Space Vector Pulse Width Modulation with Reduced Common Mode Voltage and Current Losses for Six-Phase Induction Motor Drive with Three-Level Inverter

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Abstract

Common-mode voltage (CMV) generated by the inverter causes motor bearing failures in multiphase drives. On the other hand, presence of undesired z-component currents in six-phase induction machine (SPIM) leads to extra current losses and have to be considered in pulse width modulation (PWM) techniques. In this paper, it is shown that the presence of z-component currents and CMV in six phase drive system are two major limiting factors in space vector selection. The calculated voltage space vectors for both symmetrical and asymmetrical SPIM drive system with three-level inverter are illustrated in the decoupled subspaces and described in terms of undesirable voltage components and CMV value. Several space vector pulse width modulation (SVPWM) techniques are investigated based on CMV and z-component currents generation. Then, a modified SVPWM technique with minimum current distortion, undesired current components and CMV with a modest torque ripple is proposed based on the simulation results.


NOMENCLATURE

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_s, V_r$</td>
<td>Stator and rotor voltage vectors</td>
</tr>
<tr>
<td>$R_s, R_r$</td>
<td>Stator and rotor resistance matrices</td>
</tr>
<tr>
<td>$I_s, I_r$</td>
<td>Stator and rotor current vectors</td>
</tr>
<tr>
<td>$L_s, L_r$</td>
<td>Stator and rotor inductance matrices</td>
</tr>
<tr>
<td>$L_{ds}, L_{dr}$</td>
<td>Stator and rotor leakage inductances</td>
</tr>
<tr>
<td>$T_e$</td>
<td>Electromagnetic Torque</td>
</tr>
<tr>
<td>$\omega_r$</td>
<td>Rotor angular speed</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Phase difference between two three phase windings</td>
</tr>
<tr>
<td>$p$</td>
<td>Induction machine Pole number</td>
</tr>
<tr>
<td>$p$</td>
<td>Derivative operator</td>
</tr>
<tr>
<td>$V_{dc}$</td>
<td>DC link voltage</td>
</tr>
<tr>
<td>$V_{cm}$</td>
<td>Common mode voltage</td>
</tr>
</tbody>
</table>

Subscripts

- $a, b, c, d, e, f$: Stator phase name
- $\alpha$ and $\beta$: Alpha and beta axis
- $z_1, z_2, z_3$ and $z_4$: z-components axis

1. INTRODUCTION

Multiphase induction motor drives have been widely explored in recent years because of their advantages compared with traditional three phase systems such as higher torque density, lower torque pulsation and fault tolerance ability [1]. They are being considered as a proper alternative in marine electric propulsion [2], electric or hybrid vehicles, and high power applications [3].

The six-phase induction machine (SPIM) is one of the most popular configurations, and is divided in two types...
of symmetrical and asymmetrical with 60 and 30 electrical degrees spatially phase shifted between two sets of three-phase windings.

Presence of z-component currents in SPIM due to PWM voltage generated by the inverter produces harmonic currents that do not contribute to the air gap flux and generate additional losses. This can reduce the system efficiency and increase size and cost of the machine drive system [4].

Many researches have been done to reduce harmonic currents by applying proper PWM techniques. In these PWM methods, the voltage space vectors are selected based on minimum undesirable z-component voltage production and as a result, a few number of vectors can be selected despite the large number of space vectors in compared with three phase drives [5-9].

Common-mode voltage (CMV) is one of the problems associated with variable speed drive. The high frequency, high amplitude and large step size (dv/dt) of CMV produces bearing currents via parasitic capacitances which may lead to insulation and bearing failures [10], mechanical vibration [11] and also bearing undesirable electromagnetic interference (EMI) [12]. Several PWM techniques based on sinusoidal PWM [13-15] or SVPWM [16-20] strategies have been applied to reduce the CMV in multiphase systems. The researchers tried to reduce the CMV by phase shifting of carrier signal in control system of five-phase and six-phase (both symmetrical and asymmetrical) two-level voltage source inverters feeding R-L load, which leads to higher phase current harmonic and distortion [13]. In literature, [14-15] three career based PWM methods were analyzed in terms of CMV generation on a five phase coupled-inductor inverter with R-L load. Chen and Hsieh [16] proposed a pulse-width modulator with minimum CMV for generalized two-level N-phase voltage source inverters with odd phase numbers, and used vectors with minimum CMV. A multidimensional strategy was proposed in literature [17] to eliminate the CMV in generalized multiphase multilevel inverter by selecting switch states with zero CMV. But this method is suitable only for multilevel inverters with a level number higher than three. Two PWM techniques were proposed in literature [18, 19] for the linear and over-modulation regions that result in 40 and 80% CMV peak-to-peak reduction in five-phase inverter respectively. Dabour et al. [20] mitigated the CMV by classifying the space vectors based on their corresponding CMV level in a three to five-phase matrix converter and reduced the peak amplitude of CMV to 48%.

