

# AN ADAPTIVE IMPEDANCE CONTROLLER FOR ROBOT MANIPULATORS

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**Abstract** A desired dynamic behavior of constrained manipulators can be achieved by means of impedance control and various implementations of fixed controllers have been proposed. In this paper, an adaptive implementation is presented as an alternative to reduce the design sensitivity due to manipulator mismatch. The adaptive controller globally achieves the impedance objective for the nonlinear dynamic model of rigid robot manipulators.

**چکیده** رفتار دینامیکی مطلوب دست‌های مصنوعی محدودیت دار را می‌توان از طریق روش کنترل امپدانس بدست آورد. روشهای اجرایی مختلفی برای کنترل کننده‌های ثابت پیشنهاد شده است. در این مقاله کاربرد یک روش وقتی به عنوان انتخاب دیگری برای کاهش حساسیت طراحی در اثر عدم تطابق مدل با دست مصنوعی پیشنهاد شده است. کنترل کننده وقتی در مجموع، هدف امپدانس برای مدل دینامیکی غیر خطی دست مصنوعی صلب در رباتها را برآورده می‌سازد.

## INTRODUCTION

Constrained motion control of robot manipulators is concerned with the control of a robot whose end-effector interacts mechanically with the environment. Most assembly operations and manufacturing tasks require mechanical interactions with the environment or with the object being manipulated, along with fast motion in free and unconstrained space. Several approaches to constrained motion control have been suggested as impedance control, force control and hybrid position/force control.

The fundamental philosophy of impedance control, due to Hogan [1], is that the manipulator control system should be designed, not to track a motion or force trajectory alone, but rather to regulate the mechanical impedance of the manipulator

[2]. The impedance control specifications consist of a desired motion trajectory and a desired dynamic relationship between the motion trajectory of the end-effector and measured end-effector applied forces. Two common approaches to controlling impedance via feedback control are the so-called position-based impedance control [3, 4] and the torque-based impedance control [1, 5]. Stability analysis and comparison of the behavior of both approaches are presented by Lawrence [6].

The dynamic behavior of rigid manipulators is described by a set of complex nonlinear differential equations. Most high performance model-based control strategies rely on the exact cancellation of the nonlinear dynamic. The uncertainty in some robot parameters, as link inertia and payload, has motivated

many researches in the last ten years to consider the design of globally stable adaptive controllers for robots.

The adaptive constrained motion control is not yet well developed. Adaptive force controllers have been designed for simple linear model arms [7], or without rigorous stability analysis for full nonlinear dynamic manipulator models [8, 9]. An adaptive hybrid position/force controller was proposed by Slotine and Li [10]. Where adaption is only driven by the position errors.

In this paper an adaptive impedance controller for constrained robots is presented. It is motivated by the position-based impedance control approach which consists of two loops. An external one generates a modified motion reference by adding a term obtained by filtering the measured interacting force by the inverse of the impedance transfer function. This modified motion reference is applied to an internal loop which consists of an adaptive motion controller based on an inverse dynamics plus an additional compensation. This adaptive structure has been presented previously [11]. Compared with a recent work on adaptive impedance controllers [12] the controller presented here has the advantage that all controller gains have a direct interpretation and can be defined independently of the impedance parameters, which specify the control objective.

## ROBOT MODEL AND PROBLEM FORMULATION

In the absence of friction and other perturbations, the Cartesian-space dynamics of an n-link constrained rigid robot manipulator can be written as [5]:

