BOUNDARY LAYERS AND HEAT TRANSFER ON A ROTATING ROUGH DISK

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Abstract The study of flow and heat transfer over rotating circular disks is of great practical importance in understanding the cooling of rotary machinery such as turbines, electric motors and design and manufacturing of computer disk drives. This paper presents an analysis of the flow and heat transfer over a heated infinite permeable rough disk. Boundary-layer approximation reduces the elliptic Navier-Stokes equations to parabolic equations, where the Keller-Cobeci method of finite-difference solution is used to solve the resulting system of partial-differential equations. The surface roughness is assumed to influence the turbulent boundary layer by adding a roughness parameter height K, while a variable surface temperature induces heat transfer into the flow of fluid over the rotating disk. Blowing and suction is also considered as a means of varying the surface shear distribution. The resulting curve-fit equations to the numerically calculated results of the skin-friction coefficient for three regions of laminar, transition and turbulent flow are shown to be consistent to those obtained for flow over a flat plate or a circular cylinder. To study the influence of surface roughness, calculations for various surface roughness parameters are made and results are presented. Velocity and temperature profiles and the shear stress and heat flux at the surface of the rotating disk are presented for a range of the above parameters.

Key Words Rotating Disk, Boundary Layer Flow and Heat Transfer, Finite Difference Scheme

INTRODUCTION

The purpose of this paper is to investigate the problem of a disk rotating in an infinite quiescent fluid with emphasis on the effects of roughness on the convective heat transfer coefficient under the condition of variable wall temperature and transpiration. Depending on the radius, rotational velocity or the kind of fluid (Reynolds number of the flow), laminar, transitional or turbulent may exist. The problem arises in many industrial applications such as disk-drive industry where minimum spacing between head and medium is the objective for increasing the recording density. Investigation of the entire flow requires solution of
the elliptic equations of motion, while the flow close to the disk can be approximated by parabolic boundary-layer equations.

Review of the literature indicates that the studies of fluids flow and heat transfer over a rotating disk have been the subject of several works [1-3]. Borisevich and Potanin [1] analyzed the boundary layer flows induced by rotation of a disk with surface inspirations on the basis of an approximate integration of the equations of motion. The solution of laminar convective heat transfer over a rotating disk with sinusoidal-shaped surface roughness is presented by Palec et al. [2,3]. On the other hand, there has been no work reported on investigating the phenomenon of convective heat transfer over a rough rotating disk with surface inspirations for laminar and turbulent regions of the flow. This paper, therefore, addresses this issue.

In this work, the boundary-layer equations describing the three-dimensional flow over the rotating disk is solved using an implicit finite difference scheme as described by Cebeci & Smith [4]. The boundary-layer approximation reduces the Navier-Stokes equations to a parabolic set of nonlinear partial differential equations. The resulting system of PDEs is then solved using an efficient implicit finite difference scheme. A nonuniform mesh is used and eddy viscosity concept models the turbulent Reynolds stress and heat flux terms. The model is validated using available experimental data.

**METHOD OF SOLUTION**

The time-averaged conservation equations for mass, momentum and energy in a stationary cylindrical co-ordinates for the steady incompressible axisymmetric boundary-layer on a rotating disk and in the absence of radial pressure gradient are:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( ru \right) + \frac{\partial v}{\partial y} = 0 \quad (1)
\]

\[
u \frac{\partial u}{\partial r} + v \frac{\partial u}{\partial y} - \frac{w^2}{r} = \frac{1}{\rho} \frac{\partial}{\partial y} \left( \mu \frac{\partial u}{\partial y} - \rho \frac{\partial \bar{u}}{\partial \bar{y}} \right) \quad (2)
\]

\[
u \frac{\partial w}{\partial r} + v \frac{\partial w}{\partial y} + u \frac{w}{r} = \frac{1}{\rho c_p} \frac{\partial}{\partial y} \left[ \frac{\partial \phi}{\partial y} - \rho \frac{\partial \bar{w}}{\partial \bar{y}} \right] \quad (3)
\]

\[
u \frac{\partial T}{\partial r} + v \frac{\partial T}{\partial y} = \frac{1}{\rho c_p} \frac{\partial}{\partial y} \left[ \frac{\partial \phi}{\partial y} - \mu \frac{\partial \phi}{\partial \bar{y}} \right] \quad (4)
\]

\[
q = - \left( k \frac{\partial T}{\partial y} - \rho c_p \frac{\partial \bar{v}}{\partial \bar{y}} \right) \quad (5)
\]

\[
\mu \phi = \tau_r \frac{\partial u}{\partial r} + \tau_\phi \frac{\partial w}{\partial y} \quad (6)
\]

\[
\tau_r = \mu \frac{\partial u}{\partial y} - \rho \frac{\partial \bar{u}}{\partial \bar{y}} \quad (7)
\]

\[
\tau_\phi = \mu \frac{\partial w}{\partial y} - \rho \frac{\partial \bar{w}}{\partial \bar{y}} \quad (8)
\]

Here, \( u, v, \) and \( w \) are the mean radial, axial, and tangential components of velocity, respectively.