Although intensive researches have been done on modifying PWM methods to reduce the CMV, not enough work has been done on PWM methods with CMV reduction for six-phase drives. Moreover, most of the mentioned researches for reducing the CMV were done on multiphase inverter with an R-L, and the machine performance like electromagnetic torque ripple is not analyzed. Also, the presence of undesirable z-component currents in SPIM imposes a considerable limitation in PWM method (because lower number of space vectors can be applied) and it may lead to a challenge with CMV eliminating or reducing techniques, and is not considered in the above mentioned studies [13-16] and [20]. On the other hand, many researchers tried to reduce z-component currents in multi-phase drives without considering CMV production [5-9].

In our previous research [21], it is shown that z-component mitigation and CMV elimination problems cannot be achieved simultaneously in a single neutral SPIM drive system with two-level inverter despite having larger space vectors number than three-phase drive. Hence, it is tried to overcome the limitations in this paper by exploring space vectors in a three-level inverter. For this purpose, the calculated voltage space vectors in three-level inverter are categorized in decoupled subspaces and described in terms of undesirable voltage components and CMV value. The selected space vectors (based on the above limitations) is applied in several SVPWM strategies and a comprehensive comparison is performed including current distortion, CMV production and electromagnetic torque ripple. The SVPWM strategies analysis is performed on both symmetrical and asymmetrical SPIM configurations, and finally the best SVPWM strategy is proposed for both SPIM configurations based on theoretical analysis and simulation results. The CMV generated by the inverter has a considerable amplitude and high frequency waveform. Hence, removing it by the proposed method in higher frequency applications may be more useful but inverter switches with higher switching frequency must be applied.

2. MODEL OF SPIM

The circuit diagram of SPIM is shown in Figure 1. The voltage equations of stator and rotor are as follow:

\[
\begin{align*}
\begin{bmatrix} \vec{v}_s \end{bmatrix} &= \begin{bmatrix} [R_s & \begin{bmatrix} \vec{l}_s \end{bmatrix} \end{bmatrix} + \frac{d}{dt} (L_{ls} \begin{bmatrix} \vec{l}_s \end{bmatrix}) + \begin{bmatrix} L_{ss} \end{bmatrix} \begin{bmatrix} \vec{l}_s \end{bmatrix} \\
\begin{bmatrix} \vec{v}_r \end{bmatrix} &= \begin{bmatrix} [R_r & \begin{bmatrix} \vec{l}_r \end{bmatrix} \end{bmatrix} + \frac{d}{dt} (L_{lr} \begin{bmatrix} \vec{l}_r \end{bmatrix}) + \begin{bmatrix} L_{rr} \end{bmatrix} \begin{bmatrix} \vec{l}_r \end{bmatrix}\end{align*}
\] (1)

where the voltage and current vectors are:

\[
\begin{align*}
\begin{bmatrix} \vec{v}_s \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
\begin{bmatrix} \vec{v}_r \end{bmatrix} &= \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \\
\begin{bmatrix} \vec{l}_s \end{bmatrix} &= \begin{bmatrix} i_{sa} & i_{sb} & i_{sc} & i_{sa} & i_{sc} \end{bmatrix}^T \\
\begin{bmatrix} \vec{l}_r \end{bmatrix} &= \begin{bmatrix} i_{ra} & i_{rb} & i_{rc} & i_{rd} & i_{re} \end{bmatrix}^T
\end{align*}
\] (2)
As shown in [22], the SPIM model can be decomposed into three two-dimensional orthogonal subspaces, \((\alpha, \beta)\), \((z_1, z_2)\) and \((z_3, z_4)\), by transformation (3), where \(\gamma\) can be \(60^\circ\) or \(30^\circ\) electrical degrees for symmetrical and asymmetrical SPIM respectively.

\[
[p] = \frac{1}{\sqrt{3}} \begin{bmatrix}
1 & \cos(\gamma) & -1/2 & \cos(2/3 \gamma) & -1/2 & \cos(4/3 \gamma) \\
0 & \sin(\gamma) & \sqrt{3}/2 & \sin(2/3 \gamma) & -\sqrt{3}/2 & \sin(4/3 \gamma) \\
1 & \cos(\gamma - \gamma) & -1/2 & \cos(3/3 \gamma) & -1/2 & \cos(5/3 \gamma) \\
0 & \sin(\gamma - \gamma) & \sqrt{3}/2 & \sin(3/3 \gamma) & -\sqrt{3}/2 & \sin(5/3 \gamma) \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 
\end{bmatrix}
\]

