Dynamic Meshing Characteristics of Elliptic Cylinder Gear Based on Tooth Contact Analysis

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1. INTRODUCTION

Generally, when the pitch curve is circular, it is called cylinder gear, while when its non-circular is called non-circular cylinder gear. Non-cylinder gear includes non-circular cylinder gear, non-bevel gear, non-circular face gear and so on. As one of the simplest noncircular cylinder gears, elliptic cylinder gears are widely used in automatic machinery, printers, fans, packers, hydraulic pumps, hydraulic motors and flow meters because of their compact structure and variable-ratio transmission. In recent years, tooth contact analysis (TCA) technology for tooth contact analysis has developed rapidly in the field of gear, while the traditional TCA technology only considers the normal meshing condition of gear pair under theoretical contact condition, and does not think about the influence of load on gear meshing. In view of this situation, loaded tooth contact analysis (LTCA) technology has been widely used, which is a bridge connecting geometric design and mechanical analysis in the field of gear research. This method mainly considers the change of load in the process of gear meshing, which is more in line with the actual working conditions of gears [1].

Every tooth on the elliptical cylinder gear is different, but each tooth can be regarded as a tooth on the equivalent cylinder gear, so the contact analysis method of the cylinder gear can be used to analyze the elliptic cylinder gear. At present, a good quantity of research results have been accumulated in the research of tooth surface contact. Among them, Cao [2] took the spiral bevel gear as an example, and proposed a new method of tooth contact analysis for the problem that the mathematical model of tooth surface contact and edge contact is not uniform at present. He and Yan [3] obtained the tooth surface contact trajectory, the area and the shape of the contact area when the face gear meshed with the spur gear, and the results show that the
transmission ratio and manufacturing precision have a certain influence on the transmission performance of the face gear. Yan [4] studied the tooth surface contact stress and distribution of point contact surface gears, pointed out that surface roughness has a certain influence on the size and distribution of tooth surface contact stress. For more complex planetary gears, Mo [5-6] studied the dynamic load sharing characteristics and dynamic meshing characteristics by simulating gear meshing, which provided a new idea for subsequent planetary gear research. Sanchez [7] proposed a new method of tooth surface contact analysis, which discretizes the tooth contact surface and geometrically adaptive refinement to solve the contact problem and calculate the instantaneous contact area of the gear during the meshing process. Wang [8] proposed a calculation method of tooth profile modification based on tooth surface contact analysis technology, in which the modified parameters of the rack tool obtained by TCA technology can be transformed into the shape modification parameters of tooth profile. Chen [9] established a gear transmission dynamics model, considering the contact relationship of the tooth surface, to study the influence of the meshing phase and operating conditions of the gear on the contact characteristics and dynamic characteristics. Then, many studies focused on ANSYS LS-DYNA analysis software to obtain the meshing characteristics and contact characteristics of gears [10-11].

Based on the above-mentioned research, the contact characteristics of non-circular gears have been studied by many scholars. Among them, Marius [1] proposed the non-circular gear pitch curve and the tooth profile generation method, simulated the tooth meshing in the 2D and 3D environments, and elaborated the meshing path and the size of the contact area and its changes. Based on the predetermined kinematics, Cristescu [12] designed the pitch curve of multi-stage gears and applied finite element analysis to the gear solid model as a criterion for further optimization of multi-stage gear design. In reference [13], the dynamic meshing characteristics of elliptic cylinder gears under different load conditions are obtained through simulation analysis.

The above-mentioned researches have important significance for analyzing the meshing characteristics of non-circular cylinder gears. However, there are few researches on the tooth contact analysis of the non-circular cylinder gears in the dynamic meshing process. Therefore, the article takes a pair of elliptic cylinder gear pairs in the reversing device of the new drum type pumping unit as the research object, and its precise finite element analysis model is established. Based on LS-PREPOST software and LTCA technology, the dynamic meshing process of elliptic cylinder gears is simulated, and the distribution law of stress and strain during the meshing process is studied. Figure 1 shows the elliptic cylinder gear reversing device model.

