



## Modelling and Compensation of Uncertain Time-delays in Networked Control Systems with Plant Uncertainty Using an Improved Robust Model Predictive Control Method

F. Pouralizadeh Moghaddam\*, H. Gholizade Narm

Faculty of Electrical Engineering & Robotic, Shahrood University of Technology, Shahrood, Iran

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### ABSTRACT

Control systems with digital communication between sensors, controllers and actuators are called Networked Control Systems (NCSs). In general, NCSs encounter with some problems such as packet dropouts and network induced delays. When plant uncertainty is added to the aforementioned problems, the design of the robust controller that is able to guarantee the stability becomes more complex. In this paper, a method based on Robust Model Predictive Control (RMPC) is proposed to overcome model uncertainty together with unknown delay caused in NCSs. The previous RMPC methods, called Normal RMPC, was proposed to compensate delay or model uncertainty, individually. Hereby, we propose a method, named Improved RMPC, to compensate the effects of delay and model uncertainty on NCSs, simultaneously. The proposed method is based on uncertainty polytope and LMIs (Linear Matrix Inequalities). Also, an experimental evaluation of time-delays in MODBUS networked control systems is proposed. The simulation results show the ability of the proposed method. For comparison, the Normal RMPC is applied as well. The results show that the Normal RMPC has an acceptable performance for time delay compensation, but its performance gets destroyed or even get unstable when the model uncertainty is also considered.

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## 1. INTRODUCTION

A control system communicating with sensors and actuators over a communication network is called NCS [1]. Several theories and research methods have been developed on NCSs in recent years [1]. Many types of digital industrial networks are available in the market. In modern industrial control systems, various protocols such as Profibus, Foundation Fieldbus (FF), and Modbus are included as a part of control loop. Despite the great benefits of networks in control loops, several problems arise in NCSs such as packet dropouts and network induced delays [2]. One of the major problems in NCSs is network-induced delay. Network delays degrade the performance and it may destabilizes the control system for a large amount of delays [3, 4].

For an effective compensation, it is necessary to evaluate delay at first. In fieldbus control systems, the

delay induced by network is unknown, but it is in a bounded interval [5]. The delay is estimated and evaluated for PROFIBUS PA (Process Automation) control loops in literature [5]. In literature [6], delay is evaluated in FF (Foundation Fieldbus) H1 networks analytically. Steam generator control loops with FF H1 networks are analysed for estimation and compensation of delays in refrence [7]. Implementation and evaluation of network induced delays and packet dropouts of wireless networked control systems are presented in literature [8].

After precise evaluation of delays, based on the amount and behavior of delays, the compensation methods are applied to reduce their effects. Predictive control is a well-known candidate for this goal [7, 9]. In literature [10], NCSs with random time-delays are compensated using a modified MPC. In literature [11], a developed model predictive controller is used for

\*Corresponding Author Email: [pouralizadeh.f@gmail.com](mailto:pouralizadeh.f@gmail.com)  
(F. Pouralizadeh Moghaddam)

nonlinear NCSs with delays and packet dropouts. A networked MPC is utilised to compensate constant delays in literature [12]. Because of the uncertainty in delay evaluation, the robust control approaches are applied for compensation of delays in NCSs [4, 13]. In literature [14] a model predictive control algorithm is presented for linear parameter varying systems with both state delays and randomly occurring input saturation. Various approaches are used in robust MPC for delay compensation. Some of them are based on Tube-Based methods and the others are LMI-based convex optimization. In literature [15] a robust tube-based MPC is proposed to compensate delays and disturbances in discrete-time linear systems. It is demonstrated in literature [16] that, the tube based MPC (Model Predictive Control) is an effective method to handle parameter uncertainty in a path tracking application. Uncertainty polytope is used to describe NCSs with delay uncertainty [17]. In literature [18], the robust model predictive control is used only for delay compensation in FF H1 Fieldbus control loops. A robust approach based on  $H_\infty$  control is used in literatures [19, 20] to compensate delays. Finite – time  $H_\infty$  control is developed in literature [21] for NCSs with random time variable delays. Although, several methods have been proposed to compensate time delays, few of them concentrate on time-delays and model uncertainty simultaneously. In reference [22], a completely model-free adaptive controller is designed in the presence of parametric uncertainties and time varying delays .

