



Sensitivity Analysis of Vibration Response of Railway Structures to Velocity of Moving Load and Various Depth of Elastic Foundation

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ABSTRACT

Railway structures are one of the most important structures in transportation. So the lack of precise study of their dynamic behavior leads to irreparable damages. The significant factors contributing to the accurate analysis of the dynamic behavior of railways are the type of load and foundation used in it. In this study, an Euler-Bernoulli beam subjected to a moving load on a finite depth foundation is presumed. According to the feature of finite beams, just the dynamic equilibrium in the vertical direction is regarded. In this paper, by using equilibrium equations and considering the influence of soil and structure interaction, the physical problem is simulated and by using Fourier transform method the governing differential equations are obtained. Then, the mathematical model based on suggested models is expanded and verified. By assessing the efficiency of the recommended method, the dynamic behavior of the beam is specified and the deflection ratios for various foundations are illustrated. The sensitivity analysis is provided to study the influence of various parameters such as velocity of moving load, elastic foundation depth and damping. Eventually, by considering the sequences of shear waves, critical velocity, which is dependent on the mass ratio, and various kinds of damping, deflection shapes of the beam are attained for the different velocities of the moving load, and the effect of soil depth on the dynamic behavior of the beam is discussed. It is indicated that, foundation inertia leads to a considerable reduction in critical velocity and can also intensify the response of the beam.

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1. INTRODUCTION

In some cases structural loads can vary with time and location which lead to complicated analysis and estimation. Hence, the behavior of the structure "deflection shapes, internal forces and stresses" will be time-dependent; Therefore, the behavior of the structure in this case, in comparison with the static behavior, does not have an absolute response, though, at any moment in time, there will be a particular response to it. Dynamic behavior of structures under the influence of moving loads is an important engineering issue. So many researches have been conducted in this regard. Problems that are caused by these types of loads cannot be neglected in the behavior of structures. For instance, the problems caused by train force on the railways affect the displacement by considering speed and acceleration. Over the past few years, several

prominent papers different types of physical assumptions related to the problems of accelerated moving loads are derived. As the matter of fact, the basic problems appear in the mathematical operations rather than physical occasions. Most of the approaches used in the past were not just in the base of the approximation, actually they were very dreary in the mathematical operations system. Therefore, providing a simple, direct, short and functional method, in the mathematical system, is necessary. Several studies have been conducted in the dynamic behavior of the beam under different kinds of loads. One of the first researches on the elastic foundation is performed by Timoshenko [1]. His work relates to the response of the railway under the constant speed of moving load [1]. Kenney [2] obtained a steady-state solution and provided that the critical velocity is really effective on the deformation of the beam. The frequency of the beam vibrations by using finite element method was investigated by Györgyi [3]. Li [4] presented a

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simple and unified approach for analyzing the free vibration of the generally supported Euler-Bernoulli beam. The linear association of the Fourier series and an auxiliary polynomial function are used to specify the displacement of the desired beam. Hillal and Zibdeh [5] suggested the vibration of the Euler-Bernoulli beam under the influence of moving load as a closed form solution. Also, an approach for extracting the dynamic behavior of damped Euler-Bernoulli beam excited by concentrated and distributed forces is provided by Abu-Hillal [6]. Kargarnovin and Younesian [7] investigated the dynamic response of Timoshenko beam subjected to harmonic moving load with infinite length in the visco-elastic Pasternak foundation. Ying et al. [8] studied the rough solutions for bending behavior and free vibration on the Winkler-Pasternak elastic foundation. Mehri et al. [9] using the Green function derived the dynamic behavior of the Euler-Bernoulli beam excited by moving load. Also, spectral analysis of the beam under the influence of load is recommended by Gladysz and Sniady [10]. The desired beam is contemplated orthotropic at any point, whereas the properties of different materials in the thickness of the beam are exponential. In addition, by using differential transform method the vibration of the Timoshenko and Euler-Bernoulli beam on elastic soil is predicted by Balkaya et al. [11]. In this suggested method, accurate solutions without the serious analysis necessity are attained. Motaghian et al. [12] investigated the problem of free vibration of the Euler-Bernoulli beam on the elastic foundation. Also, the nonlinear vibration of the Euler-Bernoulli beam with fixed ends under the influence of axial loads is derived by Barari et al. [13] subjected to a bending load excitation while the damping effect has been taken into account. A new analytical solution to predict the free lateral vibration of a Timoshenko beam with different kinds of boundary conditions is employed by Bazehhour et al. [14]. Also, the influence of the axial load on the natural frequencies is examined. Simultaneously, Prokić et al. [15] illustrated a numerical approach to clarify the free vibration of Timoshenko beams with optional boundary conditions. The numerical approach is fundamentally attributed to numerical integration instead of the numerical differentiation. A proficient analytical approach to analyze the vibration of the Euler-Bernoulli beam on Winkler foundation is presented by Yayli et al. [16]. To attain the free vibration response of the beam on Winkler foundation, the Stoke's transformation with Fourier sine series is utilized.