$$\ddot{F}_a = H(x)\ddot{x} + c(x, \dot{x})\dot{x} + g(x) + F$$

where  $x$  is the  $n \times 1$  vector of Cartesian positions and Euler angles of the end effector related to a fixed frame of reference  $R_0$  on the robot base.  $F_a$  is the  $n \times 1$  vector of forces/moments due to actuators but referred to the end-effector, and  $F$  is the vector of forces/moments at the end effector due to interaction.  $H(x)$  is the  $n \times n$  symmetric positive definite manipulator inertia matrix.  $C(x, \dot{x})\dot{x}$  is the  $n \times 1$  vector of centripetal and Coriolis forces, and  $g(x)$  is the  $n \times 1$  vector of gravitational forces. It is considered that the manipulator is non-redundant. It is assumed that the robot is equipped with joint position and velocity sensors and a force sensor at its end-effector. The relationship between joint positions and end-effector configuration is:

$$X = f(q)$$

with the corresponding velocity relation:

$$\dot{X} = J(q)\dot{q} \quad (2')$$

with  $q$  the  $n \times 1$  vector of joint displacements and  $J(q) = \partial f(q) / \partial q$  the Jacobian matrix. Also, the relationship between force/moments at the end effector and at the joints is given by:

$$\tau_a = J^T(a)F_a$$

$$\tau = J^T(q)F$$

Some important properties of the Cartesian arm dynamics are given below.

Property 1 [10]. Matrices  $H(x)$  and  $C(x, \dot{x})$  in (1) satisfy

$$\frac{d}{dt} [H(x) - 2C(x, \dot{x})]z = 0$$

for all  $z \in \mathbb{R}^n$ .

Property 2 [10]. A part of the dynamics (1) is linear in terms of a suitable selected set of robot and load parameters. i. e.

$$H(x)\ddot{x} + C(x, \dot{x})\dot{x} + g(x) = \Omega(x, \dot{x}, \ddot{x})\theta$$

where  $(x, \dot{x}, \ddot{x})$  is an  $n \times m$  matrix and an  $m \times 1$  vector containing the selected set of parameters.

Property 3.  $H(x)$  is an  $n \times n$  symmetric positive definite matrix and there is a constant  $\alpha > 0$  such that

$$\alpha I \leq H(x) \quad \text{for all } x \in \mathbb{R}^n$$

For revolute joint robots, if in addition  $J(q)$  is a bounded  $n \times n$  matrix, then there is a  $\beta (\alpha < \beta < \infty)$  such that

$$\alpha I \leq H(x) \leq \beta I \quad \text{for all } x \in \mathbb{R}^n.$$

Now, the adaptive impedance control problem can be formulated. Consider the robot manipulator described by (1). The dynamic vector parameter  $\theta$  of the manipulator and payload is constant but unknown. The Jacobian matrix  $J(q)$  is assumed to be nonsingular and known. Knowledge of  $J(q)$  is not restrictive because it does not depend on the dynamic parameters. The specifications of the impedance control problem are given in this paper in terms of a desired motion trajectory and (eventually) a desired force function and a desired dynamic relationship between the position error and the force (or force error) at the end-effector. The robot arm may or may not interact with the environment. The impedance control problem can be stated as

that of designing a controller to compute the torques  $T_a$  applied to the joints, so that the following control arm is verified.

$$x(t) - x_d(t) \rightarrow [Mp^2 + Bp + K]^{-1} (F(t) - F_d(t)) \quad \text{as } t \rightarrow \infty$$

where  $X_d(t)$  is the desired motion trajectory in the Cartesian space,  $F_d(t)$  the desired interaction force at the end-effector and  $p = d/dt$ ,  $M$ ,  $B$ ,  $K$  are arbitrary  $n \times n$  positive definite matrices. Let us define the impedance error as:

$$\xi = e + [Mp^2 + Bp + K]^{-1} \tilde{F} \quad (4)$$

where  $e(t) = x(t) - x_d(t)$ ,  $\tilde{F}(t) = F(t) - F_d(t)$ . Hence the control aim is verified provided that  $\xi(t) \rightarrow 0$  as  $t \rightarrow \infty$ . A technical lemma is now established.