By applying Equation (3) into the voltage Equation (1), a new decoupled system is obtained for SPIM. The stator and rotor voltage equations for \((\alpha, \beta)\), \((z_1, z_2)\) and \((z_3, z_4)\) subspaces are achieved as below:

\[
\begin{align*}
v_{\alpha} &= \begin{bmatrix} r_s + L_s p & 0 & M_p & 0 \end{bmatrix} i_{\alpha} \\
v_{\beta} &= \begin{bmatrix} 0 & r_s + L_s p & 0 & M_p \end{bmatrix} i_{\beta} \\
v_{d} &= \begin{bmatrix} 0 & \omega_M & r_s + L_s p & \omega_L \end{bmatrix} i_{d} \\
v_{q} &= \begin{bmatrix} -\omega_M & M_p & -\omega_M & r_s + L_s p \end{bmatrix} i_{q} \\
v_{z1} &= \begin{bmatrix} r_s + L_{q1} p & 0 \end{bmatrix} i_{z1} \\
v_{z2} &= \begin{bmatrix} 0 & r_s + L_{q2} p \end{bmatrix} i_{z2} \\
v_{z3} &= \begin{bmatrix} r_s + L_{q3} p & 0 \end{bmatrix} i_{z3} \\
v_{z4} &= \begin{bmatrix} 0 & r_s + L_{q4} p \end{bmatrix} i_{z4}
\end{align*}
\]

in which \(M = 3L_{mm} \), \(L_s = L_{q1} + M\) and \(L_r = L_{q2} + M\).

Based on Equations (4)-(8) the electromechanical torque of machine can be calculated as follows [9]:

\[
T_e = \frac{p}{2} (i_{\alpha} i_{\alpha} - i_{\beta} i_{\beta})
\]  

According to Equation (9) the \(z_1\) to \(z_4\) current components have no effect on electromagnetic torque, and as can be seen in Equations (5)-(8) the machine model in \((z_1, z_2)\) and \((z_3, z_4)\) subspaces are presented as a R-L circuit with no back EMF (ElectroMotive Force) voltage term. These current components can be so high in amplitude, depending on applied PWM technique, and may lead to extra copper losses and undesired current distortion. So, the \(z\)-component currents must be considered in SPIM drive control system.

3. THREE-LEVEL 6-PHASE INVERTER

3.1. Stator Phase Voltages

Based on Figure 1, the output voltage per phase (with respect to midpoint of the DC bus) in three-level six-phase inverter can be written as a function of switching states:

\[
\begin{align*}
v_{oa} &= S_a v_{dc} \\
v_{ob} &= S_b v_{dc} \\
v_{oc} &= S_c v_{dc} \\
v_{od} &= \frac{V_{dc}}{2} S_d v_{dc} \\
v_{oe} &= S_e v_{dc} \\
v_{of} &= S_f v_{dc}
\end{align*}
\]

where switching states \(S_a, S_b, S_c, S_d, S_e, \) and \(S_f\) can be 0, -1 or 1 depending on the switches state shown in Table 1.

So, there are \(3^6 = 729\) switching states for a three-level six-phase inverter. According to Figure 1, The stator phase voltages (with respect to point ‘n’) can be calculated as below:

\[
\begin{align*}
v_{oa} &= v_{oa} - v_{on} \\
v_{ob} &= v_{ob} - v_{on} \\
v_{oc} &= v_{oc} - v_{on} \\
v_{od} &= v_{od} - v_{on} \\
v_{oe} &= v_{oe} - v_{on} \\
v_{of} &= v_{of} - v_{on}
\end{align*}
\]

