 20, 21].

### 3. PROBLEM DESCRIPTION AND FORMULATION

In this section, a specific problem is formulated with flow time approach. The problem was defined in a heavy equipments company. Consider a product that has deterministic demand and because of economy of scale is produced in batches and each batch satisfies several customer orders. Because of relatively long lead times, supply of a batch usually starts based on demand forecasting. If a batch is completed before receiving customer orders for it, inventory carrying cost is incurred but if it is completed after receiving customer orders, a due date is quoted to each customer and a shortage cost is charged (which is also called backlogging cost in inventory systems literature and reveals in the forms of sales price reduction, customer goodwill ...). If an order is delivered after its due date, tardiness cost is incurred.

To formulate the problem symbolically, some definitions and notations will be required. Consider a product with an interval time between two successive orders of  $\lambda$ . After receiving each order, a due date is quoted to its customer, if it could not be delivered simultaneously. Let  $L(i)$  be function which determines the lead time quoted to customer  $i$ . Due date of order  $i$  will be  $i\lambda + L(i)$  since order  $i$  is received at  $i\lambda$ . Quoting a lead time  $x$  to a customer, will charge  $w \cdot x$  cost. Supply cost function is  $S(Q)$  in which  $Q$  is the batch size. Let  $\pi_1$  be tardiness cost per unit of product that is delivered after its due date (independent of the duration of lateness) and  $\pi_2$  be unit time tardiness

cost rate per unit of product. Inventory carrying cost per unit per unit time is  $h$ . Consider  $d$  to be the batch due date that is used in supply planning process. Because of uncertainty in supply process, batch completion time is a random variable (showed by  $X_d$ ) with a probability density function  $f$  which is a function of the batch due date. In other words, if the supply due date of a batch is set at  $d$ , the batch completion time will be a random function of  $d$ . So decision variables are:

- Batch size ( $Q$ )
- Batch due date ( $d$ )
- Customer due date assignment function,  $L(i)$
- If the batch size is  $Q$ , shortage cost of a cycle will be

$$w \sum_{i=1}^Q L(i)P(X_d > i\lambda) \quad (1)$$

In which  $X_d$  is batch receive time and  $P(X_d > i\lambda)$  is the probability that the batch is not received until order receiving time,  $i\lambda$ .

Inventory carrying cost is incurred for products that are not sold until batch completion. Inventory carrying cost of a cycle will be

$$h \sum_{i=1}^Q \left[ \int_{i\lambda}^{i\lambda + L(i)} (i\lambda + L(i) - x) f_{X_d}(x) dx + \right. \quad (2)$$

$$\left. \int_{-\infty}^{i\lambda} (i\lambda - x) f_{X_d}(x) dx \right]$$

Where  $i\lambda + L(i) - x$  is the duration of holding product  $i$  when supply completion time is  $x$  and the product due date is  $i\lambda + L(i)$ . Inventory carrying cost expression has two parts because it could happen to be in two different situations. Firstly, it incurs if the batch is received before customer order,  $i\lambda$ . Secondly, it happens if the product is not received before customer order so a lead time is quoted to him, but it is received before due date quoted to its customer.

Tardiness cost occurs for products that are delivered beyond their assigned due date.

Tardiness cost of a cycle will be

$$\pi_1 \sum_{i=1}^Q P[X_d > i\lambda + L(i)] + \pi_2 \sum_{i=1}^Q \int_{i\lambda + L(i)}^{\infty} [x - i\lambda - L(i)] f_{X_d}(x) dx \quad (3)$$

Where  $P[X_d > i\lambda + L(i)]$  is the probability of tardiness and  $[x - i\lambda - L(i)]$  is tardiness of product  $i$ , if batch is received at time  $x$ . It is obvious that tardiness occurs when the batch is received in range  $[i\lambda + L(i), \infty]$ .

Hence total cost of a cycle is

$$Z(Q) = S(Q) + w \sum_{i=1}^Q L(i)P(X_d > i\lambda) + h \sum_{i=1}^Q \left[ \int_{i\lambda}^{i\lambda + L(i)} (i\lambda + L(i) - x) f_{X_d}(x) dx + \int_{-\infty}^{i\lambda} (i\lambda - x) f_{X_d}(x) dx \right] + \pi_1 \sum_{i=1}^Q P[X_d > i\lambda + L(i)] + \pi_2 \sum_{i=1}^Q \int_{i\lambda + L(i)}^{\infty} [x - i\lambda - L(i)] f_{X_d}(x) dx \quad (4)$$

Average cost of unit product is  $\frac{1}{Q}Z(Q)$  which must be minimized.

For further elaboration of the model,  $L(i)$  is considered to be a linear function of  $i$ , i.e.  $L(i) = a - ib$ . Although with respect to the proposed solution method, there is no limitation in considering other structures for function  $L(i)$ . Parameters  $a$ ,  $b$  together with  $d$  and  $Q$  are the decision variables of the model.