Lemma 1. Let the transfer function  $H(s) \in \mathbb{R}^{n \times n}(s)$  be exponentially stable and strictly proper. Let  $u$  and  $y$  be its input and output respectively. i) If  $u \in \mathbb{L}^n$  then  $y, \dot{y} \in \mathbb{L}^n_\infty$ . ii) If  $u \in \mathbb{L}^n_2 \cap \mathbb{L}^n_\infty$  then  $y \in \mathbb{L}^n_2 \cap \mathbb{L}^n_\infty$  and  $y(t) \rightarrow 0$  as  $t \rightarrow \infty$ .

## ADAPTIVE IMPEDANCE CONTROLLER

### Controller

The adaptive impedance controller proposed to solve the problem formulation consists of a two loops controller structure with a parameter estimator or adaptive law, as shown in Figure 1.

Considering the desired motion trajectory specification, the external loop generates a modified motion reference by adding a term which is obtained by filtering the measured interacting force by the inverse of the specified impedance transfer function. Also, a modified reference is obtained for velocity

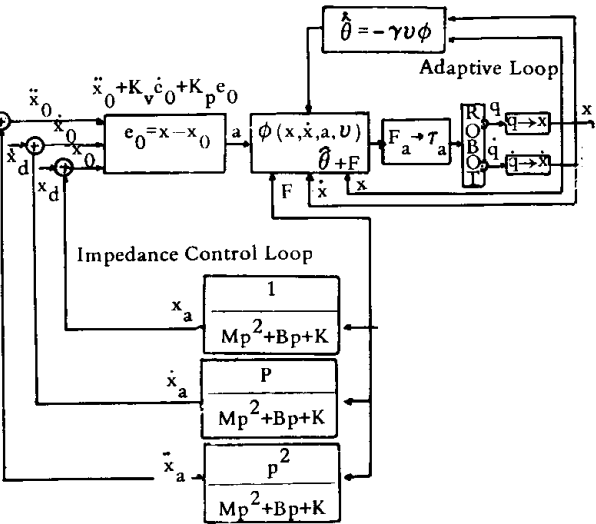


Figure 1. Block diagram for the adaptive impedance controller

and acceleration functions. These modified motion references are applied to the internal loop. This loop consists of an adaptive motion controller based on an inverse dynamics law plus an additional compensation and a parameter update law. The controller equations are given in the following.

External (impedance) loop:

$$x_a(t) = [M_p^2 + B_p + K]^{-1} \tilde{F}(t) \quad (5)$$

Modified motion references:

$$x_0(t) = x_d(t) + x_a(t) \quad (6)$$

$$\dot{x}_0(t) = \dot{x}_d(t) + \dot{x}_a(t)$$

$$\ddot{x}_0(t) = \ddot{x}_d(t) + \ddot{x}_a(t)$$

Internal (adaptive motion) loop:

$$F_a = \hat{H}a + \hat{C}[\dot{x} - v] + \hat{g} + F \quad (7)$$

$$a = \ddot{x}_0 - K_v \dot{e}_0 - K_p e_0 \quad (8)$$

$$v = (1/(p+\lambda))[\ddot{e}_0 + K_v \dot{e}_0 + K_p e_0]$$

$$e_0 = x - x_0$$

where  $\hat{H}$ ,  $\hat{C}$ ,  $\hat{g}$  have the same functional form as  $H(x)$ ,  $C(x)$ ,  $g(x)$ , respectively, with estimated parameters  $\hat{\theta}$  (see property 2),  $\lambda$  a positive scalar and  $K_v$ ,  $K_p$  are positive definite gain matrices. Notice from (9) that

$$\ddot{v} + \lambda v = \ddot{e}_0 + K_v \dot{e}_0 + K_p e_0$$

From a practical point of view,  $v$  can be implemented by

$$v = (p/(p+\lambda))\dot{e}_0 + (1/(p+\lambda))[K_v \dot{e}_0$$

where measurement of joint acceleration was obviated. Due to property 2, the motion control law (7) can be written as

$$F_a = \phi(x, \dot{x}, a, v)\hat{\theta} + F$$

with  $\phi$  an  $n \times m$  matrix.