**TABLE 1.** \(S_i\) values \((i = a, b, c, d, e, f)\) and corresponding output voltages for different switches states

<table>
<thead>
<tr>
<th>(S_i)</th>
<th>(V_{oa})</th>
<th>(V_{oc})</th>
<th>(V_{od})</th>
<th>(V_{oe})</th>
<th>(V_{of})</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(\frac{v_{dc}}{2})</td>
<td>on</td>
<td>on</td>
<td>off</td>
<td>off</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>off</td>
<td>on</td>
<td>on</td>
<td>off</td>
</tr>
<tr>
<td>-1</td>
<td>(-\frac{v_{dc}}{2})</td>
<td>off</td>
<td>off</td>
<td>on</td>
<td>on</td>
</tr>
</tbody>
</table>
where \( V_{cm} = V_a - V_b. \)
Assuming six phase balance of stator windings results in:
\[
 v_{ab} + v_{bc} + v_{ca} + v_{ad} + v_{db} + v_{fa} = 0
\]
(12)
So, \( V_{cm} \) can be calculated using Equations (11) and (12):
\[
 V_{cm} = \frac{1}{6} (v_{ao} + v_{bo} + v_{co} + v_{do} + v_{eo} + v_{fo})
\]
(13)
By manipulating Equations (10), (11) and (13) the stator phase voltages can be shown by Equation (14).
\[
 V_a = \frac{1}{6} V_{dc} \begin{bmatrix}
 5 & -1 & -1 & -1 & -1 & -1 \\
 -1 & 5 & -1 & -1 & -1 & -1 \\
 -1 & -1 & 5 & -1 & -1 & -1 \\
 -1 & -1 & -1 & 5 & -1 & -1 \\
 -1 & -1 & -1 & -1 & 5 & -1 \\
 -1 & -1 & -1 & -1 & -1 & 1
\end{bmatrix} S_i
\]
(14)
The voltage vector calculated by Equation (14) can be projected on the \( \alpha-\beta \), \( z_1-z_2 \) and \( z_3-z_4 \) planes using the transformation (3). By adding “1” to all characters of each switching state and converting the resulted six-bit ternary number to decimal, the number of corresponding space vector can be determined. For example, switching state (1 0 -1 -1 0 1) is changed into ternary number of (2 1 0 0 1 2) corresponding to the space vector with number 572.

3.2. Common Mode Voltage Calculation
The common mode voltage can be considered as the voltage between the load neutral point and the midpoint of the DC bus [13]. So, the CMV is the same as \( V_{cm} \) in Equation (13) and is given by:
\[
 V_{cm} = \frac{1}{6} \sum v_{io} = \frac{1}{6} \frac{V_{dc}}{2} \sum S_i, \quad i = a, b, c, d, e, f
\]
(15)
The space vectors are categorized in Table 2 based on CMV generation. As can be seen, the 13-level of CMV is generated in a three-level inverter and there is a symmetry in the number of vectors with opposite sign of CMV. Hence, the property of the CMV waveform directly depends on the selected space vectors in the applied PWM technique.

4. SPACE VECTOR PWM

4.1. Symmetrical SPIM
Since projected model on the \( \alpha-\beta \) plane is associated with the electromechanical energy conversion, it is clear that the applied PWM strategy must generate maximum voltage in the \( \alpha-\beta \) plane while having zero or minimum amplitude in the \( z_1-z_2 \) and \( z_3-z_4 \) planes.

The first vector group with highest amplitude in the \( \alpha-\beta \) plane (1.155 pu) is depicted in Figure 2. The selected space vectors have 0 and 0.408 pu amplitude in the \( z_1-z_2 \) and \( z_3-z_4 \) planes respectively and adjacent vectors in the \( \alpha-\beta \) plane have opposite direction in the \( z_3-z_4 \) plane. The sum of switches state for all the selected space vectors is zero. So, based on Equation (15), no CMV is produced by applying them. The space vector number 364 with switching state of \((0 \ 0 \ 0 \ 0 \ 0 \ 0)\) can be considered as zero vector which produces no CMV and \( z \)-component voltage.

According to the above space vector selection, the \( \alpha-\beta \) plane is divided into six sectors. The balanced six-phase sinusoidal reference voltages have no \( z \)-component in the orthogonal subspaces and just consists of \( \alpha-\beta \) components. So, to generate the reference voltage in the orthogonal subspaces, balanced volt-second described in Equation (16) must be considered based on reference voltage position in the \( \alpha-\beta \) plane. To equalize the number of equations and unknown variables and achieving unique answer for time intervals, four active vectors must be selected.

\[
\begin{align*}
 V_{1a} T_1 + V_{2a} T_2 + V_{3a} T_3 + V_{4a} T_4 + V_{0a} T_0 &= V_{\alpha} T_1 \\
 V_{1b} T_1 + V_{2b} T_2 + V_{3b} T_3 + V_{4b} T_4 + V_{0b} T_0 &= V_{\beta} T_1 \\
 V_{1c} T_1 + V_{2c} T_2 + V_{3c} T_3 + V_{4c} T_4 + V_{0c} T_0 &= 0 \\
 V_{1d} T_1 + V_{2d} T_2 + V_{3d} T_3 + V_{4d} T_4 + V_{0d} T_0 &= 0 \\
 T_1 + T_2 + T_3 + T_4 + T_0 &= T_i
\end{align*}
\]
(16)