The aim of this paper is to improve RMPC algorithm to deal with delay and model uncertainty simultaneously. To this end, a discrete model description of the continuous system with both the model and delay uncertainty is presented using an extended definition of uncertainty polytope. Then, the RMPC algorithm is improved to compensate their side effects. The superiorities of this article compared to [23] and [18] are to extend model description and to consider both the delay and model uncertainties in the modified RMPC design. The robust stability is guaranteed in this method. In addition, an experimental evaluation of delays is done to obtain bounds of delays in similar applications. The evaluated delay with known upper and lower bounds can be used in simulation results with real values. It can also help in applying the proposed method on an experimental NCS in the future works as discussed in [5, 18, 24]. To support this investigation, potential sources of delays are identified using a timing diagram. The experimental test bench includes a Modbus communication processor module, a SIEMENS PLC (Programmable Logic Controller) and an electronic board as an actuator to evaluate the origin of the network induced delays

This paper is organized as follows. Introduction is the first section, problem statement, objectives and definition of the assumed Modbus based control loop are presented

in Section 2. In Section 3, evaluation of network-induced delays is investigated in an experimental test bench with Modbus network. In Section 4, the extended description of the system and the proposed RMPC are presented. Simulation results are presented in Section 5 and finally, the conclusion is given in Section 6.

## 2. MATERIALS AND METHODS

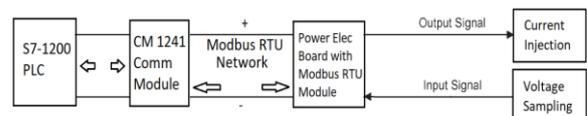
**2.1. Problem Statement** Modbus RTU is a digital, two-way serial network. It can connect multiple devices (up to 127). The original intent of Modbus is to replace multiple traditional analogue and digital signal lines between the controller and devices by a single digital communication channel. In this paper, a test bench of NCS with Modbus network is considered. The block diagram of the test bench is illustrated in Figure 1.

The NCS includes a SIEMENS S7-1200 PLC, a SIEMENS CM 1241 Modbus communication module and a power electronic board with current injection and voltage sampling units. The purpose of this system is to regulate the voltage of the anodes in a cathodic protection system.

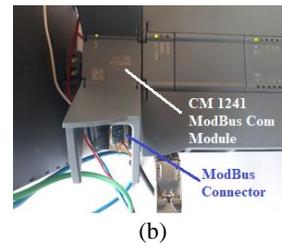
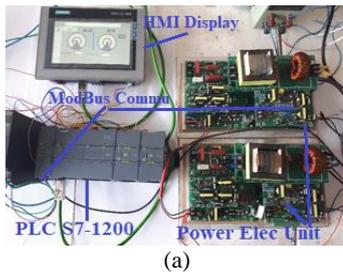
As shown in Figure 1, the PLC receives measured signals and sends control signals to actuator through the network. Therefore, the network-induced delay is appeared in transmission and sending procedure. Another problem in practice is that the plant is unknown. Therefore we faced with two problems: first, the bounds of delay and the model have to be obtained and second, the effects of delay and model uncertainty should be compensated. The goal of this paper is to propose a method for compensating uncertain time-delays together with model uncertainty.

### 2.1. Experimental Evaluation of Network Induced Delays in Modbus Control Loop

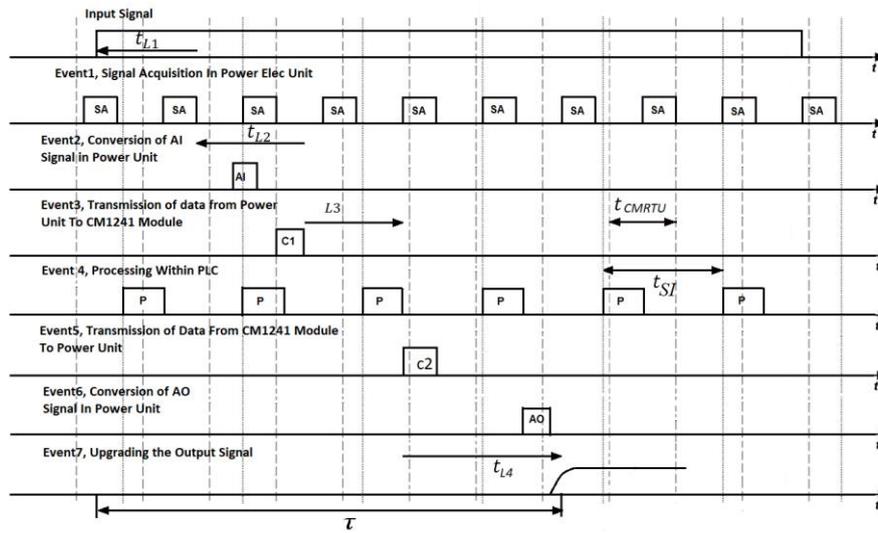
In this section, the delays in Modbus-based NCSs are evaluated. The method can be applied on other NCSs. Figure 1 illustrates an NCS with Modbus network. By using CM1241, the control signals are sent to the electronic unit from the PLC over the Modbus RTU Protocol. The CM1241 module serves as a link active scheduler (LAS) that schedules and manages the transmission of packets. Figure 2 (a, b) illustrates the test bench used to evaluate the induced delay. In this experimental setup, the bounds of delay are obtained.