#### 4. MODEL SOLUTION USING GENETIC ALGORITHMS

Genetic algorithms (GA) are computational models that solve a given problem by maintaining a changing population of individuals, each with its

own level of “fitness”. The change in the population is achieved by the selection, reproduction and mutation procedures within the method. The operation of these three procedures is dependent upon the fitness of the individuals concerned.

Every good GA needs to balance the extent of exploration of information obtained up until the current generation through recombination and mutation operators with the extent of exploitation through the selection operator. If the solutions obtained are exploited too much, premature convergence is expected. On the other hand, if too much stress is given on a search (i.e. exploration), the information obtained thus far has not been used properly. Therefore, the solution time may be enormous and the search exhibits a similar behavior to that of a random search. The issue of exploration and exploitation makes a recombination and mutation operator dependent on the chosen selection operator for successful GA run.

**4.1. Encoding** Genetic algorithms are characterized by the fact that all the information for any individual in the population is encoded using some linear encoding system. This (usually binary) encoding is intended to be analogous to natural DNA consisting of a string of four kinds of chromosomes. The standard encoding technique for applying genetic algorithms to non-linear optimization problems (where the parameters are continuous and real), is a concatenation of all the binary approximations to each number. Handling continuous search space with binary coded genetic algorithm has several difficulties [40]. Real coded genetic algorithm represents parameters without coding, which makes representation of the solutions very close to the natural formulation of many problems. Then a chromosome is a vector of floating point numbers, the precision of which will be restricted to that of the computer with which the algorithm is carried out. The size of the chromosomes is kept the same as the length of the vector which is the solution to the problem; in this way, each gene represents a variable of the problem. The values of the chromosome genes are forced to remain in the interval established by the variables they represent so the genetic operators must observe this requirement. In real coded GA (RCGA) recombination and mutation operators are

designed to work with real parameters. In this paper, RCGA is applied for optimization.

**4.2. Cross-Over** The means by which two different chromosomes (i.e. members of the population) can combine to form some new “offspring” is cross-over. Here, some well-known or recently proposed crossover operators are presented and considered in experiments.

**4.2.1. Blend crossover (BLX- $\alpha$ ) [46]** Assume that  $C_1 = (c_1^1, \dots, c_n^1)$  and  $C_2 = (c_1^2, \dots, c_n^2)$  are two chromosomes that have been selected to apply the crossover operator to them. An offspring,  $H = (h_1, \dots, h_1, \dots, h_n)$  is generated, where  $h_i$  is a randomly (uniformly) chosen number of the interval  $[c_{\min} - I\alpha, c_{\max} + I\alpha]$ ,  $c_{\max} = \max(c_1^1, c_1^2)$ ,  $c_{\min} = \min(c_1^1, c_1^2)$ ,  $I = c_{\max} - c_{\min}$ . The BLX-0.0 and BLX-0.25 are also called flat crossover and extended intermediate crossover respectively.

**4.2.2. Heuristic crossover (HX) [49]** With this operator, also called direction based crossover, from a pair of parents  $x^{(1)} = (x_1^{(1)}, x_2^{(1)}, \dots, x_n^{(1)})$  and  $x^{(2)} = (x_1^{(2)}, x_2^{(2)}, \dots, x_n^{(2)})$  an offspring  $y = (y_1, y_2, \dots, y_n)$  is generated in the following manner:

$$y_i = u(x_i^{(2)} - x_i^{(1)}) + x_i^{(2)}$$

Where  $u$  is a uniformly distributed random number in the interval  $[0,1]$  and the parent  $x^{(2)}$  has a fitness value no worse than that of parent  $x^{(1)}$ . If the offspring generated lies outside the feasible region, a new random number  $u$  is generated to produce another offspring. If required, this process is repeated up to  $k$  times. If it fails to produce a feasible offspring, like laplace operator, a random point in the feasible region is chosen. In this paper  $k$  is fixed to be 4 (as in [50,45]).

Direction based crossover uses the values of the objective function in determining the direction of genetic search.

**4.2.3. Laplace crossover (LX) [45]** Laplace crossover is based on laplace distribution. Distribution function of Laplace distribution is given by

$$F(x) = \begin{cases} \frac{1}{2}e^{-\frac{|x-a|}{b}}, & x \leq a \\ 1 - \frac{1}{2}e^{-\frac{|x-a|}{b}}, & x > a \end{cases} \quad (5)$$

Where  $a \in \mathbb{R}$  is called the location parameter and  $b > 0$  is termed as scale parameter.

Using laplace crossover, two offspring  $y^{(1)} = (y_1^{(1)}, y_2^{(1)}, \dots, y_n^{(1)})$  and

$y^{(2)} = (y_1^{(2)}, y_2^{(2)}, \dots, y_n^{(2)})$  are generated from a pair of parents  $x^{(1)} = (x_1^{(1)}, x_2^{(1)}, \dots, x_n^{(1)})$  and

$x^{(2)} = (x_1^{(2)}, x_2^{(2)}, \dots, x_n^{(2)})$  in the following way.