For a given variable disk surface temperature, the appropriate boundary conditions for the above equations considering the effect of surface inspiration denoted by surface normal velocity \( v_s \) are:

\[
y = 0, \ u = 0, \ v = v_s(r), \ w = \omega r \quad (9)
\]

\[
y \rightarrow \infty, \ u = 0, \ w = 0, \ T = T_0 \quad (10)
\]

**EDDY-VISCOSITY FORMULATION**

The turbulent boundary-layer formulation follows that of Cebeci & Smith [4] in which the boundary-layer is divided into inner and outer layer regions.

In the inner-region of the boundary-layer, the
turbulent viscosity is given by:

$$\varepsilon = L \left[ \frac{\partial u^1}{\partial y} + \frac{\partial \bar{w}}{\partial y} \right]^{\frac{1}{2}} 0 \leq y \leq y_c$$  \hspace{1cm} (11)

with

$$\bar{w} = \omega r - w \quad \text{and} \quad L = \kappa y \left[ 1 - \exp \left( - \frac{y}{A} \right) \right]$$  \hspace{1cm} (12)

And, the outer region of boundary-layer,

$$\varepsilon = 0.01681 \left[ \int_0^y \left[ \omega r - (u^1 + \bar{w}) \right] dy \right]_{y_c}^{y} \leq y \leq \delta$$ \hspace{1cm} (13)

here $y_c$ is where $\varepsilon = \varepsilon_c$.

For an aerodynamically unsmooth rotating disk, employing the concept of equivalent sand-grain roughness as a means of characterizing the surface roughness of the disk, and assuming that the plate has a uniform average roughness height $\kappa$, based on a roughness Reynolds number $\kappa^* = \kappa u_c / \nu$, we distinguish three regions of, aerodynamically-smooth wall:

$$\kappa^* < 5$$ \hspace{1cm} (14)

transitional-roughness regime:

$$5 \leq \kappa^* \leq 70$$ \hspace{1cm} (15)

fully-rough flow:

$$70 < \kappa^* < 2000$$ \hspace{1cm} (16)

To account for the fact that $L_c$ and $\varepsilon_c$ cannot go to zero at the wall, i.e., the eddy diffusivity and mixing length must be finite at $y=0$, we denote $L$ in Equation 11 as:

$$L = \kappa (y + \Delta y) \left[ 1 - \exp (-(y+\Delta y)/A) \right]$$  \hspace{1cm} (17)

with

$$\kappa = 0.4$$  \hspace{1cm} (18)

and $\Delta y$ is expressed as a function of equivalent sand-grain-roughness parameter $\kappa^*$, by defining $\Delta y^*$ as;

$$\Delta y^* = \frac{\Delta y u_c}{v}$$  \hspace{1cm} (19)

$$\Delta y^* = 0 \quad \kappa^* < 5$$  \hspace{1cm} (20)

$$\Delta y^* = 0.9 \left( \sqrt{\kappa^*} - \kappa^* \exp (-\kappa^* / 6) \right) \quad 5 \leq \kappa^* \leq 70$$ \hspace{1cm} (21)

$$\Delta y^* = 0.7 (\kappa^*)^{0.58} \quad 70 \leq \kappa^* \leq 2000$$ \hspace{1cm} (22)

The effect of the surface inspiration is accounted for in the inner layer by the following relation;

$$A = \frac{A^* \sqrt{u_c}}{N}$$ \hspace{1cm} (23)

where;

$$A^* = 26, \quad N = \exp (11.8 \sqrt{\nu^*_c})^{\frac{1}{2}}, \quad \nu^*_c = \frac{\sqrt{\nu}}{u_c}$$  \hspace{1cm} (24)

In case there is no suction and blowing;

$$A = A^* \sqrt{u_c}$$  \hspace{1cm} (25)