**Figure 1.** Block diagram of the test NCS with Modbus communication



**Figure 2.** The experimental setup: a) Modbus RTU-based control loop, b) PLC and CM1241 communication modules



**Figure 3.** Timing diagram of the Modbus network-based test loop

**TABLE 1.** Execution time of the events in Figure 3

Event	1	2	3	4	5	6	7
Execution time (s)	0.1	0.15	0.45	0.1	0.45	0.15	0.2

**TABLE 1.** Range of delay for various cycle time ( $t_{CMRTU}$ ) of Modbus RTU Communication Module

$t_{CMRTU}$	0.5	0.75	1	1.25	1.5
$[\underline{\tau}, \bar{\tau}]$	[1.2, 1.55]	[1.4, 1.75]	[1.7, 2.25]	[2.05, 2.9]	[2.35, 3.5]

There are several terminologies in the Modbus and PLC literature. The scan time specifies the execution period of the controller algorithm in the PLC. In order to measure the loop time delay, a -5 to +5 V step input is applied to the voltage sampling section of the power electronic unit, and the SIEMENS S7-1200 PLC forwards it to the actuator. Figure 3 shows the timing diagram associated with seven events. These events are: 1) SA: input signal acquisition; 2) AI: AI signal conversion; 3) transmission of the data from power electronic board to the CM1241 communication module; 4) SI (Scan Interval time): controller execution time in

SIEMENS S7-1200 PLC; 5) data transmission from CM1241 to the power electronic board; 6) AO: Modbus signal conversion to analogue equivalence; and 7) UP: upgrading the output signal.

The scan period of the SIEMENS S7-1200 PLC is set to 100ms. In Figure 3, signal acquisition, scan interval, and cycle time of Modbus RTU ( $t_{CMRTU}$ ) are usually not synchronized in practical NCSs. Thus, the time delay is variable. However, the bounds can be established in terms of the execution time of each event in a given network. From the experimental evaluation and PLC settings, the mean value of time required in each event is given in Table 1.

Due to the direct impact of the  $t_{CMRTU}$  on the time delay, different values of this parameter lead to different higher and lower bounds of time delay. Table 2 shows the experimental results of delay bounds for various cycle time of Modbus RTU.

**2. 2. Proposed Method** In this section, the discrete model of the system is extended to deal with the model and delay uncertainty simultaneously. Then the RMPC algorithm is improved to compensate both the unknown delay and model uncertainty.

**2. 2. 1. Model Description for Uncertain Systems Without Delay** Based on reference [25] the discrete time model of the uncertain continuous LTI system is an LTV system as follow:

$$\begin{cases} x(k+1) = A(k)x(k) + B(k)u(k) \\ y(k) = Cx(k) \end{cases}; \quad (1)$$

$[A(k), B(k)] \in \Omega$

where  $x \in R^n$ ,  $u \in R^m$ , and  $y \in R^r$  are state, input and output vectors, respectively. The set  $\Omega$  is the polytope

$$\Omega = \text{CO}\{[A_1 \ B_1], [A_2 \ B_2], \dots, [A_L \ B_L]\} \quad (2)$$

where CO indicates the convex hull. In other words, if  $[A \ B] \in \Omega$  then we have:

$$[A \ B] = \sum_{i=1}^L \lambda_i [A_i \ B_i] \quad (3)$$

**2. 2. 2. Model Description for Uncertain Systems with Unknown but Bounded Delays** The open loop state space model of the physical system which is illustrated in Figure 1 with Modbus network and network interfaces is as follow:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t - \tau) \\ y(t) = Cx(t), \end{cases} \quad (4)$$

where  $x \in R^n$ ,  $u \in R^m$ , and  $y \in R^r$  are state, input and output vectors, respectively; and A and B are uncertain matrices. The parameter  $\tau \in [\underline{\tau}, \bar{\tau}]$  represents delay with constant lower and upper bounds. The sampling period of the signal acquisition is  $t_{PSA}$  in which  $t_{PSA} \leq T/10$ ;  $T$  is the lowest time constant of the plant model.