Update law

To update the parameter vector  $\hat{\theta}$  consider an integral adaptive law [13].

$$\dot{\hat{\theta}} = -\Gamma \phi^T v$$

where  $\Gamma = \Gamma^T$  is an  $m \times m$  positive definite adaptation gain matrix.

### Main Result

The main properties of the proposed adaptive impedance controller are summarized in the following.

Proposition. For the controller described previously, in closed-loop with the manipulator (1), the following holds:

$$(a) \tilde{\theta} \in \mathbb{L}_\infty^n$$

$$(b) v \in \mathbb{L}_2^n \cap \mathbb{L}_\infty^n$$

$$(c) e_o \in \mathbb{L}_2^n \cap \mathbb{L}_\infty^n$$

$$(d) x(t) \rightarrow x_o(t) \text{ as } t \rightarrow \infty$$

$$(e) \dot{\xi}(t) \rightarrow 0 \text{ as } t \rightarrow \infty$$

Now, from the update law (13),

$$\dot{\tilde{\theta}} = -\Gamma \phi^T v \quad (16)$$

where the fact that  $\theta$  is constant has been used.

Consider the non-negative function

$$V(t) = (1/2)[\tilde{\theta}^T \tilde{\theta} + v^T H v] \quad (17)$$

whose time derivative along the trajectories of (15) and (16) is

$$\dot{V}(t) = -\lambda v^T H v \leq 0 \quad (18)$$

where property 1 has been used to eliminate the term  $v^T (H/2 - C)v$ . Equations (17) and (18) imply that  $\tilde{\theta} \in \mathbb{L}_\infty^m$  and  $v \in \mathbb{L}_\infty^n$ . Using property 3 and (18), It can be concluded that  $v \in \mathbb{L}_2^n$ . Now, as  $v \in \mathbb{L}_2^n \cap \mathbb{L}_\infty^n$  and considering (9) with  $e_o$  as the output of an exponentially stable and strictly proper linear filter, from lemma 1 it is concluded that  $e_o, \dot{e}_o \in \mathbb{L}_2^n \cap \mathbb{L}_\infty^n$  and  $e_o(t) \rightarrow 0$  as  $t \rightarrow \infty$  ( $x(t) \rightarrow x_o(t)$ ). Now consider the impedance error (4). Using (5) and definition of  $e(t)$ .

or

$$\xi(t) = X(t) - X_d(t) - X_a(t)$$

or, considering (6)

$$\xi(t) = x(t) - x_d(t) - x_o(t) + x_d(t)$$

$$\xi(t) = x(t) - x_o(t)$$

As  $x(t) \rightarrow x_o(t)$ , then  $\xi(t) \rightarrow 0$  as  $t \rightarrow \infty$ . Additional convergence properties (f) and (g) related to error derivatives, are established in the following. Under the assumption of  $F$  and  $F_d$  bounded,  $X_a, \dot{X}_a$  and  $\ddot{X}_a$  are also bounded (refer to lemma 2).

If in addition  $F, F_d, X_d, \dot{X}_d, \ddot{X}_d$  are bounded.

$$(f) \dot{x}(t) \rightarrow \dot{x}_o(t) \text{ as } t \rightarrow \infty$$

$$(g) \dot{\xi}(t) \rightarrow 0 \text{ as } t \rightarrow \infty$$

Proof. First, let us consider the adaptive motion controller whose inputs are the modified references and the outputs are the position and velocity at the end-effector, (Figure 1). The closed-loop system is obtained by equating (1) and (12)

$$H\ddot{x} + C\dot{x} + g = \phi(\tilde{\theta} + \theta) \quad (14)$$

where the parameter error vector  $\tilde{\theta}$  is defined as  $\tilde{\theta} = \hat{\theta} - \theta$ ,  $\theta$  contains the unknown dynamic parameters, which are assumed to be constants. From (7) it can be written.

