RESEARCH NOTE

M1, M2, ..., Mk/G1, G2, ..., Gk/l/N QUEUE WITH BUFFER DIVISION AND PUSH-OUT SCHEMES FOR ATM NETWORKS

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Abstract In this paper, loss probabilities and steady state probabilities of data packets for an asynchronous transfer mode (ATM) network are investigated under the buffer division and push-out schemes. Data packets are classified in classes k which arrive in Poisson fashion to the service facility and are served with general service rate under buffer division scheme, finite buffer space N is divided into N1, N2, …, Nk such that N = N1 + N2 +…+ Nk. Under push-out scheme if upon arrival of class 1 packet to the system finds that there are less than N1-1 class 1 packets waiting for service and there is no unoccupied buffer space then one of the packets of other classes is pushed out. The pushed out packet is lost. The packets are served under general service discipline.

Key Words Steady State Probability, ATM, Finite Buffer, Push-Out Scheme

1. INTRODUCTION

In this investigation, M1, M2, ..., Mk/G1, G2, ..., Gk/l/N queuing system with buffer space division and push-out schemes for an asynchronous transfer mode (ATM) network is developed. ATM is a high bandwidth, low-delay switching technique that can switch all types of traffic in a packet format of fixed length. Queuing model under study has its widespread applications to evaluate the performance measures of computer network, ATM network broadband integrated service digital networks (B-ISDN).

Several authors studied various queuing systems, which are well employed in the communication networks. Doshi and Heffes [1] studied overload performance of several processor queuing discipline For M/M/1 queue. Wassal and Hasan [2] proposed architecture for the ATM switches used on board satellites and obtained loss delay in such architecture. Wang and Silvester [3] presented two-state Markov modulated Poisson process bulk (BMMPP) queuing system and studied the performance of ATM multiplexer loaded with various types of traffic. Deng and Chen [4] suggested a multicast bounded-fanout principle to reduce the overflow probability of copy network to an acceptable level. Chaudhry and Gupta [5] obtained the various performance measures for Gk/Geom/l/N queue. Jain and Ghimire [6] analyzed resource sharing finite queue and
obtained various performance measures. A channel grouping technique and virtual FIFO architecture was used in ATM switching network in [7].


2. MATHEMATICAL DESCRIPTION OF THE MODEL

In our queuing model, we allow buffer management, push-out schemes according to which total space N is divided as $N = \sum_{i=1}^{k} N_i$. There cannot be packets of class i more than $N_i-1$ ($i = 1, 2, \ldots, k$) waiting for services.

We have the following notations for our model:

- $\lambda_i = \text{Poisson arrival rates of packets of class } i$ ($i = 1, 2, \ldots, k$).
- $l_i = \text{Loss probability of a packet of class } i$ ($i = 1, 2, \ldots, k$).
- $l = \text{The loss probability for a packet}$.

$b_i = \text{Probability density function for service time of class } i \text{ packets.}$

$R = \text{Ratio of packets lost during a long period of time to packets served in the same period of time.}$

$T = \text{Time period when the system reaches the steady state.}$

$R_i = \text{Average number of class I packets lost per packet served.}$

$b(x) = \text{Probability density function of time.}$

$r_i = \text{Number of packets of class } i$ ($i = 1, 2, \ldots, k$) at the departure time.

$\alpha_i(x) = \text{Probability that classes } i$ ($i = 1, 2, 3, \ldots, k$) packet is served when there are $r_1, r_2, \ldots, r_k$ packets of classes 1, 2, ..., k respectively present in the system.

$P(x) = \text{Steady state probability that the system is in state } r \text{ at the departure time of a packet.}$

$P(r; m) = \text{One step transition probability from state to state } r \text{ to state } m.$

3. LOSS PROBABILITY OF PACKETS

Loss of packets takes place if either there is no waiting space available in the buffer upon its arrival or it is pushed out from buffer while waiting for service. Number of packets lost during a busy period of server is dependent so that

$$l = \frac{\sum_{i=1}^{k} \lambda_i T(1-l)R}{\sum_{i=1}^{k} \lambda_i T} = 1-l$$

Equation 1 can be written as

$$l = \frac{R}{1+R} \quad (2)$$

Loss probability of a class 1 packet is given by

$$\frac{\sum_{i=1}^{k} \lambda_i T(1-l)R}{\lambda_1 T} = \frac{\sum_{i=1}^{k} \lambda_i (1-l)R}{\lambda_i} \quad (3a)$$
Similarly the loss probabilities of classes 2, 3, ..., k packets are obtained by

\[
I_u = \left( \sum_{i=1}^{k} \lambda_i \right) - \left( \sum_{j=1}^{k} \lambda_j I_j \right) \frac{1}{\lambda_u}, \quad u = 2, 3, ..., k
\]