Based on the uncertainty polytope, the discrete time model of Equation (1) can be obtained as follow [17]:

$$x_{k+1}^a = A_k^\alpha x_k^a + B_k^\alpha \Delta u_k, \quad (5)$$

where  $x_k^a$  is the state vector with definition as follow:

$$x_k^a = [x_k, x_{k-1}, \Delta u_{k-\bar{d}}, \dots, \Delta u_{k-\underline{d}}, \dots, \Delta u_{k-1}] \quad (6)$$

and  $[A_k^\alpha, B_k^\alpha] \in \Omega$

where  $\Omega$  is a prespecified set. For polytopic systems, the set  $\Omega$  is a polytope. With combination of polytopic sets in [17], [25], when both  $\tau$  and the plant matrices  $A, B$  are uncertain, the set  $\Omega$  can be obtained as:

$$\Omega = \left\{ (A_k^\alpha, B_k^\alpha) \mid (A_k^\alpha, B_k^\alpha) = \sum_{j=1}^L \sum_{i=\underline{d}}^{\bar{d}} \lambda_{ij} (A_{ij}^\alpha, B_{ij}^\alpha) \right\} \quad (7)$$

$$\sum_{j=1}^L \sum_{i=\underline{d}}^{\bar{d}} \lambda_{ij} = 1; \lambda_{ij} \geq 0 \quad (8)$$

$$\forall i \in [\underline{d}, \dots, \bar{d} - 1] \text{ and } \forall j \in [1, \dots, L]$$

where

$$A_{ij}^\alpha = \begin{bmatrix} I + \bar{A}_j, -\bar{A}_j, 0_{n \times (\bar{d}-i)}, \bar{B}_j, 0_{n \times (\bar{d}-1)} \\ I_{n \times n}, 0_{n \times (\bar{d}+n)} \\ 0_{(\bar{d}-1) \times (2n+1)}, I_{(\bar{d}-1) \times (\bar{d}-1)} \\ 0_{1 \times (2n+\bar{d})} \end{bmatrix} \quad (9)$$

$$B_{ij}^\alpha = \begin{bmatrix} 0_{(2n+\bar{d}-1) \times 1} \\ 1 \end{bmatrix} \quad (10)$$

$$i = \underline{d}, \dots, \bar{d}, j = 1, \dots, L$$

A and B are functions of uncertain parameter  $\beta$ , where  $\beta_{min} \leq \beta \leq \beta_{max}$ ,  $A_{ij}^\alpha$  and  $B_{ij}^\alpha$  are the convex coordinates which define a precisely known system inside the polytope  $\Omega$  described by the convex combination of its  $(\bar{d} - \underline{d} + 1) \times L$  vertices. Therefore, the continuous system (4) with lower and upper limits (lie in uncertainty range  $\tau \in [\underline{\tau}, \bar{\tau}]$  and  $\beta_{min} \leq \beta \leq \beta_{max}$ ) can be discretized and represented by the uncertain sets of Equation (7). Symbols in Equations (9), (10) are defined in Table 3.

**2. 2. 3. Designing a Robust MPC for an NCS with Uncertainties in Delay and model** The normal MPC is unable to explicitly deal with model uncertainty [25]. In robust model predictive control, the online constrained minimization problem is modified to a min-max problem (minimizing the worst case, where the worst case is taken over the set of uncertain models in uncertainty polytope). The RMPC method proposed in [18] guarantees the stability of systems with uncertain delay but certain model. We call this method as Normal RMPC (NRMPC) in this paper. In this section, the RMPC method is improved based on the model of Equation (5) to compensate systems with both model and delay uncertainties. This improved RMPC (IRMPC) guarantees the stability for systems with both unknown delay and model uncertainty. In IRMPC algorithm, the state feedback controller of Equation (11) minimizes the upper bound of the objective function in Equation (12) at each time instant  $k$ .