$$\phi\theta = Ha + C[\dot{x} - v] + g$$

Hence, (14) becomes

$$H[\ddot{e}_o + K_v \dot{e}_o + K_p e_o] + Cv = \phi\tilde{\theta}$$

Substituting (9),

$$H[\ddot{v} + \lambda v] + Cv = \phi\tilde{\theta} \quad (15)$$

Considering that  $\ddot{X}_d$ ,  $\dot{X}_d$ ,  $X_d$  are also bounded, it results  $\ddot{X}_o$ ,  $\dot{X}_o$ , an  $X_o$  to be bounded. Also  $\ddot{x}$ ,  $\dot{x}$  are bounded because  $e_o \in \mathbb{L}_\infty^n$ . Now consider equation (14) written as,

$$\ddot{x} = H^{-1}(x)(\phi(x, \dot{x}, a, v)(\tilde{\theta} + \theta) - C(x, \dot{x})\dot{x} - g(x)) \quad (19)$$

Observing that  $a$  - eq. (8) - and  $v$  are bounded signals, it results  $\phi \in \mathbb{L}_\infty^{n \times m}$  and from (19)  $\ddot{x}$  is also bounded because  $H^{-1}$  is bounded by property (3). Hence  $\ddot{e}_o \in \mathbb{L}_\infty^n$ . The facts that  $\dot{e}_o \in \mathbb{L}_\infty^n$  and  $e_o \in \mathbb{L}_2^n \cap \mathbb{L}_\infty^n$  imply that  $\dot{e}_o(t) \rightarrow 0$  or  $\ddot{x}(t) \rightarrow \ddot{x}_o(t)$  as  $t \rightarrow \infty$ . Finally,

$$\ddot{\xi}(t) = \ddot{x}(t) - \ddot{x}_o(t) \rightarrow 0 \text{ as } t \rightarrow \infty.$$

**Remark.** Force reference  $F_d$  can be used to induce a desired force during interaction of manipulator with the environment. For small values of  $k$  and error  $e$ , a constant  $F_d$  can be attained with practical accuracy.

## SIMULATION RESULTS

Computer simulations have been carried out to show the stability and performance of the proposed adaptive controller. The manipulator used for the simulations is the two - degrees - of - freedom arm moving in a vertical plane as shown in Figure 2. The manipulator is modeled as two rigid links of unitary length with point masses  $m_1$  and  $m_2$  at the distal ends of the links. Friction is not considered in the model.

The simulation experiment is designed as follows (see Figure 2). The desired trajectories last for ten seconds, the first five seconds for free space motion, the remaining for interactive impedance control. The interaction with the environment is modeled in this experiment as

$$f_1 = (b_e \dot{x}_1 + k_e(x_1 - x_{1e})) \text{ if } x_1 \geq x_{1e} \\ \text{otherwise } f_1 = 0$$

$$f_2 = 0 \text{ (no interaction in the } x_2 \text{ axis)}$$

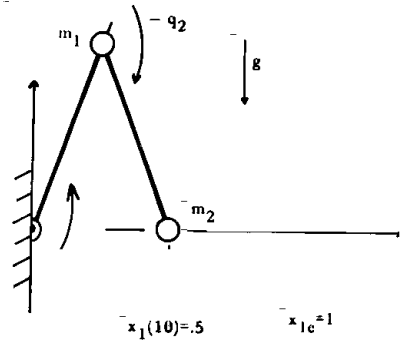


Figure 2. Two link manipulator and the environment

with  $b_e$  the damping and  $k_e$  the stiffness coefficients of the environment. Simulation is carried out using the following desired impedance parameters:

$$M = \text{diag}(m), \quad m = 1$$

$$B = \text{diag}(b), \quad b = 10$$

$$K = \text{diag}(k), \quad k = 25$$

Initial conditions for the manipulator are

$$X(0) = [0.5 \quad 0]^T, \quad \dot{X}(0) = 0.$$

**Controller design**

Parameters are chosen to be.