Now, a similarity variable;

$$\eta = \frac{1}{2} \left( \frac{\omega}{v} \right) y$$ \hspace{1cm} (26)

and a non-dimensional stream function $f(r, \eta)$, defined by:

$$f(r, \eta) = \frac{\psi (r, \eta)}{r^2 (\nu \omega)^{0.5}}$$ \hspace{1cm} (27)
are introduced. Here, the stream function \( \psi (r, \eta) \) is defined as:

\[
ur = \frac{\partial \psi}{\partial z} \quad \text{wr} = -\frac{\partial \psi}{\partial r}
\]

(28)

Using the concept of eddy viscosity, the Reynolds stress terms can be written as:

\[
- \rho \bar{u} \bar{v'} = \rho e_m \left( \frac{\partial u}{\partial y} \right) v_r
\]

(29)

\[
- \rho \bar{v} \bar{w'} = \rho e_m \left( \frac{\partial w}{\partial y} \right) v_r
\]

(30)

\[
- \rho e_r \bar{u} \bar{v'} = \rho e_r \left( \frac{\partial u}{\partial y} \right) v_r
\]

(31)

Here the primes indicate differentiation with respect to \( \eta \). In terms of the transformed variables, the boundary conditions, become:

\[
\eta = 0 \quad f' = 0 \quad g' = 1 \quad f_\infty = \frac{-1}{r^2 (v \omega)^{0.5}} \int_0^r v r \, dr \quad \text{(38)}
\]

\[
S = S_{w}^{(0)} = \frac{\rho q}{T} \quad \text{or}
\]

\[
\eta = 0, \quad \frac{\partial S}{\partial \eta} = -\frac{1}{K T} \left( \frac{v}{\omega} \right)^{1/2} q_{r} w (r)
\]

\[
\eta \to \infty, \quad f' = 0 \quad g' = 0 \quad S = 1
\]

(39)

### NUMERICAL METHOD OF SOLUTION

Equations 32-34 subject to boundary Conditions 38 and 39 are transformed into a system of eight first-order differential equations. The complete system of first order differential equations are then discretized using the semi-implicit Keller-Cebeci scheme[5] where the derivatives are approximated by centered difference quotients and averages centered at the mid-points of net rectangles.

Linerization of the system of nonlinear difference equations is then obtained by Newton's method. The computations are performed using a nonuniform mesh with a geometrically increasing grid system in the axial direction. Therefore, a fine mesh is obtained near the wall and a coarse one away from the solid wall and near the outer edge of the boundary layer. Using the above method a second-order accurate solution is obtained.

In order to quantify the above procedure, the data of Cham and Head [6] is chosen. The measurements were made on a 3 ft. in diameter and 1/2 in. thick steel smooth rotating disk. Traverses of the boundary-layer were made at three
rotational speeds of 515, 1000 and 1550 rev/min and at three radii on the disk for each speed. The range of Re \(_t\) (=\(\omega R_t^2/\nu\)) covered in the Cham & Head data was from \(3 \times 10^5\) to \(2 \times 10^6\). Figure 1 compares the experimentally obtained values of the \(C_{r_\theta}\), the circumferential skin-friction coefficient, defined as:

\[
C_{r_\theta} = \frac{2 \mu}{\rho \omega R_t^2} \frac{\partial}{\partial y} \left( \frac{\partial \omega}{\partial y} \right)_{\text{wall}}
\]  

(40)

The Figure indicates that there exists a fair agreement between the predictions and the experimentally obtained values.

Based on the calculated values of \(C_{r_\theta}\) vs. Reynolds number, and in the spirit of curve-fit equations given for a flow inside a cylinder or over a flat plate, the following equations are obtained by passing a smooth curve through the calculated values:

**A - Laminar Region**

Similar to the skin friction coefficient equation for a laminar flow over a flat plate, the following equation is derived by curve-fitting the results of the numerical model shown in Figure 1:

\[
C_{r_\theta} = \frac{1.23}{\sqrt{\text{Re}_t}}
\]

(41)

**B - Transition Region**

For the transition region, curve-fitting the calculated results in a fashion similar to that suggested by Schlichting [7], we have:

\[
C_{r_\theta} = \frac{0.039 \cdot 1413}{\text{Re}_t^{1/7} \cdot \text{Re}_t}
\]

(42)

**C - Turbulent Region**

For the turbulent region, an equation similar to that suggested by White [8] for a smooth surface is as follows:

\[
C_{r_\theta} = \frac{0.448}{\ln^2 (0.096 \text{Re}_t)}
\]

(43)

**HEAT TRANSFER CALCULATIONS**

The results of the heat transfer calculations of the scheme is shown in two parts. First, simulating the laminar, constant wall temperature conditions of Mccomas & Hartnett [9], yields Figure 2, where there exists an excellent agreement.
between the experimental and the numerically obtained values of the dimensionless laminar temperature profiles. The experimental data of McComas & Hartnett is obtained for the conditions of a smooth rotating disk of 483 mm, where build-in-electric heaters were used to maintain a uniform temperature on the disk. In their tests, McComas and Hartnett measured average Nusselt numbers for a range of $2 \times 10^4 < Re_c < 6 \times 10^5$.