\( V_1, \ V_2, \ V_3 \) and \( V_4 \) are four adjacent active vectors in each sector; and \( T_1, \ T_2, \ T_3 \) and \( T_4 \) are their corresponding time intervals. Also \( V_0 \) and \( T_0 \) are zero vector and its corresponding time interval respectively. For example, for the first sector \((0 < \theta < 60)\) \( V_1, \ V_2, \ V_3 \) and \( V_4 \)

\begin{table}[h]
\centering
\caption{Categorization of space vectors based on CMV value}
\begin{tabular}{|c|c|}
\hline
Number of switching states & CMV Value \\
\hline
141 & 0 \\
252 & \( \pm \frac{V_v}{12} \) \\
180 & \( \pm \frac{V_v}{6} \) \\
100 & \( \pm \frac{V_v}{4} \) \\
42 & \( \pm \frac{V_v}{3} \) \\
12 & \( \pm \frac{5V_v}{12} \) \\
2 & \( \pm \frac{V_v}{2} \) \\
\hline
\end{tabular}
\end{table}
corresponds to $V_{404}$, $V_{650}$, $V_{702}$ and $V_{234}$ respectively. Based on Equation (16), the time intervals can be calculated as follow:

$$
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_{0}
\end{bmatrix} =
\begin{bmatrix}
V_{1a} & V_{2a} & V_{3a} & V_{4a} & V_{0a} \\
V_{1b} & V_{2b} & V_{3b} & V_{4b} & V_{0b} \\
V_{1c} & V_{2c} & V_{3c} & V_{4c} & V_{0c} \\
V_{1d} & V_{2d} & V_{3d} & V_{4d} & V_{0d} \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}^{-1}
\begin{bmatrix}
V_aT_x \\
V_bT_x \\
V_cT_x \\
V_dT_x \\
T_x
\end{bmatrix} \tag{17}
$$

As shown in Figure 2, all the selected space vectors in the $z_3$-$z_4$ plane are located in the second and fourth quarters which $z_3$ and $z_4$ components have same amplitude with opposite sign. So, the 3rd and 4th rows of 5x5 matrix in Equation (17) are symmetric and it cannot be reversed. Therefore, the time intervals cannot be calculated by considering zero average voltage in the $z_3$-$z_4$ plane during $T_x$. So, Equation (16) can be rewritten as Equation (18) and the time intervals is calculated by Equation (19).

$$
\begin{align*}
V_{1a}T_1 + V_{2a}T_2 + V_{3a}T_3 + V_{4a}T_0 &= V_aT_x \\
V_{1b}T_1 + V_{2b}T_2 + V_{3b}T_3 + V_{4b}T_0 &= V_bT_x \\
T_1 + T_2 + T_3 + T_0 &= T_x \tag{18}
\end{align*}
$$

$$
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_{0}
\end{bmatrix} =
\begin{bmatrix}
V_{1a} & V_{2a} & 0 & 0 \\
V_{1b} & V_{2b} & 0 & 0 \\
0 & 0 & 0 & 0 \\
1 & 1 & 1 & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
V_aT_x \\
V_bT_x \\
V_cT_x \\
T_x
\end{bmatrix} \tag{19}
$$

The selected space vectors can be applied according to state sequence shown in Figure 3 for the first sector ($0 < \theta < 60$). Although the adjacent active vectors in the $\alpha$-$\beta$ plane have opposite direction in the $z_3$-$z_4$ plane, during time interval $T_x$ the closer space vector to the reference vector has bigger time interval and consequently the average voltage on the $z_3$-$z_4$ plane is not zero. So, these nonzero voltages generate harmonic currents in the $z_3$-$z_4$ plane. These currents may be too much large if the machine leakage inductance is little.

The second group with 1 pu amplitude in the $\alpha$-$\beta$ plane and no $z$-component and CMV generation is shown in Figure 4. In other words, the only vectors with no $z$-component voltage and CMV between all 729 vectors belong to this group.

Figure 5 illustrates the state sequence of each phase for using second group space vectors, and shows zero value for CMV. Whereas the selected space vectors have no projection in the $z_3$-$z_2$ and $z_3$-$z_4$ planes, it needs to consider just volt-second balance in the $\alpha$-$\beta$ plane. Therefore, the time intervals $T_1$, $T_2$ and $T_0$ can be calculated by Equation (19).