**TABLE 2.** Definitions of the symbols in (9), (10)

Symbols	Details
$\bar{A}_j$	$\bar{A}_j = e^{A_j \times t_{PSA}}$
$\bar{B}_j$	$\bar{B}_j = \int_0^{t_{PSA}} e^{A_j \times s} B_j ds$
$t_{PSA}$	$t_{PSA} = t_k - t_{k-1}$ , the time in seconds between two consecutive samples
$k$	Time index
$\underline{d}$	$\underline{d} = fl(\underline{\tau}/t_{PSA})$ ,
$\bar{d}$	$\bar{d} = cl(\bar{\tau}/t_{PSA})$ ,
$fl(\cdot)$	A function in which the nearest integer less than $(\cdot)$ is selected
$cl(\cdot)$	A function where the nearest integer larger than $(\cdot)$ is selected
$I_{n \times n}$	Identity Matrix with dimension $n \times n$
$0_{n \times m}$	Zero Matrix with dimension $n \times m$
$L$	Maximum vertices in plant uncertainty

$$\Delta u_k = F_k x_k^a \tag{11}$$

$$J_k = \sum_{i=0}^{\infty} ((x_{k+i}^a)^T Q_1 (x_{k+i}^a) + (\Delta u_k)^T R \Delta u_k) \leq \gamma \tag{12}$$

where  $\Delta u_k = u_k - u_{k-1}$  and  $F_k$  is the state feedback gain,  $Q_1 \geq 0$  and  $R > 0$  are weighting matrices with proper dimensions, scalar  $\gamma$  is the upper bound of the objective function. The design procedure is as follow:

*step1:* Drive upper bound of  $\max J_k$ : Given quadratic function  $V(x_k^a) = (x_k^a)^T P_k x_k^a > 0, V(0) = 0$ . Suppose  $V(x_k)$  satisfies the following inequality (guaranteeing stability):

$$V(x_{k+i+1}^a) - V(x_{k+i}^a) \leq -((x_{k+i}^a)^T Q_1 (x_{k+i}^a) + (\Delta u_k)^T R \Delta u_k) \tag{13}$$

for  $x(\infty) = 0 \Rightarrow V(x(\infty)) = 0$ , summing Equation (13) from  $i = 0$  to  $i = \infty$  yields  $-V(x_k^a) \leq -J_k$

$$[A_{k+i} \max_{B_{k+i} \in \Omega, i \geq 0} J_k \leq V(x_k^a)]$$

Then, the upper bound of  $J_k$  is  $V(x_k^a)$ . With substituting the original optimization problem, the minimization of the objective function  $J_k$  is as follow:

$$\min_{u_{k+i}, i=0, 1, \dots} \left( \max_{[A_{k+i} \max_{B_{k+i} \in \Omega, i \geq 0} J_k \leq V(x_k^a)]} \right) \Rightarrow \min_{u_{k+i}, i=0, 1, \dots} V(x_k^a) \xleftrightarrow{\text{with } V(x_k^a) = (x_k^a)^T P_k x_k^a \leq \gamma} \min \gamma$$

the infinite horizon in (12) guarantees the asymptotic stability based on [25].

*Step2:* minimize the upper bound of  $\max J_k$  with a state-feedback control law using LMIs.

**Theorem 1.** For the system in Equation (5) with uncertainty in the form of Equations (7)-(8) at each time instant  $k$ , a state feedback gain in the control law (11) which is able to minimize the upper bound of objective function of Equation (12) can be obtained by  $F_k = Y_k Q_k^{-1}$  where  $Q_k > 0$  and  $Y_k$  is obtained from the solution of the following linear minimization problem:

$$\min \gamma \quad \text{s.t.} \quad \begin{bmatrix} 1 & (x_k^a)^T \\ x_k^a & Q_k \end{bmatrix} \geq 0 \tag{14}$$

$$\begin{bmatrix} Q_k & * & * & * \\ A_{ij}^a Q_k + B_{ij}^a Y_k & Q_k & * & * \\ Q_1^{\frac{1}{2}} Q_k & 0 & \gamma I & * \\ R^{\frac{1}{2}} Y_k & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \tag{15}$$

**Proof.** Defining  $Q_k = \gamma P_k^{-1} > 0$  and SCHUR complement:

$$\begin{bmatrix} Q(x) & S(x) \\ S(x)^T & R(x) \end{bmatrix} > 0 \Leftrightarrow R(x) > 0,$$

$$Q(x) - S(x)R(x)^{-1}S(x)^T > 0$$

$$\Rightarrow V(x_k^a) = (x_k^a)^T P_k x_k^a \leq \gamma \Leftrightarrow 1 - (x_k^a)^T \gamma^{-1} P_k x_k^a > 0$$

$$\Leftrightarrow 1 - (x_k^a)^T Q_k^{-1} x_k^a \Leftrightarrow \begin{bmatrix} 1 & (x_k^a)^T \\ x_k^a & Q_k \end{bmatrix} \geq 0$$

$$\min \gamma \quad \text{s.t.} \quad V(x_k^a) = (x_k^a)^T P_k x_k^a \leq \gamma$$

$$\Leftrightarrow \min \gamma \quad \text{s.t.} \quad \begin{bmatrix} 1 & (x_k^a)^T \\ x_k^a & Q_k \end{bmatrix} \geq 0$$