First, a uniformly distributed random number  $u \in [0,1]$  is generated. Then, a random number  $\beta$  is generated which follows the laplace distribution by simply inverting the distribution function of Laplace distribution as follows:

$$\beta = \begin{cases} a - b \cdot \ln(u), & u \leq \frac{1}{2} \\ a + b \cdot \ln(u), & u > \frac{1}{2} \end{cases} \quad (6)$$

The offspring are given by the equations

$$\begin{aligned} y_i^{(1)} &= x_i^{(1)} + \beta \left| x_i^{(1)} - x_i^{(2)} \right| \\ y_i^{(2)} &= x_i^{(2)} + \beta \left| x_i^{(1)} - x_i^{(2)} \right| \end{aligned}$$

From the above equations it is clear that both the offspring are placed symmetrically with respect to the position of the parents. For smaller values of  $b$ , the offspring are likely to be produced near the parents and for larger values of  $b$  offspring are expected to be produced far from the parents. For fixed values of  $a$  and  $b$ , LX dispenses offspring proportional to the spread of parents i.e. if the parents are near each other the offspring are also expected to be near each other and if the parents

are far from each other then the offspring are likely to be far from each other.

**4.2.4. Average-bound crossover (ABX) [42]** The two parents will eventually produce two offspring. The ABX comprises two operations: average crossover and bound crossover which produce 4 offspring in the following manner:

$$y_i^1 = \frac{x_i^1 + x_i^2}{2} \quad (7)$$

$$y_i^2 = \frac{(x_i^1 + x_i^u)(1 - \omega_a) + (x_i^1 + x_i^2)\omega_a}{2} \quad (8)$$

$$y_i^3 = x_i^u(1 - \omega_b) + \max(x_i^1, x_i^2)\omega_b \quad (9)$$

$$y_i^4 = x_i^1(1 - \omega_b) + \min(x_i^1, x_i^2)\omega_b \quad (10)$$

$\omega_a, \omega_b \in [0,1]$  Denote the user defined weight for average crossover and bound crossover, respectively. As the value of  $\omega_a$  tends to 1, the offspring tends to be the average of the selected parents. As the value of  $\omega_a$  tends to 0, the offspring tends to be the average of the domain boundary. For many optimization problems, the value of the weight  $\omega_a$  can be set as 0.5.

Changing the value of the weight  $\omega_b$  in the ABX will change the characteristics of the bound crossover operations. It is also chosen by trial and error, which depends on the kind of the optimization problem. A value of  $\omega_b$  approaching 1 will make the offspring to be near the selected parents. As the value of  $\omega_b$  tends to 0, the offspring will become near the domain boundary. Among  $y_i^1$  to  $y_i^4$ , two with the best fitness values are used as the offspring of the crossover operation.

It seems that ABX is some what like a fuzzy connectives based crossover [51] in which the interval of the action of each gene is divided into three regions and demonstrate exploration and exploitation zones. The first two offspring of ABX are generated in the exploitation zone and the last two are generated in the exploration zone.

The offspring have to be feasible with respect to

their genes' values:  $h_i \in [lb_i, ub_i], i=1, \dots, n$ . If not feasible, the offspring are regenerated following the same rule with other randomly picked numbers.

**4.3. Mutation** The role of mutation in GA is to restore lost or unexpected genetic material into a population to prevent the premature convergence of GA to sub-optimal solutions.

Any good searching algorithm must explore large search space at the beginning and the search should then narrow down as it converges to the solution.

Basically each bit within the population is flipped with a certain (small) probability. This probability is usually less than one in one thousand. Some propose that the mutation is simply a means of preventing an early convergence (and hence allowing the algorithm enough time to complete its work). However others claim that it is, in fact, an essential means of introducing genetic diversity into the population. That is, allowing genes which contain desirable characteristics to be formed, which would not otherwise be formed. Here some of the well-known or recently developed mutation operators are introduced:

**4.3.1. Non-uniform mutation (NUM) [47]**

Randomly selects one gene  $c_i$  and sets its value according to the following rule:

$$c_i = \begin{cases} c_i + (ub_i - c_i) \cdot \Gamma(t) & \text{if } a_1 < 0.5 \\ c_i - (c_i - lb_i) \cdot \Gamma(t) & \text{if } a_1 \geq 0.5 \end{cases} \quad (11)$$

$$\Gamma(t) = 1 - a_2 \left( 1 - \frac{t}{t_{\max}} \right)^b \quad (12)$$

With  $a_1$  and  $a_2$  two random numbers in  $[0,1]$ ,  $b$  a constant parameter,  $t$  the time or generation number,  $t_{\max}$  the maximum number of generations the GA is allowed to run.

