DETERMINATION OF RESONANCE FREQUENCY OF DOMINANT AND HIGHER ORDER MODES IN THIN AND THICK CIRCULAR MICROSTRIP PATCH ANTENNAS WITH SUPERSTRATE BY MWM

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Abstract

An accurate model named as the Modified Wolff Model (MWM) is presented as an efficient CAD tool for determination of resonant frequency of the dominant and higher order modes under the multi-layer condition in thin and thick circular microstrip patch antennas. The effects of dielectric cover on the resonant frequency obtained from MWM have been compared against the result of theoretical method of Spectral Domain Analysis (SDA). The accuracy of calculated resonant frequency by the MWM for dominant mode is about 0.5% and the average error for higher order modes is about 0.88%. Sensitiveness of higher order modes and also uncertainty effect on the resonant frequency are calculated by the MWM.

Key Words

Modified Wolff Model, MWM, Microstrip Antenna, Microstrip Patch Antenna, Superstrate, Dynamic Dielectric Constant, Transverse Transmission Line, TTL

1. INTRODUCTION

Accurate determination of the resonant frequency is very important for the design of microstrip patch antennas because they have narrow bandwidths and can only operate effectively in the vicinity of the resonant frequency. The circular patch operating at higher order modes has been used as the radiating element with higher gain to control the radiation pattern from broad side radiation to end fire radiation and the conical beam has been obtained with the help of the higher order modes [1-12]. To protect a microstrip antenna from environmental effect, a dielectric cover is usually added at the top of patch [14].

However most of the previous theoretical and
experimental work has been carried out only on electrically thin microstrip patch antennas normally of the order of $0.02 > h_1 / \lambda$ where $h_1$ is the thickness of dielectric substrate.

In this paper a unified model MWM is presented to determine the resonant frequency of both thin and thick circular microstrip patch antennas for dominant and higher order modes. In such patches the resonant frequency depends strongly on the effective dynamic dielectric constant and effective radius. Therefore models of Wolff and Knoppick [15] has been modified by using the Transverse Transmission Line (TTL) technique [16] to compute the effective dynamic dielectric constant under multi-layer condition.

The resonant frequency of dominant and higher order modes calculated by the MWM shows good agreement with compare to experimental and available numerical results. The importance of the model is that none of the above-mentioned models is applicable to determine the effect of dielectric cover on the resonant frequency of various patches. Experimentally it is verified that a dielectric cover on a rectangular patch antenna decreases the resonant frequency significantly [16]. Therefore theoretical method for predicting the effect of superstrate on the resonant frequency is of considerable interest. Here we have used MWM to calculate the effect of cover on the resonant frequency and compared the results obtained from SDA [14]. The percentage error calculated by the MWM is 0.88% whereas that of the Antosziewicz method is 1.3%. The error in the higher order mode in resonating structures may arise due to uncertainty in the fabrication process of microstrip patch resonators and antennas. We have realized that the uncertainty of 1% in increase in radius $R$ of circular patch results into uncertainty 0.9% to 1% in resonant frequency. Likewise 1% uncertainty in $\varepsilon_r$ and $h_1$ results into 0.63%. Deviation in resonant frequency of higher order modes is more sensitive to the variation in $\varepsilon_r$. For instance 2% change in $\varepsilon_r$ results in 0.83% change in resonant frequency of the higher order modes.

The resonant frequency of higher order modes in circular microstrip patch has been calculated by J. Q. Howell [17] and the minimum percentage error of resonant frequency is 1.39% and maximum is 10.6% and the average is 4.87%, whereas the minimum percentage error of resonant frequency for TM$_{11}$ mode obtained by the MWM is zero and for higher order maximum is 9.4%. Meanwhile the resonant frequency of higher order modes calculated by the MWM compared against the results obtained from Antoszkiewicz [6].

2. FORMULATION OF THE MODEL

The MWM is basically a cavity model. The original Wolff Model was developed for a single layer open resonating structure [15]. To generalize the Wolff model to multilayer resonating structure as shown in Figure 1, we have adopted the variational method to calculate dynamic dielectric constant $\varepsilon_{dy}$ Determination of $\varepsilon_{dy}$ takes into account the charge distribution along both the longitudinal and transverse directions of the patch. Thus, even though the MWM uses static variational method, it simulates the effect of full wave analysis [8].