(3b)

\[ \text{4. STEADY STATE PROBABILITIES} \]

The quantity \( r = (r_1, r_2, ..., r_k) \) constitutes an imbedded Markov chain where \( 0 \leq r_i < N_1, r_2, r_3, ..., r_k \geq 0 \) and \( r_1 + r_2 + ... + r_k \leq N-1 \). We may define the following probabilities, which may help to facilitate the expressions for \( P(r) \) and \( P(r; m) \).

1. \( I(n, \lambda, b(x)) \) is the probability that there are exactly \( n \) arrivals with rate \( \lambda \) and is given by

\[
I(n, \lambda, b(x)) = \int_{0}^{\infty} \frac{(\lambda x)^n}{n!} e^{-\lambda x} b(x) dx
\]

\[
= \frac{\lambda^n}{n!} \int_{0}^{\infty} x^n e^{-\lambda x} b(x) dx
\]

(4)

2. \( I(\geq n, \lambda, b(x)) \) is the probability that there are at least \( n \) arrivals with rate \( \lambda \) during service time of which probability density function is \( b(x) \) and it is given by

\[
I(\geq n, \lambda, b(x)) = 1 - \sum_{r=0}^{n-1} I(r_1, \lambda, b(x))
\]

(5)

3. \( I(n, \Lambda, b(x)) \) is the probability that there are exactly \( r_1, r_2, ..., r_k \) arrivals with rates \( \lambda_1, \lambda_2, ..., \lambda_k \) respectively and it may be expressed as

\[ I(n, \Lambda, b(x)) = \int_{0}^{\infty} \prod_{i=1}^{k} \frac{1}{n_i!} (\lambda_i x)^{n_i} e^{-\lambda_i x} b(x) dx \]

\[
= \prod_{i=1}^{k} \lambda_i^{n_i} \left( \sum_{i=1}^{k} n_i \right)! I \left( \sum_{i=1}^{k} n_i, \sum_{i=1}^{k} \lambda_i, b(x) \right) \]

\[
= \prod_{i=1}^{k} \lambda_i^{n_i} \left( \sum_{i=1}^{k} n_i \right)! \left( \sum_{i=1}^{k} \lambda_i \right)^{\sum_{i=1}^{k} n_i} \prod_{i=1}^{k} \left( n_i! \right)
\]

(6)

4. \( I(\geq n, \Lambda, b(x)) \) is the probability that there are at least \( n_1 \) and exactly \( n_2, n_3, ..., n_k \) arrivals with rates \( \lambda_1, \lambda_2, ..., \lambda_k \) respectively. Now

\[ I(\geq n, \Lambda, b(x)) = I(n, \Lambda, b(x)) - \sum_{r_i} I(r_i, n, \Lambda, b(x)) \]

(7)

Also

\[ I(\geq n_1, \geq n_2, ..., \geq n_k, \Lambda, b(x)) = \int_{0}^{\infty} \prod_{i=1}^{k} \frac{1}{n_i!} (\lambda_i x)^{n_i} e^{-\lambda_i x} b(x) dx \]

\[ = \prod_{i=1}^{k} \lambda_i^{n_i} \left( \sum_{i=1}^{k} n_i \right)! I \left( \sum_{i=1}^{k} n_i, \sum_{i=1}^{k} \lambda_i, b(x) \right) \]

\[ = \prod_{i=1}^{k} \lambda_i^{n_i} \left( \sum_{i=1}^{k} n_i \right)! \left( \sum_{i=1}^{k} \lambda_i \right)^{\sum_{i=1}^{k} n_i} \prod_{i=1}^{k} \left( n_i! \right) \]

(8)