This function gives value in the range  $[0,1]$  such that the probability of returning a number close to zero increases as the algorithm advances. The size of the gene generation interval shall be lower with the passing of generations. This property causes

this operator to make a uniform search in the initial space when  $t$  is small and very locally at a later stage, favoring local tuning.

**4.3.2. Wavelet mutation (WAV) [42]**

$$\hat{x}_i = \begin{cases} x_i + \delta \cdot (x_i^u - x_i) & , \delta > 0 \\ x_i + \delta \cdot (x_i - x_i^l) & , \delta \leq 0 \end{cases} \quad (13)$$

By using the Morlet wavelet,

$$\delta = \frac{1}{\sqrt{a}} e^{-\frac{(\varphi/a)^2}{2}} \cos\left(5\frac{\varphi}{a}\right) \quad (14)$$

Where  $\varphi \in [-2.5, 2.5]$  is randomly generated. If  $\delta$  is positive ( $\delta > 0$ ) approaching 1, the mutated gene will tend to the maximum value of  $x_i$ . Conversely, when  $\delta$  is negative ( $\delta \leq 0$ ) approaching -1, the mutated gene will tend to the minimum value of  $x_i$ . A larger value of  $|\delta|$  gives a larger searching space for  $x_i$ . When  $|\delta|$  is small, it gives a smaller searching space for fine-tuning the gene.

The value of the dilation parameter  $a$  can be set to vary with the value of  $\tau/T$  in order to meet the fine-tuning purpose, where  $T$  is the total number of iteration and  $\tau$  is the current number of iteration. In order to perform a local search when  $\tau$  is large, the value of  $a$  should increase as  $\tau/T$  increases so as to reduce the significance of the mutation. Hence, a monotonic increasing function governing  $a$  and  $\tau/T$  is proposed as

$$a = e^{-\ln(g)(1-\tau/T)^\zeta} + \ln(g) \quad (15)$$

Where  $\zeta$  is the shape parameter of the monotonic increasing function,  $g$  is the upper limit of the parameter  $a$ . In this paper,  $g$  is set as 10,000. The value of  $a$  is between 1 and 10,000. The maximum value of  $\delta$  is 1 when the random number of  $\varphi = 0$  and  $a = 1(\tau/T = 0)$  and it ensures that a large search space for the mutated gene is given. When the value  $\tau/T$  is near to 1, the value of  $a$  is so large that the maximum value of  $\delta$  will become very small. For example, at  $\tau/T = 0.9$  and  $\zeta = 1$ , the

dilation parameter  $a = 4,000$ . If the random value of  $\varphi$  is zero, the value of  $\delta$  will be equal to 0.0158; a small searching space for the mutated gene is given for fine tuning.

A larger  $\zeta$  ( $\zeta = 5$ ) means that the RCGA will go to perform fine-tuning faster. On the other hand, a smaller  $\zeta$  ( $\zeta = 0.2$ ) means the mutation operation is playing a significant role at a later stage. When  $\zeta$  becomes larger, the decreasing speed of the step size ( $\delta$ ) of the mutation becomes faster.

**4.4. Reproduction** This is one of the most important, and surprisingly, least controversial of the operations. This is where it is decided which members of the population will be allowed to survive, and which will perish. It is usually done via a weighted random selection. The weighing is done on the fitness of each individual.

That is, the more fit members of the population have a greater chance of progressing to the next generation than those less fit.

Elitism is a technique to preserve and use previously found best solutions in subsequent generations of GA. In an elitist GA, the statistic of the population's best solutions cannot degrade with the generation. Maintaining archives of non-dominated solutions is an important issue. The final contents of archive represent (usually) the result returned by optimization process. It is common (and highly effective) to employ an archive as a pool from which to guide the generation of new solutions. Some algorithms use solutions in the archive exclusively for this purpose, while others tend to rely on the archive to varying degrees according to parameter settings.

**4.4.1. Tournament selection** In tournament selection, a number  $Tour$  of individuals is chosen randomly from the population and the best individual from this group is selected as a parent. This process is repeated until the required numbers of parents are selected. These selected parents produce uniform offspring at random. The parameter for tournament selection is the tournament size  $Tour$ .  $Tour$  takes values ranging from 2 to  $popsize$ , the number of individuals in the population.

**4.4.2. Normalized geometric ranking** Rank-

based methods order the chromosomes according to their fitness value. Since the selection here depends on the degree the fitter chromosomes are favored, the GA will produce improved population fitness over succeeding populations. The normalized geometric ranking scheme [41] is used in this work. Individuals in the population are ranked in decreasing order according to their fitness value. Then each individual is assigned a probability of selection based upon a triangular or geometric distribution. Michalewicz [47] and Joines and Houck [48] have shown that GAs incorporating ranking methods based upon the geometric ranking distribution outperform those based on the triangular distribution. Thus for a finite population size the probability assigned to the  $i$ th individual or chromosome will be  $P_i = q(1-q)^{r-1}$ .