2 ELLIPTIC CYLINDER GEAR MESHING THEORY AND FINITE ELEMENT MODEL

2.1 Tooth Surface Model of Elliptic Cylinder Gear
The curvature radius of the pitch curve of elliptic cylinder gear is a variable, and each tooth profile is different. In order to analyze tooth contact characteristics, tooth surface model should be established. The pitch curve equation of elliptical cylindrical gear is:

\[ r = \frac{A(1 - e^2)}{1 - e \cos \varphi} \]  

(1)

The vector equation of the tooth profile is:

\[ r_i = r + an \]  

(2)

where A is the radius of the long axis of elliptic cylinder gear, e the eccentricity of elliptic cylinder gear, r the pitch curve radius of elliptic cylinder gear, \( \varphi \) the rotational angle of elliptic cylinder gear, \( r_p \) the radial path of any point n on the tooth profile, \( r_e \) the diameter of the pitch curve at the intersection point of the normal and pitch curve of the n-point on the tooth profile, \( an \) a vector whose direction is the same as the normal direction of the tooth profile and whose length is equal to the distance between the pitch curve and the tooth profile. The tooth profile of elliptic cylinder gears can be divided into two parts: the point higher than the pitch curve and the point lower than the pitch curve, and there are different methods for solving the two-part tooth profile equation.

a. For points above pitch curve profile, the angles between the vector \( an \) and the polar axis are \( \theta - \mu + \alpha_e \) (right profile angle) and \( \mu - \theta + \alpha_e \) (left profile angle), as shown in Figure 2.

Figure 1. Reversing device of planetary gear train with elliptic cylinder gears
The equation of the right tooth profile is:
\[
\begin{align*}
 x_r &= r_s \cos \alpha + a_n \cos(\alpha - \alpha_u) \\
 y_r &= r_s \sin \alpha + a_n \sin(\alpha - \alpha_u)
\end{align*}
\]  
(3)

The equation of the left tooth profile is:
\[
\begin{align*}
 x_l &= r_s \cos \alpha + a_n' \cos(\alpha - \alpha_u) \\
 y_l &= r_s \sin \alpha + a_n' \sin(\alpha - \alpha_u)
\end{align*}
\]  
(4)

b. For points on the tooth profile below the pitch curve, the angles between the vector \(a_n\) and the polar axis are \(\alpha - \alpha_u\) (right profile angle) and \(\mu - \alpha_u\) (left profile angle), as shown in Figure 3.

The equation of the right tooth profile is:
\[
\begin{align*}
 x_r &= r_s \cos \alpha + a_n \cos(\alpha - \alpha_u) \\
 y_r &= r_s \sin \alpha + a_n \sin(\alpha - \alpha_u)
\end{align*}
\]  
(5)

The equation of the left tooth profile is:
\[
\begin{align*}
 x_l &= r_s \cos \alpha + a_n' \cos(\alpha - \alpha_u) \\
 y_l &= r_s \sin \alpha + a_n' \sin(\alpha - \alpha_u)
\end{align*}
\]  
(6)

According to formulas (3 to 6), the three-dimensional tooth surface equation of elliptic cylinder gear can be obtained, in which the right tooth surface equation of elliptic cylinder gear is:
\[
\begin{align*}
 x_r &= r_s \cos \theta \pm a_n \cos(\theta - \alpha_u) \\
 y_r &= r_s \sin \theta \pm a_n \sin(\theta - \alpha_u)
\end{align*}
\]  
(7)

The left tooth surface equation of elliptic cylinder gear is:
\[
\begin{align*}
 x_l &= r_s \cos \theta + a_n' \cos(\mu - \theta - \alpha_u) \\
 y_l &= r_s \sin \theta + a_n' \sin(\mu - \theta - \alpha_u)
\end{align*}
\]  
(8)

where \( \pm a_n \) refers to the direction of the tooth line, and is equal to the width of the tooth.