For the covered circular patch shown in figure 1, the resonant frequency could be obtained from [9] as:

$$f_{nm} = \frac{X_{mn} V_0}{2\pi R_{eff} \sqrt{\varepsilon_{dy}}}$$  \hspace{1cm} (1)

![Figure 1. Geometry of circular microstrip patch antenna with cover: a cross-sectional view.](image-url)
Where $X_{nm}$ is the mth root of derivative of Bessel function of nth order, the value of which for some m's and n's ($X_{01} = 3.832$, $X_{11} = 1.841$, $X_{21} = 3.054$, $X_{31} = 4.201$) and $V_0$ is the velocity of light in free space. $R_{\text{eff}}$ is the effective radius obtained from expression of Chew and Kong [18].

\[
R_{\text{eff}} = R \left[ 1 + 2h_i / R (\varepsilon_{\text{rl}} (\ln(R/2h_i)) + (1.41(\varepsilon_{\text{rl}} + 1.77)) \right]^{1/2} + h_i (0.2668(\varepsilon_{\text{rl}} + 1.65))^{1/3}
\]

(2)

Where $R$ is the radius of the patch, $h_i$ is the height of the substrate and $\varepsilon_{\text{rl}} = \varepsilon_r$ is relative dielectric.
constant of substrate. $\varepsilon_{\text{dyn}}$ is the dynamic dielectric constant which takes into account the fringe field. The dynamic dielectric constant obtained with help of variation method in the Fourier Domain and using the Greens function problem has determined by the TTL method [16].

$$\varepsilon_{\text{dyn}} = \frac{C_{\text{dyn}}(2R, h_1, h_2, \varepsilon_{r_1}, \varepsilon_{r_2})}{C_{\text{dyn}}(2R, h_1, h_2, \varepsilon_{r_1} = \varepsilon_{r_2} = 1)}$$

(3)

Where $\varepsilon_{r_1}, \varepsilon_{r_2}$ are the relative dielectric constant of substrate and superstrate respectively. $C_{\text{dyn}}(2R, h_1, h_2, \varepsilon_{r_1}, \varepsilon_{r_2})$ and $C_{\text{dyn}}(2R, h_1, h_2, \varepsilon_{r_1} = \varepsilon_{r_2} = 1)$ are the total dynamic capacitance of covered and air filled patch respectively. The dynamic capacitance of circular patch, which takes into account the fringe field, and modal variation of the field is obtained [16].

$$\varepsilon_{\text{dyn}} = \frac{C_{\text{static}}(2R, h_1, \varepsilon_{r_1})}{C_{\text{static}}(2R, h_1, h_2, \varepsilon_{r_1}, \varepsilon_{r_2})}$$

(4)

Where $C_{\text{static}}(2R, h_1, \varepsilon_{r_1})$ is the main static capacitance of the patch and given by:

$$C_{\text{static}} = \frac{\varepsilon_0 \varepsilon_{r_1} R^2}{\gamma h_1} \left[ 1 - \frac{J_{n-1}(K_r) J_{n+1}(K_r)}{J_n^2(K_r)} \right]$$

Figure 2. Sensitivity of modes on circular microstrip patch.

$$\gamma = \begin{cases} 1 & \text{for } n = 0 \\ 2 & \text{for } n \neq 0 \end{cases}$$

(5)

$K$ is the wave number. The dielectric cover does not influence the main capacitance of circular disk. For the fundamental TM$_{11}$ mode, static capacitance can be obtained as:

$$C_{\text{static}} = 0.3525 \varepsilon_0 \varepsilon_{r_1} \pi R^2 / h_1$$

(6)

Where $h_1$ is the substrate thickness of the patch.

To calculate fringe capacitance we have done a structural transformation by replacing $W = \pi R/2$ and $L = 2R$ which gives very accurate result of resonant frequency. The fringe capacitance can be obtained from the following expression.

$$C_{\text{static}} = \frac{1}{2} \left[ Z_0(W, h_1, h_2, \varepsilon_{r_1} = \varepsilon_{r_2} = 1) L - \frac{\varepsilon_0 \varepsilon_{r_1} A}{h_1} \right]$$

(7)

Where $A$ is the area of central patch and $Z_0(W, h_1, h_2, \varepsilon_{r_1} = \varepsilon_{r_2} = 1)$. $Z(W_1, h_1, h_2, \varepsilon_{r_1}, \varepsilon_{r_2})$ are the characteristic impedances of the patch of width $W = 2R$ on the air substrate and with dielectric respectively. The characteristic impedance can be calculated by using variational method along with TTL technique to obtain the Green’s function of the structure[16]. Therefore the capacitance per unit length of the line is obtained from expression [19].

$$\frac{1}{C} = \frac{1}{\pi \varepsilon_0} \int_0^\beta \left[ \frac{f(\beta)}{Q} \right]^2 \frac{1}{Y} d\beta$$

(8)

Where $Q$ is the total charge on the conducting patch and $f(\beta)$ is the Fourier transform of the charge distribution function and $Y$ is the admittance function [19].

$$Y = \varepsilon_{r_1} \coth(\beta h_1) + \varepsilon_{r_2} \left[ \frac{\varepsilon_{r_2} + \coth(\beta h_2)}{1 + \varepsilon_{r_2} \coth(\beta h_2)} \right]$$

(9)
3. RESULTS

This paper the MWM is applied to determine the resonant frequency of electrically thin and thick circular microstrip patch antennas and the computed results obtained from the MWM are compared experimentally theoretically verified values which are given in the literature [13] and the total values are listed in Table 1.