Where  $q$  is the probability of selecting the best individual and  $r$  is the rank of the individual (1 is the best rank). Joines and Houck [48] showed that a pure geometric distribution is not appropriate since its range is defined on the interval one to infinity. To alleviate this problem, they developed a normalized distribution such that the probability will be

$$P_i = q'(1-q)^{r-1}, \quad q' = \frac{q}{1 - (1-q)^P} \quad (16)$$

Where  $p$  is the population size.

**4.5. Experimental Setup** Table 1 show 24 different algorithms that were tested with three problem instances.

The present experiments were carried out using the following parameters: the population size is 60, the crossover and mutation probabilities are 0.8, 0.1 respectively. The number of generations in each run is 100. Each algorithm of Table 1 was run 25 times for each sample data set. The Elitism size was set to 1.  $g$  parameter in wavelet mutation was set as 10,000.

**4.6. Problem Instances** Parameters values and function forms are determined based on studies in a manufacturing environment.  $S$  function assumed to be linear piecewise functions so that the impact of batch size on procurement and assembly costs could be represented properly.

Three sample data sets were considered and

TABLE 1. Real-Coded Genetic Algorithms.

ID	Selection	Crossover	Mutation
RCGA04	Ranking ( $q = 0.09$ )	BLX ( $\alpha = 0.4, k = 4$ )	NUM ( $b = 3$ )
RCGA05	Ranking ( $q = 0.09$ )	BLX ( $\alpha = 0.4, k = 4$ )	NUM ( $b = 5$ )
RCGA06	Ranking ( $q = 0.09$ )	BLX ( $\alpha = 0.4, k = 4$ )	WAV ( $\zeta = 5$ )
RCGA07	Tournament (Tour = 4)	BLX ( $\alpha = 0.4, k = 4$ )	NUM ( $b = 3$ )
RCGA10	Tournament (Tour = 4)	HX ( $k = 4$ )	NUM ( $b = 3$ )
RCGA13	Ranking ( $q = 0.05$ )	HX ( $k = 4$ )	NUM ( $b = 3$ )
RCGA19	Ranking ( $q = 0.09$ )	HX ( $k = 4$ )	WAV ( $\zeta = 5$ )
RCGA20	Ranking ( $q = 0.05$ )	HX ( $k = 4$ )	WAV ( $\zeta = 5$ )
RCGA21	Ranking ( $q = 0.05$ )	HX ( $k = 4$ )	WAV ( $\zeta = 1$ )
RCGA22	Ranking ( $q = 0.05$ )	HX ( $k = 4$ )	WAV ( $\zeta = 0.2$ )
RCGA23	Tournament (Tour = 4)	HX ( $k = 4$ )	WAV ( $\zeta = 1$ )
RCGA24	Ranking ( $q = 0.05$ )	BLX ( $\alpha = 0.4, k = 4$ )	NUM ( $b = 3$ )
RCGA25	Tournament (Tour = 3)	BLX ( $\alpha = 0.33, k = 4$ )	NUM ( $b = 3$ )
RCGA26	Ranking ( $q = 0.05$ )	BLX ( $\alpha = 0.4, k = 4$ )	WAV ( $\zeta = 5$ )
RCGA27	Tournament (Tour = 3)	HX ( $k = 4$ )	NUM ( $b = 3$ )
RCGA28	Ranking ( $q = 0.05$ )	HX ( $k = 4$ )	NUM ( $b = 5$ )
RCGA29	Ranking ( $q = 0.05$ )	LX ( $a = 0, b = 0.5, k = 1$ )	WAV ( $\zeta = 5$ )
RCGA30	Ranking ( $q = 0.05$ )	LX ( $a = 0, b = 0.5, k = 1$ )	NUM ( $b = 5$ )
RCGA31	Tournament (Tour = 4)	HX ( $k = 4$ )	WAV ( $\zeta = 5$ )
RCGA32	Ranking ( $q = 0.05$ )	ABX ( $\omega a = 0.5, \omega b = 0.5$ )	WAV ( $\zeta = 5$ )
RCGA33	Ranking ( $q = 0.05$ )	ABX ( $\omega a = 0.5, \omega b = 0.5$ )	NUM ( $b = 5$ )
RCGA34	Ranking ( $q = 0.05$ )	HX ( $k = 4$ )	WAV ( $\zeta = 4$ )
RCGA37	Ranking ( $q = 0.05$ )	LX ( $a = 0, b = 0.5, k = 4$ )	WAV ( $\zeta = 5$ )
RCGA38	Ranking ( $q = 0.05$ )	LX ( $a = 0, b = 0.5, k = 4$ )	NUM ( $b = 5$ )

TABLE 2. Problem Instances.