2. 2. Meshing Theory of Elliptic Cylinder Gear

Although the radius of pitch curve of the elliptic cylinder gear changes constantly, in the actual meshing process, one or two pairs of gears are mainly engaged, i.e. the contact between the involute profile and the involute profile. The calculation of the tooth contact stress is consistent with the contact stress of the involute cylinder gear. According to the formula of Hertz theory, the contact stress of the two contact tooth surfaces is: [13]
\[
\sigma_c = \frac{P_{ca}}{\mu \rho \frac{1}{E_1} + 1 - \mu_2^2} \sum \rho
\]  
(9)

where \( P_{ca} \) is the calculated load per unit length, \( B \) representing the tooth width and \( Pn \) the tooth surface normal force, \( E_1, E_2 \) and \( \mu_1, \mu_2 \) the elastic modulus and Poisson's ratio of the two gears that are in contact with each other and \( \sum \rho \) the combined radius of curvature at the two contact faces.

A diagram of the force of a pair of inter-meshing elliptic cylinder gear pairs in the drive wheel is shown in Figure 4. The force \( F_t \) of the non-circular involute spur gear in the tangential direction of the pitch curve and the normal force \( F_n \) of the tooth surface of the gear tooth are:

\[ F_t = \frac{P_{ca}}{B} \]  
(10)

where \( P_{ca} \) is the calculated load per unit length, \( B \) representing the tooth width and \( Fn \) the tooth surface normal force, \( E1, E2, \mu1, \mu2 \) the elastic modulus and Poisson's ratio of the two gears that are in contact with each other and \( \sum \rho \) the combined radius of curvature at the two contact faces.

A diagram of the force of a pair of inter-meshing elliptic cylinder gear pairs in the drive wheel is shown in Figure 4. The force \( F_t \) of the non-circular involute spur gear in the tangential direction of the pitch curve and the normal force \( F_n \) of the tooth surface of the gear tooth are:
where \( a \) is the center distance of the elliptic cylindrical gear, \( r_1 \), \( r_2 \) the pitch curve radius of the driving wheel and driven wheel, \( T_1(t), T_2(t) \) input torque and output torque.

A pair of tooth profiles are meshed at the pitch curve of a non-circular involute spur gear. The meshing force is large. When the gear materials are the same, the nominal value of contact stress and the calculated value of contact stress are respectively:

\[
\sigma_{n0} = \frac{F_E}{2 \pi b (1 - \mu)} \left( \frac{1}{\rho_a} + 1 \right)
\]

\[
\sigma_m = \sigma_{n0} \sqrt{K_A K_h K_{\mu} K_{ha}}
\]

where \( \sigma_{n0} \) is the nominal value of contact stress, \( \sigma_m \) the calculated value of contact stress, \( K_h \) the meshing stiffness coefficient, \( K_A \) the usage coefficient, \( K_v \) dynamic load coefficient, \( K_{\mu} \) tooth load distribution coefficient for contact stiffness calculation and \( K_{ha} \) representing the load distribution coefficient between teeth calculated by contact stiffness.

The elliptic cylinder gear is complicated and time-varying during the meshing process. The above formulas can calculate the force during the tooth meshing process, but some of the coefficients need to be selected empirically, and the error is large. While the software such as LS-DYNA and LS-PREPOST can fully simulate the actual meshing process of the gear and implement the loaded tooth contact analysis (LTCA). The elliptic cylinder gear parameters studied in the paper are shown in Table 1.

The mesh element size needs to be considered when meshing the elliptic cylinder gear model with Hypermesh software. Since the tooth meshing process is mainly analyzed, the tooth and the middle part should be set separately to reduce the analysis time. The finite element meshing model of the elliptic cylinder gear is shown in Figure 5.

### TABLE 1. Elliptic cylinder gear design parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Module ( m/(\text{mm}) )</td>
<td>3</td>
</tr>
<tr>
<td>Number of teeth ( Z )</td>
<td>47</td>
</tr>
<tr>
<td>Center distance ( d/(\text{mm}) )</td>
<td>145</td>
</tr>
<tr>
<td>Addendum coefficient ( h^* )</td>
<td>1</td>
</tr>
<tr>
<td>Top clearance coefficient ( C^* )</td>
<td>0.25</td>
</tr>
<tr>
<td>Tooth width ( B/(\text{mm}) )</td>
<td>30</td>
</tr>
<tr>
<td>Eccentricity ( e )</td>
<td>0.3287</td>
</tr>
<tr>
<td>Pressure angle(^(*) )</td>
<td>20</td>
</tr>
</tbody>
</table>

\[
\text{Pitch curve equation } r = \frac{64.667}{1 \pm 0.3287 \cos \varphi}
\]
avoid the negative volume. The debug time step scale factor TSSFAC is taken the value of 0.5, the time step DT2MS values $-2 \times 10^{-7}$ can complete the analog tooth engagement. After gear meshing, the number of driver wheel nodes is 205716, the number of units 210211, the number of driven wheel nodes 181740, and the number of units 186180.