Substituting  $V(x_k^a) = (x_k^a)^T P_k x_k^a$  in Equation (13)

$$(x_{k+i+1}^a) = (x_{k+i}^a)^T P_k x_{k+i}^a \text{ and } \Delta u_{k+i} = F_k x_{k+i}^a$$

$$(x_{k+i}^a)^T H x_{k+i}^a \leq 0 ;$$

$$H = \begin{pmatrix} (A_{k+i}^a + B_{k+i}^a F_k)^T P_k (A_{k+i}^a + B_{k+i}^a F_k) \\ -P_k + F_k^T R F_k + Q_1 \end{pmatrix}$$

That is satisfied for all  $\geq 0$  , if

$$((A_{k+i}^a + B_{k+i}^a F_k)^T P_k (A_{k+i}^a + B_{k+i}^a F_k) - P_k + F_k^T R F_k + Q_1) \leq 0$$

Substituting  $P_k = \gamma Q_k^{-1}, Q_k > 0$  and  $Y_k = F_k Q_k$ , then pre and post multiplying by  $Q_k$

$$\begin{bmatrix} Q_k & * & * & * \\ A_{k+i}^a Q_k + B_{k+i}^a Y_k & Q_k & * & * \\ Q_1^{\frac{1}{2}} Q_k & 0 & \gamma I & * \\ R^{\frac{1}{2}} Y_k & 0 & 0 & \gamma I \end{bmatrix} \geq 0 \tag{16}$$

The inequality Equation (16) is satisfied for all  $[A_{k+i}^a \ B_{k+i}^a]$ , hence it is satisfied for all

$$[A_k^a \ B_k^a] \in \Omega = \{(A_k^a, B_k^a) | (A_k^a, B_k^a) =$$

$$\sum_{j=1}^L \sum_{i=\underline{d}}^{\bar{d}} \lambda_{ij} (A_{ij}^a, B_{ij}^a);$$

$$\sum_{j=1}^L \sum_{i=\underline{d}}^{\bar{d}} \lambda_{ij} = 1 ; \lambda_{ij} \geq 0\}$$

If and only if  $Q_k > 0, Y_k = F_k Q_k$  and  $\gamma$  such that:

$$\begin{bmatrix} Q_k & * & * & * \\ A_{ij}^a Q_k + B_{ij}^a Y_k & Q_k & * & * \\ Q_1^{\frac{1}{2}} Q_k & 0 & \gamma I & * \\ R^{\frac{1}{2}} Y_k & 0 & 0 & \gamma I \end{bmatrix} \geq 0$$

$$; i = \underline{d}, \dots, \bar{d} ; j = 1, \dots, L$$

The feedback matrix is then given by  $F_k = Y_k Q_k^{-1}$  □.

### 3. SIMULATION

In this section, the simulation is performed on the second-order approximation of cathodic protection system with unknown but bounded delay and uncertainty in the model. The upper and lower bounds of delays are

evaluated experimentally in this section. These upper and lower bounds are used for modelling the system. The simulations are performed for two scenarios: small delay and large delay.

### 3. 1 Second-order Cathodic Protection Model

The second-order approximation of the plant controlled through MODBUS is considered as follow:

$$G_0(s) = \frac{k}{s(s+\alpha)} e^{-\tau s}$$

This model obtained from identification procedure, where step input is applied to the system and second order approximation is obtained from system identification toolbox in MATLAB. For simulation, we assume two scenarios: in scenario I (small delay):  $\tau \in [0.15 \ 0.35]$ , and in scenario II (large delay):  $\tau \in [1.4 \ 1.75]$ .

### 3. 2. Simulation Results

The main parameters of scenario I are shown in Table 4.