Function/Parameter	Problem Instance 1 (P1)	Problem Instance 2 (P2)	Problem Instance 3 (P3)
Batch Receive Time Distribution	Beta (a,b,p,q): a = d-0.1, b = d + 0.3 p = 2, q = 6	Triangular (a,c,b): a = d-0.05, b = d + 0.15, c = d	Uniform (a,b): a = d-0.05, b = d + 0.15
s(n)	$s(n) = \begin{cases} 2+20n & , n \leq 4 \\ 3+19n & , 5 \leq n \leq 10 \\ 3.5+18.5n & , 11 \leq n \leq 25 \\ 4+18.3n & , 26 \leq n \end{cases}$	$s(n) = \begin{cases} 20n & , n \leq 10 \\ 19.5n & , 11 \leq n \leq 25 \\ 19.2n & , 26 \leq n \leq 45 \\ 19n & , 46 \leq n \end{cases}$	$s(n) = \begin{cases} 20n & , n \leq 10 \\ 19.5n & , 11 \leq n \leq 25 \\ 19.2n & , 26 \leq n \leq 45 \\ 19n & , 46 \leq n \end{cases}$
w	12	12	12
P <sub>1</sub>	2	2	2
P <sub>2</sub>	15	15	15
h	15	15	15
λ	0.01	0.005	0.005

solved using different algorithms. As Table 2 shows, three different probability distribution functions (including beta, triangular and uniform) have been used. The Beta distribution can be used to model events which are constrained to take place within an interval defined by a minimum and maximum value. For this reason, the Beta distribution-along with the triangular distribution-is used extensively in PERT, CPM and other project management/control systems to describe the duration of an activity. Beta probability density function with parameters p, q distributed over the interval a, b is:

$$f(x) = \frac{(x-a)^{p-1}(b-x)^{q-1}}{(b-a)^{p+q-1} \hat{a}(p,q)}, \quad a < x < b, \quad p, q > 1 \quad (17)$$

In which a, b are lower and upper limits of the beta random variable. Parameters p and q determine the shape of beta distribution. Mean and variance of beta are calculated by:

$$\mu = \frac{p}{p+q} b + \frac{q}{p+q} a, \quad \sigma^2 = \frac{pq(b-a)^2}{(p+q-1)(p+q)^2} \quad (18)$$

Figure 1 shows a sample beta pdf in which a, b are 0.3, 0.8 respectively. Mean of this random variable is 0.425. In the data set 1, p, q were considered to be 2, 6 and a, b to be functions of batch due date (d) which means that batch receive time is a beta random variable whose upper and lower limits are a = d - 0.1, b = d + 0.3 respectively.

The triangular distribution is a continuous

distribution defined on the range  $x \in [a, b]$  with probability density and distribution functions

$$f(x) = \begin{cases} \frac{2(x-a)}{(b-a)(c-a)} & a \leq x \leq c \\ \frac{2(b-x)}{(b-a)(b-c)} & c < x \leq b \end{cases}, \quad (19)$$

$$F(x) = \begin{cases} \frac{(x-a)^2}{(b-a)(c-a)} & a \leq x \leq c \\ 1 - \frac{(b-x)^2}{(b-a)(b-c)} & c < x \leq b \end{cases}$$

Figure 2 depicts the triangular pdf.

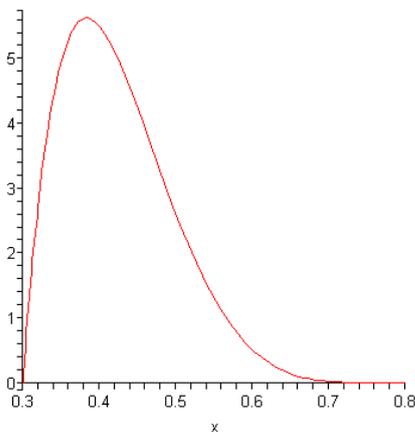


Figure 1. Beta probability density function with parameters 2, 6.

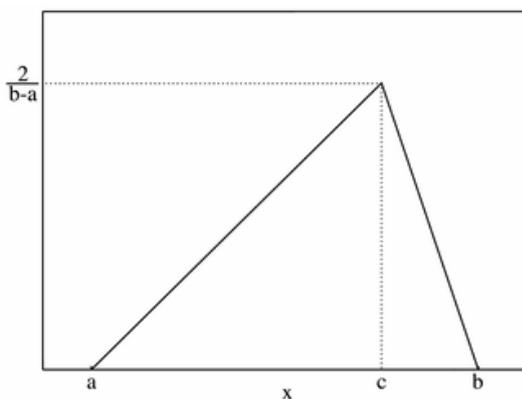


Figure 2. Triangular probability density function.