The variation of load applied to driven gear is shown in Figure 6. After setting the above parameters, the model is solved to obtain the load step, the effective plastic strain, the effective stress and the surface pressure of the tooth line direction and the tooth profile direction respectively of the driven wheel during the tooth meshing process, and the meshing force of the teeth under different speed conditions.

3. 1. Tooth Meshing Load Step

The meshing simulation of gear is carried out for 0.1s, and six time points are randomly selected to observe the change of stress load step of gear, as shown in Figure 5. During the meshing simulation, the loads on the gear vary with time. The maximum loads on the gear in Figure 7 are 1.104 MPa, 0.9724 MPa, 0.9005 MPa, 0.7403 MPa, 0.6217 MPa and 0.6233 MPa, respectively. It can be concluded that the contact area of the tooth surface is elliptical, which has the same shape as the contact area of the spur gear, and the maximum load on the teeth occurs at the middle section of the gear. During the gradual transition from the middle section to the ends of teeth, the load is continuously reduced, and the elliptical contact area changes during the tooth meshing process, which is generally symmetrically distributed at the middle section of the gear. When the thickness of the two teeth of the intermeshing is the same, the distance between the elliptical contact area and the end surface of the gear is about 5%~10% of the thickness of the tooth. If the tooth widths of the two meshing teeth are inconsistent, the gear with a smaller tooth width has a larger elliptical contact area.
3.2. Stress and Strain Analysis of Tooth Line Direction
Tooth profile is generally composed of the top part, the root part and the working area [14]. The elliptic cylinder gear generally has a working area near the pitch curve. In order to study the stress and strain distribution law of the tooth line direction, it is necessary to perform equidistant data acquisition on the working area near the tooth line direction curve. The position of the data collection point and the number of the gear are shown in Figure 8. By collecting the data of the tooth surface, the effective plastic strain, effective stress and surface pressure of the working area of No. 1 tooth and No. 24 tooth are obtained. The specific change trend is shown in Figure 9. Because there is a certain collision between the teeth during the meshing, there are shocks on the effective stress and surface pressure curves in the figure. Among them, the effective plastic strain of the point C on the No. 1 tooth and the No. 24 tooth are the largest, followed by points B and D, and the smallest are A and E. The effective stress and the surface pressure of the teeth also exhibit the same distribution law, which means that the effective plastic strain, effective stress and surface pressure of the center position of the elliptical contact area of the tooth surface are the largest. When transitioning from the center position to the two sides, the above three are reduced to varying degrees. The reason is that the power is transmitted through the working area of the surface of the driving and driven wheel teeth during the tooth meshing process. In the process of gear meshing, besides the sliding gear, the part of the middle section and the pitch surface in the elliptical contact area (point C) will mesh at any time, which will wear more than other parts.

3.3. Stress and Strain Analysis of Tooth Profile Direction
Due to the constant change of the radius of the elliptic cylinder gear, the tooth profiles are different. In order to study the variation of the effective plastic strain, effective stress and tooth surface pressure in the tooth root to the tooth tip range, the meshing data of the No. 1 tooth, No. 12 tooth and No. 24 tooth were collected separately, as shown in Figure 10. The root of the No. 1 tooth has the largest effective plastic strain, followed by the pitch curve, and the deformation of the tooth tip position is the smallest. The distribution law of

![Figure 8. Tooth surface data collection point and tooth number](image)

![Figure 9. Distribution of stress and strain along tooth line](image)
effective stress is that the root position is the largest, then the tooth tip, and the pitch curve is the smallest. The surface pressure distribution is the largest near root and the pressure near the tooth tip and the pitch curve are basically the same. The effective plastic strain near the root of the 12th tooth is the largest, followed by the top of the tooth, and the root is the smallest. The effective stress gradually decreases from the top to the root, while the pressure distribution on the surface of the tooth is the largest near the pitch curve, followed by the top of the tooth and the smallest near the root. The effective plastic strain of the root of the No. 24 tooth is the smallest, and the position of the tooth tip is the largest. The effective stress decreases gradually from top to root, and the pressure of tooth surface is the largest near the pitch curve, and then followed by the top and the root is the smallest. From the above analysis, it is known that during the tooth meshing process, the effective plastic strain, effective stress and surface pressure between different teeth are alternately changed.