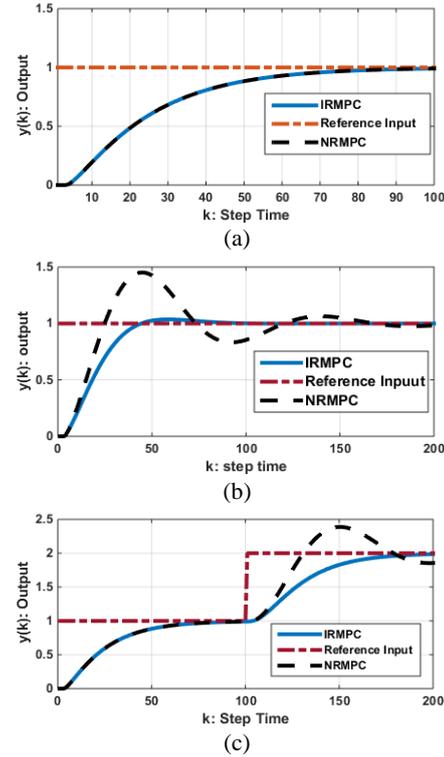
Figure 4 (a, b, c) illustrates the output signals of two methods (NRMPC and IRMPC) for scenario I. Figure 4 (a) shows the simulation result when the model of the system is certain but, the delay is uncertain. Figure 4.b shows the simulation result when both the model and delay of the system are uncertain. One can see that the performance of the proposed improved RMPC is much better than the Normal RMPC. However, the performance gets worse for NRMPC when both the model and delay are uncertain, but it remains stable for small delay. Figure 4 (c) contains two parts. The first part is like Figure 4 (a), and the second part is like Figure 4.b, but the input reference is changed from one to two at step time  $k=100$ .

In order to investigate the effects of larger time delays on the certain/uncertain system controlled by NRMPC and IRMPC, the simulation is performed for scenario II. The main parameters of scenario II are shown in Table 5.

Figure 5 (a, b, c) illustrates the step responses of two methods for scenario II. Figure 5 (a) shows the simulation results when the model of the system is certain but, the delay is uncertain. It is obvious that the performance of the two methods are the same. However, as shown in Figure 5 (b), for a large delay, when both the model and delay are uncertain, the system is unstable using NRMPC. Figure 5 (c) shows the simulation result for the proposed IRMPC method when both the model and delay of the system are uncertain. Although the erformance gets worse for a large delay, the proposed p IRMPC

**TABLE 4.** The main parameters used in the simulation for scenario I

Scenarios I	Parameter ( $\underline{d}$ , $\bar{d}$ )	Sample time (h)	(Q, R)	k	$\alpha$
$\tau \in [0.15 \ 0.35]$	(2, 4)	100 ms	(1, 0.5)	0.9	[0.1 1]



**Figure 4.** Comparison of step responses in scenario I for two control algorithms NRMPC and the proposed IRMPC: a) certain model but uncertain delay, b) uncertainty in both model and delay, c) a combination of a and b in addition to step change at 100

**TABLE 5.** The main parameters used in the simulation for scenario II

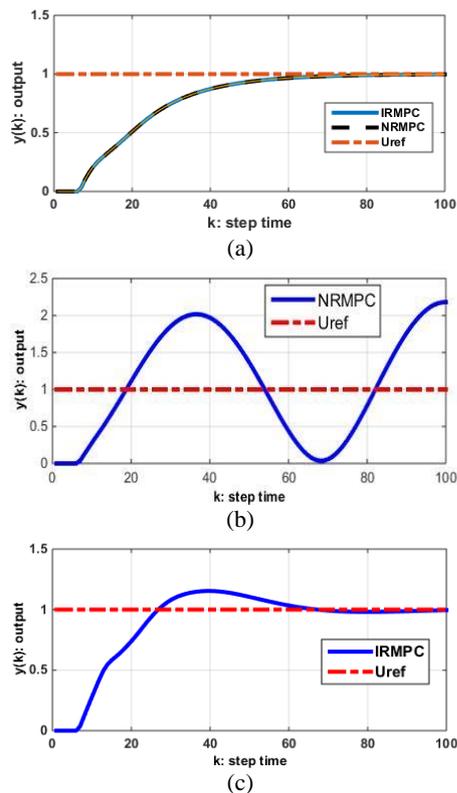
Scenarios I	Parameter ( $\underline{d}$ , $\bar{d}$ )	Sample time (h)	(Q, R)	k	$\alpha$
$\tau \in [1.4 \ 1.75]$	(7, 9)	200ms	(1, 0.5)	0.9	[0.1 1]

guarantees the stability, and the output tracks the command without error.

It can be seen in Figure 4 (b) and Figure 4 (c) that for model uncertainty, the performance degrades using normal RMPC comparing to using IRMPC for small delay (scenario I). For larger values of delay (scenario II) together with model uncertainty, the controlled system using NRMPC may not be stable (Figure 5 (b)) and the performance of the controlled system degrades using IRMPC method but the system is still stable (Figure 5 (c)). For comparison of the two control methods, the following index is selected [18]:

$$E_{perf} = \left(\frac{1}{n}\right) \sqrt{\sum_{k=1}^n (|xref_k - y_k|^2 + u_k^2)}, \quad (17)$$

where n is the last step of the simulation (100 in this case), Equation (17) states the overall performance on the



**Figure 5.** Comparison of step responses in scenario II for two control algorithms, NRMPC and the proposed IRMPC: a) compensation of the two methods for certain model but uncertain delay, b) NRMPC method for both as uncertain model, and uncertain delay, c) IRMPC method for both uncertain model and uncertain delay

simulation interval. In both cases, the reference signal is unit step. Table 6.gives the performance index of the two control methods.