**4.7. Model Verification** Model verification is often defined as “ensuring that the computer program of the computerized model and its implementation are correct” and the definition is adopted here. The following techniques are used for model verification:

**4.7.1. Structured programming and program modularity** The computer program has been designed, developed, and implemented using techniques found in software engineering. These include such techniques as structured programming and program modularity. Totally 24 functions and 15 procedures constitute the overall program that could be partitioned in 2 main modules (objective function calculation and genetic algorithm). This modularity makes the program more traceable.

**4.7.2. Testing** Three testing techniques were used: Structured walk-through: These are step-by-step reviews of program to assure that it behaves correctly.

Unit test: this type of testing is oriented to discovering defects in the individual functions and procedures making up the program. For example, functions written for beta distribution function were tested in this step.

Integration test: These are tests at the module and program level. As higher levels of integration are reached the purpose of testing moves from predominantly verification to validation. These groups should be tested to see if they work together properly as the program is built. For example, objective function is a module that was tested in this way.

**4.8. Experimental Results** The best solution found for test problems are presented in Table 3.

For each test problem a table is presented below (Tables 4-6). The fitness column is the average of best fitness values found in each run. Initial and last population average columns show the average of fitness of elements in the initial and last populations. Total Average is the average of the fitness of all the elements appearing through GA’s execution. Best Solution Gen. Shows the last generation in which the best element was found. As it is expected, diversity is the initial (randomly generated) population is high and in the last

**TABLE 3. Optimum Solutions.**

Problem	n	a	b	d	Fitness
P1	13	0.0622	0.00145	0.03833	20.05777
P2	26	0.0633	0.00141	0.00136	20.2903
P3	26	0.1006	0.00356	-0.02364	20.4640

**TABLE 4. Problem Instances.**

ID	Fitness	Initial population Average	Last population Average	Total Average	Best Solution Gen.	Successful Runs
RCGA04	20.0582180	115.1006	20.058218	21.9768	97.8	10
RCGA05	20.0588102	118.8962	20.072111	21.7989	99.2	9
RCGA06	20.5909955	114.7098	20.600143	22.1423	99.8	24
RCGA07	20.0582125	104.3831	20.058224	21.9204	99.8	10
RCGA10	20.0609181	107.7236	20.060918	22.9674	68.5	19
RCGA13	20.0578337	117.4383	20.057834	24.4680	90.8	23
RCGA19	20.0639377	109.2584	20.107163	22.3512	81.2	19
RCGA20	20.0577786	115.2598	20.387223	24.5765	94.7	25
RCGA21	20.0577831	114.9840	20.448000	26.0360	93.8	23
RCGA22	20.0579195	120.8111	20.706947	25.9796	94.9	19
RCGA23	20.0608630	112.2184	20.088022	23.3292	67.2	21
RCGA24	20.0583427	112.8525	20.058356	22.6366	99.5	10
RCGA25	20.0595538	119.8462	20.059676	22.2886	98.7	2
RCGA26	20.3152544	109.2135	20.331042	22.4814	99.4	0
RCGA27	20.0578613	113.1436	20.057861	23.6181	87.8	22
RCGA28	20.0578062	109.5346	20.067484	24.3489	86.0	24
RCGA29	20.0638146	112.5194	20.291238	22.9410	98.2	22
RCGA30	20.0579165	109.9684	20.067722	22.8390	93.4	20
RCGA31	20.0578627	103.4213	20.087448	22.6567	73.8	21
RCGA32	21.4000000	111.5000	22.433333	24.1333	68.8	0
RCGA33	21.1104617	111.8037	22.042272	25.5210	41.6	0
RCGA34	20.0577790	107.9320	20.344000	24.7080	95.2	25
RCGA37	20.0577791	107.8569	20.081416	22.4287	97.8	25
RCGA38	20.0578615	111.5957	20.062774	22.7158	92.8	22

TABLE 5. Results for P2.

ID	Fitness	Initial population Average	Last population Average	Total Average	Best Solution Gen.	Successful Runs
RCGA04	20.3179178	150.7987	20.327535	22.7372	99.8	19
RCGA05	20.3722906	157.1712	20.411625	22.7302	99.2	6
RCGA06	20.9230683	149.3076	20.936868	23.1972	99.8	0
RCGA07	20.3264925	157.4748	20.326961	22.8969	99.5	12
RCGA10	20.2903147	151.9289	20.290315	24.2431	78.8	25
RCGA13	20.2903147	146.3522	20.290315	27.4624	94.6	25
RCGA19	20.3591500	151.5029	20.429075	23.5517	78.6	16
RCGA20	20.2967899	158.0778	20.864692	27.0578	93.0	23
RCGA21	20.2903155	157.8921	20.855212	29.2699	98.2	25
RCGA22	20.2903147	149.2366	21.567079	29.5289	95.2	25
RCGA23	20.2964821	153.3789	20.319879	24.9234	82.5	24
RCGA24	20.3362827	150.5686	20.340935	23.5217	99.2	9
RCGA25	20.3862906	158.7407	20.387894	23.2692	99.6	6
RCGA26	20.7682720	170.5133	20.787096	24.4654	99.5	0
RCGA27	20.2903147	161.6875	20.290315	25.4991	93.4	25
RCGA28	20.2903147	157.6269	20.290315	26.1982	92.6	25
RCGA29	20.3676368	148.9813	20.550880	24.3864	98.2	11
RCGA30	20.3289108	157.6736	20.328926	24.0559	97.1	18
RCGA31	20.3068547	153.3570	20.344982	23.9392	70.9	22
RCGA32	21.5093297	160.8750	23.299400	26.9899	81.0	0
RCGA33	21.1398177	151.2442	22.743890	27.7654	42.9	0
RCGA34	20.2903147	160.3525	20.880219	27.1151	96.5	25
RCGA37	20.3289261	145.7382	20.441083	23.9764	97.2	17
RCGA38	20.3178814	157.0021	20.328534	23.7349	96.4	20