Taking the speed of 600r/min as an example to further analyze the stress and strain of the elliptic cylinder gear tooth surface during the meshing process, the effective plastic strain, effective stress and surface pressure of the tooth top, the pitch curve and the root of the No. 1 tooth, No. 12 tooth and No. 24 tooth were analyzed, and the specific changes are shown in Figure 11. It can be seen from the figure that the effective plastic strain, effective stress and surface pressure of the No. 12 tooth are the largest. No. 12 tooth is located at the
intersection of the circular curve and the elliptical curve, and it is necessary to transition from the tooth profile on the elliptical curve to the tooth profile on the circular curve during operation. The meshing between the teeth is not smooth as before and it will produce certain impact, vibration and even noise. Therefore, all the teeth stresses and strains on the same elliptical curve will appear to increase first and then decrease. The stresses and strains of gears at both ends of the long axis and its vicinity on the elliptical curve are less than the teeth at both ends of the short axis and its vicinity.

3.4. Variation of Tooth Resultant Force at Different Speeds

The variation trend of the tooth resultant force with the rotation speed is shown in Figure 12. The maximum resultant force of the gear under three rotation speeds is 225608N, 223515N and 226300N, respectively, and the difference between the three is small. At the speed of 300r/min, the resultant force curve is smoother, while the speed increases to 900r/min, the resultant force curve has a certain impact. When the speed is 300r/min, 600r/min and 900r/min, the driven wheels are rotated by 0.5r, 1r and 1.5r, respectively. In the meshing process of 0.1s, the meshing speed curve is smooth at low speed. When the meshing time decreases, instantaneous impact vibration increases at high speed, resulting in non-smooth phenomena.

![Figure 11. Comparison of stress and strain of different teeth](image)
4. CONCLUSIONS

This paper presents an analysis method of the dynamic meshing characteristics of the elliptical cylindrical gear based on LTCA, and the effective plastic strain, effective stress, surface pressure are obtained respectively. (a) Along the tooth line direction of the elliptic cylinder gear, the effective plastic strain, effective stress and surface pressure of the center position of the elliptical contact area on the tooth surface are the largest. In the transition from the center to both sides, it decreases in varying degrees. (b) In the direction of the tooth profile, the effective plastic strain, effective stress and surface pressure between different teeth are alternately changed. And the stress and strain near the long axis of the elliptic pitch curves are smaller than those near the short axis. (c) The meshing force of elliptic cylinder gears will not change obviously with the increase of rotational speed.

5. ACKNOWLEDGEMENTS

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6. REFERENCES

چکیده

به عنوان مهم‌ترین محل کار چرخ‌دندان‌ها، دندان‌ها نقش انتقال بار و توان را ایفا می‌کنند. خط دندان‌ها و مشخصات دندان‌ها سطح دندان‌ها هستند که بر شکل سطح دندان‌ها، صفات دندان‌ها و موضع وکنش در جهت خط دندان‌ها و مشخصات دندان‌ها تأثیر می‌کنند. با استفاده از روش‌های مختلف نرم‌افزار همچون LS-PREPOST، نتایج تحقیق نشان می‌دهد که با تغییر موضع وکنش در جهت خط دندان‌ها و مشخصات دندان‌ها، می‌توان بهبود پویایی وکنش‌های مشکل‌زا را در سطح دندان‌ها داشت. نتایج تحقیق می‌تواند به این نتیجه برسد که در تغییرات وکنش‌های مشکل‌زا در سطح دندان‌ها، می‌توان بهبود پویایی وکنش‌های مشکل‌زا را در سطح دندان‌ها داشت.