$$K_p = \text{diag}(k_p), \quad k_p = 30$$

$$K_v = \text{diag}(k_v), \quad k_v = 15$$

$$\gamma = \text{diag}(\gamma), \quad \gamma = 0.5$$

$$\lambda = 15.$$

The desired trajectories as well as the actual trajectories achieved through impedance control are shown in Figure 3. It is clear that the impedance control objective is reduced to an unconstrained motion one during the first five seconds of free space motion. Then the desired  $x_{1d}$  trajectory diverges from the real one  $x_1$  so as to accomplish the impedance control objective. Convergence to zero of  $\xi$  and  $\dot{\xi}$  in both directions is shown in Figure 4. Figure 5 shows evolution of estimated parameters from an initial guess of  $\theta_1(0)=0, \theta_2(0)=0$

control problem has been presented. The dynamic parameters of the manipulator are assumed to be unknown but constant. The controller is based on the position - based impedance control structure and an adaptive motion controller. Compared with previous solutions this controller presents the advantage that all controller gains have a direct interpretation and can be assigned independently of desired impedance parameters. Future research should include comparative as well as practical robustness studies which consider the effects of joint and link flexibility, friction, sensor and actuator dynamics, digital implementation and other uncertainties and perturbations. Experimental analysis should also be considered.

## CONCLUSIONS

An adaptive solution to the impedance

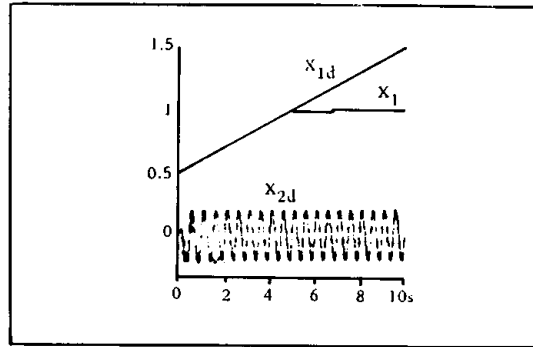


Figure 3. Desired and actual trajectories

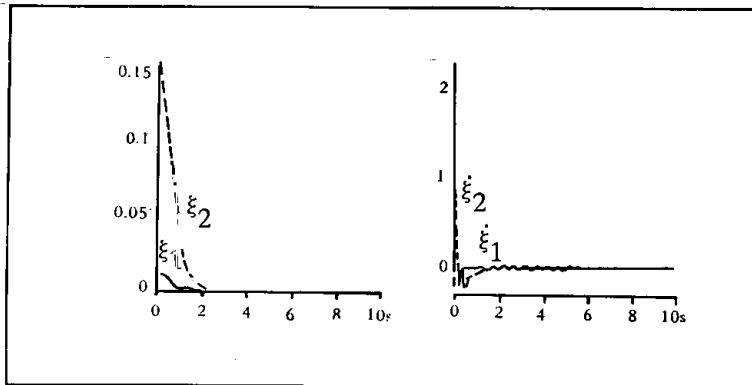


Figure 4. Impedance errors

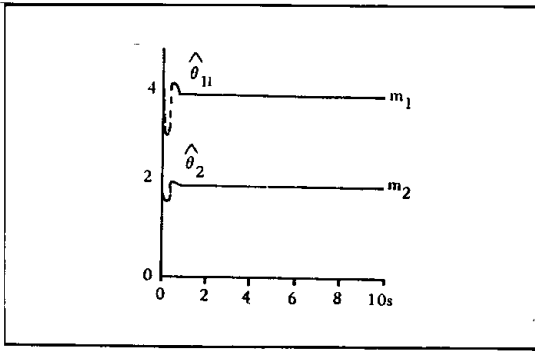


Figure 5. Estimated parameters

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