4.2. Asymmetrical SPIM

Similar to symmetrical SPIM, high amplitude in the $\alpha$-$\beta$ plane and minimum amplitude in the $z_1$-$z_2$ and $z_3$-$z_4$ planes must be considered.
for space vector selection. The first vector group with highest amplitude in the α-β plane (1.11 pu) is depicted in Figure 6. The selected space vectors have 0.3 pu amplitude in the z₁-z₂ and 0 or 0.4 pu amplitude in the z₃-z₄ plane. The sum of switches state for the selected space vectors is 0 or ±2. So, based on Equation (15), produced CMV has 0 or ±Vdc/6 amplitude.

Similar to the first space vectors group in the symmetrical SPIM (Section 4.1), the selected space vectors are located in 2nd and 4th quarters in the z₃-z₄ plane and consequently the time interval equation cannot be solved. So, volt-second balance just can be considered in α-β and z₁-z₂ planes, and four adjacent vectors can be selected for each sector to calculate the time intervals. For example, space vectors number 656, 650, 648 and 702 can be used for the first sector (−15<θ<15). Therefore, the time intervals can be calculated as follow:

\[
\begin{bmatrix}
T_1 \\
T_2 \\
T_3 \\
T_4 \\
T_0
\end{bmatrix} =
\begin{bmatrix}
V_{1α} & V_{2α} & V_{3α} & V_{4α} & V_{0α} \\
V_{1β} & V_{2β} & V_{3β} & V_{4β} & V_{0β} \\
V_{1c1} & V_{2c1} & V_{3c1} & V_{4c1} & V_{0c1} \\
V_{1c2} & V_{2c2} & V_{3c2} & V_{4c2} & V_{0c2} \\
1 & 1 & 1 & 1 & 1
\end{bmatrix}^{-1}
\begin{bmatrix}
V_{α}^* \\
V_{β}^* \\
V_{c1}^* \\
V_{c2}^* \\
T_i
\end{bmatrix}
\tag{20}
\]

The state sequence for the first sector is depicted in Figure 7. As can be seen, changing the switches state during each sampling period Tᵢ is more than two previous PWM patterns shown for symmetrical SPIM, and leads to higher switching losses.

The second vector group with 1.07 pu amplitude in the α-β plane is depicted in Figure 8. The projection of the selected space vectors in the α-β, z₁-z₂ and z₃-z₄ planes have similar characteristics to the first group, and consequently the time intervals is calculated by Equation (20). But the selected space vectors have smaller amplitude in the z₁-z₂ and z₃-z₄ planes compared to the first group. The sum of switches state for the selected space vectors is ±1, and based on Equation (15), the

Figure 5. The state sequence of inverter for first sector (−30<θ<30) by applying space vectors with 1 pu amplitude in the α-β plane for symmetrical SPIM

Figure 6. Selected space vectors in asymmetrical SPIM with highest amplitude (1.11pu) in the α-β plane: a) projection in the α-β plane, b) projection in the z₁-z₂ plane, c) projection in the z₃-z₄ plane, d) corresponding switches state for each vector

Figure 7. The state sequence of inverter for the first sector (−15<θ<15) by applying space vectors with 1.11 pu amplitude in the α-β plane for asymmetrical SPIM
produced CMV has \( \pm \frac{1}{12} V_{dc} \) amplitude. The state sequence of each phase for the first sector \((0<\theta<60)\) is depicted in Figure 9. It can be seen that the number of phases state changing is equal to the first group but the resulted CMV has lower peak to peak value.

The third space vector group is depicted in Figure 10 with 0.97 pu and 0.26 pu amplitudes in the \(\alpha-\beta\) and \(z_1-z_2\) planes respectively. The selected space vectors produce no CMV and have no projection in the \(z_3-z_4\) plane. So, the time intervals can be calculated by Equation (20). The state sequence illustrated in Figure 11 shows lower phases state changing compared to two previous space vector groups.

5. SIMULATION RESULTS

The described SVPWM techniques are simulated by Matlab/Simulink software and shown in Figures 12-16. The system parameters are given in Table 3 and the PWM techniques are named in Table 4. The modulation index and frequency is set to 1 and 50 Hz respectively, and the mechanical load is assumed to have a constant torque (40 Nm). As depicted in simulation results, all the methods have similar \(\alpha-\beta\) current component approximately. The \(\alpha-\beta\) currents in all PWM methods are balanced (same amplitude and 90° phase difference) with a small distortion which leads to torque ripple producing. But the difference in the phase currents is due to \(z\)-components as a result of different properties of the selected space vectors for each method.