The measured values [1-6] and calculated by [1,3,4-12] and the minimum percentage error and the maximum percentage error are listed in Table 2. It can be seen that in Tables 1 and 2 that two models [13] and [10] and our present model MWM have good agreement with experimental results. But the importance of the model MWM is that none of the above mentioned models are applicable to calculate the cover effect on the resonant frequency. The effect of cover on the resonant frequency is calculated by the MWM and compared against the result of SDA and are listed in the Table 3. Our results are in about 1% difference compared to the results from the SDA.

The resonant frequency calculated by the MWM is always higher than that calculated by using SDA. However the resonant frequency calculated by the SDA is between 0.5% and 1% lower than that of measured value for the uncovered patches, this is due to high value of the calculated $\varepsilon_{\text{eff}}$ using SDA. Thus, the MWM is expected to provide a resonant frequency about 0.5% of the experimental value for covered circular patches. Table 4 shows the accurate determination of resonant frequency of higher order modes as computed by the MWM and compared against experimental and theoretical results of Antoszkiewicz [6]. The average error calculated by the MWM is 0.88% where as, from the Antoszkiewicz method is 1.3%. Table 5 shows the resonant frequency computed by the MWM compared results obtained from Howell [17] and the minimum percentage error of resonant frequency computed by the MWM is zero and maximum is 9.4%, the average error for all modes is 2.2%. Whereas, the minimum percentage error calculated by Howell is 1.39%, the maximum is 10.6% and average is 4.8%. For mode (4,1.1,2), (5,1.2,2) and (1,3.7,1), in Table 5 one resonant was observed during the experiment. Thus, the average resonant frequency of two modes has been taken for error

### TABLE 3. The Effect of Cover on the Resonance Frequency.

<table>
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<th>R (cm)</th>
<th>$\varepsilon_{11}$</th>
<th>$\varepsilon_{12}$</th>
<th>$h_1$ (cm)</th>
<th>$h_2$</th>
<th>$f_{\text{res}}$ (MHz)</th>
<th>SDA [14]</th>
<th>$\text{SDA}_1$</th>
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<td>2.5</td>
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<tr>
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### TABLE 4. Resonance Frequency of Higher Order Mode for the Circular Microstrip Disk Antenna. $h_1 = 0.318$ cm, $R = 4.85$ cm, $\varepsilon_{11} = 2.5$.

### TABLE 5. Resonance Frequency of the Higher Order Mode for Circular Microstrip Patch Antennas. $R = 3.4925$ cm, $\varepsilon_{11} = 2.5$, $h_1 = 0.3175$ cm.
estimation. Thus, the MWM can be used for the accurate determination of resonant frequency of higher modes of a circular patch. The results of resonant frequency of higher modes of the circular patch from Full Wave Analysis method are not available in the literature.

4. SENSITIVITY OF THE MODES

Figure 2 shows the sensitivity of the resonant frequency of the higher order modes with respect to the variation in $\varepsilon_r$. The resonant frequency of the higher order modes are more sensitive to the variation in $\varepsilon_r$. For instance 2% change in $\varepsilon_r$ results into 0.5% change in the resonant frequency of TM$_{11}$ mode whereas, it results into 0.85% change in the resonant frequency of higher modes.

5. UNCERTAINTY EFFECT

Figure 3 gives us the deviation in the calculated resonant frequency by the MWM due to uncertainty in the fabrication process of microstrip patch resonator and antenna. Several resonating circular patches operating in TM$_{11}$ mode have been designed on microstrip thickness $h_1=0.5875$ cm dielectric constants.
6. CONCLUSION

An accurate unified model, the Modified Wolff Model (MWM), is presented to determine the resonant frequency of dominant and higher order modes under the multi-layer condition in thin and thick circular microstrip patch antennas. The importance of the model is that it is applicable to all kinds of arbitrary geometry of patches. It achieves the accuracy of full wave analysis method without any computational difficulties. Almost in all cases the calculated results of MWM comparing to the results, which obtained from other numerical methods are more closely to the experimental values. The MWM model has also been applied satisfactory for higher order modes and the results compared with the experimental values are more closely than the results obtained from numerical models and also the MWM applied to determine uncertainty effect on resonant frequency and sensitivity of modes to the resonant frequency.

7. REFERENCES


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