TABLE 6. Results for P3.

ID	Fitness	Initial population Average	Last population Average	Total Average	Best Solution Gen.	Successful Runs
RCGA04	20.5834359	161.36	20.583461	23.2912	99.2	2
RCGA05	20.6045089	158.78	20.604510	22.9444	98.4	1
RCGA06	21.0556529	159.53	21.072663	23.5274	99.2	0
RCGA07	20.5673691	157.14	20.567529	23.2800	99.2	2
RCGA10	20.4743162	148.07	20.474320	24.7782	96.8	23
RCGA13	20.4701713	161.99	20.470195	27.1952	97.5	24
RCGA19	20.5110514	162.69	20.539689	24.0052	92.6	17
RCGA20	20.4743190	156.03	20.572707	27.0499	91.4	23
RCGA21	20.4640564	159.09	20.786432	30.3777	95.9	24
RCGA22	20.4640546	161.33	21.545295	30.5182	90.6	25
RCGA23	20.4654835	166.36	20.495321	25.5842	97.0	24
RCGA24	20.5489485	156.39	20.549045	23.8561	98.6	9
RCGA25	20.5801834	168.79	20.580366	23.5307	99.5	1
RCGA26	21.0731202	153.90	21.099351	24.4922	99.2	0
RCGA27	20.4640497	162.79	20.464056	25.6914	98.2	25
RCGA28	20.4640495	156.29	20.464051	26.4296	97.7	25
RCGA29	20.5255613	152.73	20.657026	24.7777	96.4	13
RCGA30	20.5008146	160.33	20.500947	24.4406	98.2	18
RCGA31	20.5090685	163.63	20.537408	24.6862	94.9	17
RCGA32	21.4778228	155.84	23.997060	27.1444	66.2	0
RCGA33	21.2324508	157.66	23.844241	27.3270	91.2	0
RCGA34	20.4701840	156.48	20.673386	26.9250	91.5	23
RCGA37	20.5767227	161.95	20.624625	24.5855	97.8	5
RCGA38	20.5110841	154.79	20.511194	24.0719	98.0	14

population is low. Successful runs is the number of runs in which the deviation of the found solution from the optimum fitness is at most  $1 \times 10^{-5}$ . Hence, a success was counted when the following condition was met

$$f_1 - f_{\text{opt}} \leq 10^{-5}$$

Where  $f_{\text{opt}}$  is the known global minimum of the problem and  $f_1$  is the best value obtained by an algorithm.

Experiment results show that crossover operator is the most important part of the algorithm; because the crossover operator has significant effect on performance of the algorithms. The HX crossover presents the best performance (algorithms RCGA28, RCGA34, RCGA27, RCGA21 and RCGA13) and after it, the LX shows better performance (RCGA30) but the worst results are obtained from BLX and ABX (RCGA32, RCGA33). With normalized geometric ranking,  $q = 0.05$  and with tournament selection,  $\text{Tour} = 3$  have a better result.

Total average of the best algorithms is significantly higher than of other algorithms which show that these algorithms are more explorative than others.

## 5. CONCLUSIONS

In this paper, the problem of due date assignment in make-to-demand production systems were characterized and an original model were presented for it. The main focus was on products with relatively low demand rates. In such cases, the economy of scale??? could not be ignored. On the other hand, quoted lead time and due date related costs must be considered. A model was formulated for such a problem. In the lot sizing literature, the lead time and due date related costs and in the due date assignment literature, economy of scale are ignored. In this paper both were considered. A real-coded genetic algorithm was developed for solving the model.

Assumptions are based on a specific company. The approach used in this paper may be

applied to other production systems. Below are some of the possible extensions:

- Demand may be considered stochastic.
- Each customer may order more than one unit of product. The customer order quantity may follow a probability distribution.
- Long quoted lead times may lead to lost sales.
- Constraints like resource availability may be considered.

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