The result of applying the SVPWM-1 method on symmetrical SPIM is depicted in Figure 12. As explained in Section 4.1, no \(z_1-z_2\) current and CMV is produced in this method. But, having nonzero average voltage in each sampling time interval \((T_s)\) in the \(z_3-z_4\) plane results in high \(z_3\) and \(z_4\) current components with frequency of \(3f_s\).
TABLE 3. Parameters of the simulated motor [23]

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inverter DC voltage</td>
<td>600 V</td>
</tr>
<tr>
<td>Number of Pole pairs</td>
<td>4</td>
</tr>
<tr>
<td>Mutual inductance</td>
<td>51.3 mH</td>
</tr>
<tr>
<td>Stator resistance</td>
<td>2.34 Ω</td>
</tr>
<tr>
<td>Stator leakage inductance</td>
<td>6.7 mH</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>1.17 Ω</td>
</tr>
<tr>
<td>Rotor leakage inductance</td>
<td>6.7 mH</td>
</tr>
<tr>
<td>Inertia coefficient</td>
<td>0.03 kg.m²</td>
</tr>
</tbody>
</table>

TABLE 4. Naming of several SVPWM technique based on SPIM configuration and space vectors amplitude in the α-β plane

<table>
<thead>
<tr>
<th>SPIM configuration</th>
<th>Amplitude in the α-β plane</th>
<th>name</th>
</tr>
</thead>
<tbody>
<tr>
<td>Symmetrical</td>
<td>1.15 pu</td>
<td>SVPWM-1</td>
</tr>
<tr>
<td></td>
<td>1 pu</td>
<td>SVPWM-2</td>
</tr>
<tr>
<td></td>
<td>1.11 pu</td>
<td>SVPWM-3</td>
</tr>
<tr>
<td>Asymmetrical</td>
<td>1.077 pu</td>
<td>SVPWM-4</td>
</tr>
<tr>
<td></td>
<td>0.97 pu</td>
<td>SVPWM-5</td>
</tr>
</tbody>
</table>

So, the generated z₁ and z₄ current components appear as high 3rd harmonic in the stator currents and cause high current losses in the drive system. Hence, this PWM method is not applicable.

Figure 13 shows the result of applying the SVPWM-2 method on symmetrical SPIM. As can be seen, no z-component current and CMV are produced and the resulting phase currents are sinusoidal. Moreover, the electromagnetic torque has less ripple compared to the SVPWM-1 method.

The results of applying SVPWM-3 and SVPWM-4 on the asymmetrical SPIM are depicted in Figures 14 and 15, respectively, and both of them produce high z₁ and z₄ current components with frequency of 3fₛ leading to presence of high 3rd harmonic in the stator phase currents. The CMV produced by SVPWM-3 and SVPWM-4 have V₆/₆ and V₆/₆/12 peak amplitudes respectively.

As illustrated in Figure 16, applying the SVPWM-5 with smaller space vectors amplitude in the α-β plane compared to two previous methods, produces no CMV and z₁-z₄ current components and the resulted z₁-z₂ current components are insignificant.

Based on simulation results, just SVPWM-2 and SVPWM-5 respectively for symmetrical and asymmetrical SPIM configurations are applicable. So, this two PWM methods are compared in the next simulation results as proposed the PWM techniques for both symmetrical and asymmetrical SPIM configurations.

Changing of one phase switches state during Tₛ in the first sector are shown in Figures 17 and 18 for SVPWM-2 and SVPWM-5 methods respectively. In SVPWM-2 two switches in each phase have no state changing, and the state of other switches is changed through two pulses. However, in the other sectors this is different and the constant state belongs to other switches. But, the switches in SVPWM-5 have more state changing which leads to higher switching losses.
Figure 13. Simulation results of applying SVPWM-2 on the symmetrical SPIM

Figure 14. Simulation results of applying SVPWM-3 on the asymmetrical SPIM

Figure 15. Simulation results of applying SVPWM-4 on the asymmetrical SPIM

Figure 16. Simulation results of applying SVPWM-5 on the asymmetrical SPIM

Figure 17. The switching sequence of “a” phase switches for symmetrical SPIM and selected space vectors with 1pu amplitude in the α/β plane

Figure 19 shows the ratio between rms value of z-components to α-β voltages versus modulation index for SVPWM-2 and SVPWM-5 methods. The ratio is calculated by Equation (21) and describes the presence of undesired (z-components) voltages compared to the α-β components in the stator phase voltages.

\[
\frac{V_z}{V_{αβ}} = \frac{\sqrt{V_α^2 + V_β^2 + V_3^2 + V_4^2}}{\sqrt{V_α^2 + V_β^2}}
\]  