Now For \( r_1 = r_2 = ... = r_k = 0 \), the class of packet which is to be served next depends upon the class form which next packet comes. Since with the probability \( \lambda_2 / \sum_{i=1}^{k} \lambda_i \), from class 2 and so on and with the probability \( \lambda_k / \sum_{i=1}^{k} \lambda_i \), from class k, so
that state transition probability \( P(o; m) \) is given by:

\[
p(o; m) = \begin{cases} 
\frac{\beta_1 II(m, \Lambda, b_1(x)) + \beta_2 II(m, \Lambda, b_2(x)) + \ldots + 
\beta_k II(m, \Lambda, b_k(x))}{m_1 < N_1 - 1 \text{ and } \sum_{i=1}^{k} m_i < N - 1} 
\end{cases}
\]

\[
= \begin{cases} 
\frac{\beta_1 II(m_1, \Lambda, b_1(x)) + \beta_2 II(m_1, \Lambda, b_2(x)) + \ldots + 
\beta_k II(m_1, \Lambda, b_k(x))}{m_1 = N_1 - 1 \text{ and } \sum_{i=1}^{k} m_i < N - 1} 
\end{cases}
\]

\[
= \begin{cases} 
\frac{\beta_1 II(m, \Lambda, b_1(x)) + \beta_2 II(m, \Lambda, b_2(x)) + \ldots + 
\beta_k II(m, \Lambda, b_k(x))}{m_1 = N_1 - 1} 
\end{cases}
\]

where \( i = 1, 2, \ldots, k \)

\[
(10)
\]

where \( \beta_1 = \frac{\lambda_1}{\sum_{i=1}^{K} \lambda_i} ; \beta_2 = \frac{\lambda_2}{\sum_{i=1}^{K} \lambda_i} ; \beta_k = \frac{\lambda_k}{\sum_{i=1}^{K} \lambda_i} \)

For \( r_1 = r_2 = \ldots = r_{k-1} = 0 \) and \( t > 0 \), in this case packet of class \( k \) is served. Let \( \Delta_i = m_1, \Delta_2 = m_2, \ldots, \Delta_k = m_k - (r_i - 1) \). Then we have

\[
P(r_i; m) = \begin{cases} 
0 & \text{if } \Delta_i < 0 \text{ and } \sum_{i=1}^{k} m_i < N - 1 
\end{cases}
\]

\[
\forall (\Delta_i, \Lambda, b_i(x)) \text{ such that } m_i < N_{i-1} \text{ and } \sum_{i=1}^{k} m_i < N - 1
\]

\[
P(\gamma; m) = \begin{cases} 
0 & \text{if } \Delta_i < 0 \text{ and } \sum_{i=1}^{k} m_i < N - 1 
\end{cases}
\]

\[
\forall (\Delta_i, \Lambda, b_i(x)) \text{ such that } m_i < N_{i-1} \text{ and } \sum_{i=1}^{k} m_i < N - 1
\]

\[
(11)
\]

where \( e_i \) is the vector having 1 at the \( i \)th position and zero at the other positions.

For \( r_i < 0 \); a packet of class \( i \) \((i = 1, 2, \ldots, k)\), will be served with probability \( \alpha_i \) \((\gamma) \) and with the assumption that \( \Delta_i = m_i - (r_i - 1) \) and \( \Delta_i^2 = (m_i - r_i) \). \( \Delta_i^2 = m_i - (r_{i+1} - 1) \) \((i = 1, 2, \ldots, k)\). Here \( \Delta_i^2 \) indicates the number of changes of classes \( i \) packets in a class 1 packet service time, \( \Delta_i^2 \) denotes the number of changes of class \( i \) \((i = 1, 2, \ldots, k)\) packets in class 2 packet service time and so on. We notice that \( \Delta_i^2, \Delta_i^2, \Delta_i^2 \) cannot be negative.

We define

\[
\delta(\Delta) = \begin{cases} 
0 & \text{if } \Delta_i < 0 \text{ and } \sum_{i=1}^{k} m_i < N - 1 
1 & \text{otherwise}
\end{cases}
\]
Equation 12 gives complete computation for P(r;m) the steady state probabilities, P(r) obey the law of conservation [12]. So:

\[ P(r) = \sum_{m} P(m) P(r;m); \]

r_i \leq N_i, r_{i+1} \geq 0 \text{ and } \sum_{i=1}^{k} r_i \leq N - 1

(13)

and

\[ \sum_{i=1}^{k} P(r) = 1 \]

(14)

Equation 13 and 14 give the complete computation of P(r).

5. APPLICATIONS OF THE MODEL

Recently: asynchronous transfer mode (ATM) has been increasingly accepted as the basic technology for high-speed packet switching. The model under study has its widespread applications in many areas of computer communication system in modeling and designing process. Loss probabilities of traffic obtained in this research enable queuing theorists as well as practitioners to evaluate performance measures of B-ISDN, ATM, local area network (LAN), wide area network (WAN) in telephony and computer communication networks.

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7. REFERENCES

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