Table 6 shows that the transient and steady state performance of IRMPC is significantly better than NRMPC.

#### 4. CONCLUSION

Industrial networks such as Modbus introduce time delays in the control loops. When the network is a part of a control loop, the network-induced delay effects on

**TABLE 6.** The performance index of the two control methods

	NRMPC Scenario I	NRMPC Scenario II	IRMPC Scenario I	IRMPC Scenario II
$E_{perf}$	0.1971	0.3396	0.1083	0.1922
$t_{settling\ time}(s)$	15.5	unstable	4	12
Overshoot %	45	unstable	3.6	15.4

system's characteristics such as stability and performance. In real time networked control systems, due to the asynchrony between the devices and network characteristics, time delays are unknown but bounded. In this paper, the RMPC algorithm was improved for systems with model and delay uncertainties. To this end, a discrete model description of the continuous system is extended using a modified description of uncertainty polytope. The superiorities of this method are to extend model description and to consider both the delay and model uncertainty in RMPC design. The robust stability is guaranteed in this method. It was demonstrated that the proposed improved RMPC algorithm is a combination of two methods. Each of these methods used for compensation of only unknown delays or just for uncertain models. In this paper, a control loop based on Modbus network was investigated in which the architecture is prevalence in automation systems. This control loop was analysed for evaluation of delays based on previous aspects of Fieldbus control loops. Simulation results demonstrated that the proposed improved RMPC is more effective than normal RMPC when both the model and delay are uncertain. It was seen that when the uncertainty is only in delay, the results of two methods are similar. The two algorithms differ when both the model and delay are uncertain.

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### Persian Abstract

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#### چکیده

سیستم‌های کنترلی‌ای که در آن ارتباط بین سنسورها، عملگرها و کنترلرها از طریق ارتباطات دیجیتالی برقرار می‌شود، سیستم‌های کنترل تحت شبکه (NCSS) نامیده می‌شوند. در حالت کلی NCSSها با مسائل متعددی نظیر از دست دادن بسته‌های داده و تاخیرهای زمانی ناشی از شبکه مواجه می‌شوند. برطرف کردن این مشکلات، جزو موضوعات مورد توجه در تئوری کنترل می‌باشند. هنگامی که عدم قطعیت در مدل سیستم نیز به موارد فوق‌الذکر اضافه گردد، طراحی یک کنترل کننده مقاوم که پایداری سیستم را تضمین نماید، کار بسیار پیچیده‌ای می‌شود. در این مقاله روشی مبتنی بر کنترل مقاوم پیش بین مدل (RMPC) به منظور غلبه بر عدم قطعیت مدل همراه با تاخیر زمانی نامعین در سیستم‌های کنترل تحت شبکه ارائه می‌شود. روش‌های قبلی RMPC که در این مقاله تحت عنوان RMPC مرسوم (NRMPC) شناخته می‌شوند، تنها برای جبران‌سازی تاخیر زمانی یا عدم قطعیت مدل به کار گرفته می‌شدند. در اینجا ما روشی به نام کنترل مقاوم پیش بین مدل بهبود یافته (IRMP) ارائه می‌کنیم، و در آن هر دو اثر تاخیرهای زمانی نامعین و عدم قطعیت در مدل سیستم جبران و لحاظ می‌گردند. روش ارائه شده مبتنی بر چندضلعی عدم قطعیت و نامساویهای ماتریسی خطی است. همچنین یک ارزیابی تجربی از تاخیرهای زمانی در سیستم‌های کنترل تحت شبکه مدباس ارائه می‌گردد. نتایج شبیه سازی توانایی روش ارائه شده را نشان می‌دهد. همچنین روش مرسوم NRMPC نیز اعمال و با روش IRMP مقایسه می‌گردد. نتایج شبیه سازی نشان می‌دهد که اگرچه زمانی که سیستم تنها شامل تاخیرهای زمانی نامعین می‌باشد، روش NRMPC عملکرد خوبی را نشان می‌دهد اما این عملکرد زمانی که عدم قطعیت مدل نیز وارد می‌شود، نه تنها تضعیف بلکه حتی منجر به ناپایداری سیستم می‌شود.

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