(21)
The selected space vectors in SVPWM-2 have no z-component voltage and the resulted $V_z/V_{\alpha\beta}$ ratio is zero. On the other hand, the zero vector $V_{664}$ (0 0 0 0 0 0) applied for SVPWM-5 has no $\alpha$-$\beta$ and z-components, and the time interval of applying active vectors in the $\alpha$-$\beta$ and $z_1$-$z_2$ planes are the same. So, the $V_z/V_{\alpha\beta}$ ratio for this PWM method is constant and equals to the ratio between space vectors amplitude in the $z_1$-$z_2$ plane to the $\alpha$-$\beta$ as shown in Equation (22).

$$\frac{V_z}{V_{\alpha\beta}} = \frac{|V_{z1-z2}|}{|V_{\alpha-\beta}|} = \frac{0.26}{0.97} = 0.268$$ (22)

However, the SVPWM-5 produces a considerable z-component voltage compared to the SVPWM-2, but as shown in Figure 20, the phase voltage THD for both methods has a little difference in all modulation index values.

The phase currents FFT analysis in both of the methods for $m=0.6$ and $f=30$ Hz are compared in Figure 21, which shows a slight difference in current THD. Since in the SVPWM-2 method no z-component voltage is produced and has the same current distortion compared to SVPWM-5 method, it can be concluded that the former has more $\alpha$-$\beta$ current distortion than the latter. So, the resulted electromagnetic torque in SVPWM-2 method has more ripple than SVPWM-5 method, and it can be seen by comparing the torque waveforms in Figure 13 with Figure 16 in which the torque ripples for SVPWM-2 and SVPWM-5 methods are 9.12 and 3.7% respectively.

6. CONCLUSION

A comprehensive analysis of the SVPWM strategies for both symmetrical and asymmetrical six-phase induction motor drive with three-level inverter was given in this
paper. The space vectors were selected and applied in each PWM method based on maximum α-β voltage, minimum z-component current, and CMV production. Based on theoretical analysis and simulation results, it is shown that just SVPWM-2 and SVPWM-5 methods with lower amplitude in the α-β plane for symmetrical and asymmetrical SPIM respectively are applicable. Both of the PWM methods produce no CMV but SVPWM-5 produces a slight z₁-z₂ current component while no z-component current is generated by SVPWM-2 method. The comparison of the number of switches state changing during the sampling time interval shows that SVPWM-2 has lower switching compared to SVPWM-5 strategy and leads to lower switching losses. On the other hand, both of the PWM methods have the same voltage and current THD approximately, and the generated electromagnetic torque has lower ripple in SVPWM-5 method. However, high frequency electromagnetic torque ripple can be damped by the machine mechanical system specially in larger machines. So, the symmetrical SPIM with SVPWM-2 strategy can be a better configuration for six-phase drive system. However, applying three-level inverter in a six-phase drive system imposes additional costs, but it leads to longer machine life time and a considerable modification in efficiency due to CMV and z-component currents elimination and lower switching losses.

7. REFERENCES


چکیده
افزایش روز افزون در به کارگیری سیستم‌های درایو چند فاز باعث توسعه ساختار اینورترها و روشهای کنترلی به کار گرفته شده برای آنها شده است. تولید ولتاژ حالت مشترک توسط اینورتر، خرابی بیرینگ‌ها را در درایوهای چند فاز به دنبال دارد. از طرف دیگر حضور جریانهای مولفه $z$ در ماشین القایی شش فاز باعث افزایش تلفات جریانی می‌شود که باید در در روش PWM مورد استفاده در نظر گرفته شود. در این مقاله نشان داده شده است که حضور جریانهای مولفه $z$ و ولتاژ حالت مشترک دو عامل مهم در انتخاب بردارهای فضاپیمایی است. بردارهای فضاپیمایی و تازه‌برای هر دو راه اندازی ممکن و نامناسب ماشین القایی تغذیه شده به اینورتر سطحی در زیرفضاهای قائم نمایش می‌دهند. سپس روش‌های مدولاسیون SVPWM برای استفاده در انتخاب بردارهای فضاپیمایی و تازه‌برای هر دو راه اندازی ممکن و نامناسب ماشین القایی تغذیه شده به اینورتر سطحی در زیرفضاهای قائم نمایش می‌دهند. به هدف پایانی با حداکثر اغتشاش جریان، مولفه‌های جریانی نامناسب و ولتاژ حالت مشترک و با ریپل گشتاور قابل قبول ارائه شده است.