Next, shown in Figure 3 is the result of the application of the model to the experimental heat transfer data obtained for both laminar and turbulent flow conditions. The solid line compares the results of calculations with the experimental data of McComas & Hartnett [9] and Owen & Haynes [10], while the dashed line is a comparison of the numerical results to those of Cobb & Saunders [11], for the respective conditions of constant and variable wall temperatures, given as:

$$T_w(r)/T_w = \text{Const.}, \quad T_v(r)/T_w = r^{2+1} \quad (44)$$

As for the skin friction results, the proposed model makes a fair prediction of the heat transfer coefficient over a rotating disk.

**SURFACE ROUGHNESS CONSIDERATIONS**

The existence of surface roughness has no effect on the laminar and transition regions of the flow. In this model, its influence is seen only in the inner region of the turbulent flow. Figures 4-6 show the results of calculations for the radial and circumferential components of the flow, as well as the skin friction coefficient for various values of surface roughness heights. Figures 4 and 5 which show the two components of velocity plotted versus the non-

![Figure 4](image1.png)  
**Figure 4.** Calculated values of radial component of velocity for surface roughness ($k=1 \times 10^{-3}$ m) and $Re_c$.

![Figure 5](image2.png)  
**Figure 5.** Calculated values of circumferential component of velocity for surface roughness ($k=1 \times 10^{-3}$ m) and $Re_c$.  

Figure 3. Comparison of calculated values of $Nu^*$ vs. $Re$ with experimental results for a constant and variable wall temperature.
dimensional similarity parameter \( \eta \), indicate thickening of the boundary-layer for increased surface roughness. Figure 6 indicates that the wall friction dramatically increases with even a small roughness on the disk. The figure also indicates that the effect of surface roughness is similar to that of a flow inside a pipe or a flat-plate with rough walls. Figure 7 shows heat transfer results in terms of Nusselt number plotted versus Reynolds number for various values of wall roughness parameters. As in the case of a flat plate, at a given value of \( \text{Re} \), the value of \( \text{Nu} \) increases with increased wall roughness.

THE EFFECT OF SURFACE INSPIRATION

Figures 8 and 9 show circumferential surface friction and heat transfer coefficients on the surface of a rotating disk in the presence of surface suction and blowing. The results indicate that there exists an increase of about 20% in the magnitude of skin friction coefficient and 5% in the heat transfer coefficient for conditions of surface suction and blowing.

**Figure 6.** Comparison of circumferential skin-friction coefficient for various values of surface roughness.

**Figure 7.** Calculated values of Nusselt versus Reynolds number for specified surface roughness values.

**Figure 8.** Calculated values of circumferential skin friction coefficient for conditions of surface suction and blowing.

**Figure 9.** Calculated values of Nusselt number for conditions of surface suction and blowing.
coefficient when surface blowing velocity of 0.002 m/s is reduced to -0.002 m/s of surface suction at $Re_c=2000$. The figure indicates an expected decrease in the values of $C_{fl}$ and $NU$ when comparing the conditions of surface suction and blowing with decreased $Re_c$.

CONCLUSIONS

The issue of modelling of flow over a rotating disk with surface roughness has not been considered extensively in the past, with the exception of the works of Palec [2, 3]. Though, in his work, Palec models surface roughness assuming a periodical profile with sinusoids, it is merely the definition of surface waviness and not roughness.

In the present work, the problem of fluids flow over a rotating disk with a specified surface roughness is solved using the boundary-layer approximations. The equations of motion coupled with the energy equation are solved using an efficient second order implicit finite-difference scheme. Uniform surface inspiration is included as a boundary condition to the problem. The influence of the surface roughness is contained in the inner-layer of the turbulent flow. The numerical solution of the three dimensional boundary-layer flow and heat transfer is shown to be in good agreement with the available experimental data.

REFERENCES