The dynamic response of the railway track structure subjected to moving load on visco-elastic foundation is derived by Mohammadzadeh and Mosayebi [17]. An analytical method and a combined finite element for predicting the vibration of a crane system excited by suspended moving body is provided by Zrnić et al. [18]. Zakeri and Shahbabaei [19] presented the influence of elastic supports stiffness on the natural frequencies and two-span beams modes. The dynamic response of non-uniform Timoshenko beam under moving mass provided by Roshandel et al. [20]. Dimitrovová [21] investigated the

dynamic behavior of the Euler-Bernoulli beam on a finite depth base. In this case, a new formula is utilized for the critical velocity of the moving load. The influence of damping models, boundary conditions, and model size in the finite element modelling of a moving load derived by Shih et al. [22]. Bian et al. [23] presented the dynamic response of the railway under constant and accelerated moving loads with various velocities. For this purpose, the railways were modeled as the Euler-Bernoulli beam. Sheng et al. [24] studied the dynamic response of the railways under the influence of moving harmonic load by using the Fourier transform method.

By using the Green's function method, the dynamic behavior of the railway subjected to accelerated moving load investigated by Ghannadiasl. Thereby, a direct and accurate modeling technique for railway is provided as the Euler-Bernoulli beam on the elastic foundation under the moving load with various boundary conditions [25]. Ghannadiasl and Khodapanah Ajirlou [26] investigated the dynamic analysis of the Euler-Bernoulli cracked beam on the elastic foundation under the concentrated load. Using Green's function natural frequency and deflection of Euler-Bernoulli beam with several boundary conditions are obtained. They also carried out multi-span damped cracked beam by using the considered approach [27]. Forcellini et al. [28] investigated 3D homogeneous soil models by using Response Site Analyses. The considered approach has been performed on various homogeneous soil profiles, different in shear velocity. Solar energy for traction of High Speed railways derived by Nazir [29]. The solar panels have been installed along the length of a HS rail network in order to use tracks as energy carriers. Kolesnikov and Tolmacheva [30] studied ways to minimize rigid pavement weight. They provided a mathematical model of rigid pavement on an elastic foundation and investigated the effect of various parameters on displacements and the values of stresses. The influence of spatial variability of undrained shear strength on the bearing capacity of shallow strip footing on clay investigated by Azan and Haddad [31]. They presented two new equations in order to combine the influence of soil variability parameters on the bearing capacity of the strip footing on clay.

The dynamic behavior of the Euler-Bernoulli beam excited by the moving load in the previous studies is assessed. In the present paper, a precise solution in closed form is illustrated for assessing the sensitivity of vibration response of railway structures to velocity of moving load and various depths of elastic foundation. Also, it might be announced here that the previous authors did not mention the effects of foundation depth and deflection ratio for various foundations. The present paper is organized as follows. In section 2, the governing equations based on the Euler-Bernoulli beam theory is clarified. Then, in section 3, the complete solutions and some numerical examples are illustrated. In section 4, effects of the soil depth and various types of damping with some numerical examples are provided. Finally, in section 5, the conclusions are classified, in brief.

2. RESEARCH METHODOLOGY

In this study, an infinite Euler-Bernoulli beam subjected to the influence of different kinds of damping coefficients such as beam internal and viscous damping with uniform cross-section is studied as shown in Figure 1. The governing differential equation of the Euler-Bernoulli beam is illustrated as Equation (1):

$$EI \frac{\partial^4 w}{\partial x^4} + c_i \frac{\partial^5 w}{\partial x^4 \partial t} + N \frac{\partial^2 w}{\partial x^2} + m \frac{\partial^2 w}{\partial t^2} + c_b \frac{\partial w}{\partial t} + P_s = P\delta(x - vt) \tag{1}$$

where c_i is the beam internal damping coefficient, N is the axial force, which is assumed positive in compression, m is the beam mass per unit length, P_s is the pressure of foundation that will be switched later, P is the moving load, and v is its velocity. $w(x, t)$ and P are presumed positive when acting downward. Moreover, δ is the Dirac delta function and c_b is the beam viscous damping coefficient. Conversely, the dynamic equilibrium of the soil in the vertical direction illustrated in terms of Equation (2):

$$\bar{\rho} \frac{\partial^2 u}{\partial t^2} + c_f \frac{\partial u_r}{\partial t} = \bar{k}_{st} H \frac{\partial^2 u}{\partial z^2} + \bar{G}_s \frac{\partial^2 u}{\partial x^2} \tag{2}$$

where the upper bar illustrates the limitation to the finite strip b , in other words, density and moduli of soil are multiplied by b . u_r is the vertical soil displacement which is used in order to introduce the influence of foundation damping accurately, $\bar{\rho}$ is soil density, \bar{k}_{st} depicts the stiffness, H is soil depth, the expression $\bar{G}_s \left(\frac{\partial^2 u}{\partial x^2} \right)$ counts for the shear effect and c_f is the foundation viscous damping coefficient.

All variables will be utilized in dimensionless forms. The critical velocity will be specified by parametric analysis and the systems of Equations (1) and (2) will be clarified for steady-state beam deflection. Thereby, changing the equations to moving coordinate $s = x - vt$ and considering limitation to the steady state conditions gives Equations (3) and (4) as follows:

$$(EI - c_i v) \frac{\partial^4 w}{\partial s^4} + (N + mv^2) \frac{\partial^2 w}{\partial s^2} - v c_b \frac{\partial w}{\partial s} + P_s = P\delta(s) \tag{3}$$

$$\bar{\rho} v^2 \frac{\partial^2 u}{\partial s^2} - c_f v \frac{\partial u_r}{\partial s} = \bar{k}_{st} H \frac{\partial^2 u}{\partial z^2} + \bar{G}_s \frac{\partial^2 u}{\partial s^2} \tag{4}$$

Initially, Equation (4) is solved. Afterwards, the relative displacement satisfies the boundary conditions, which makes the determination easier. Thereby

$$z = \zeta H, u = u_r + (1 - \zeta)w \tag{5}$$

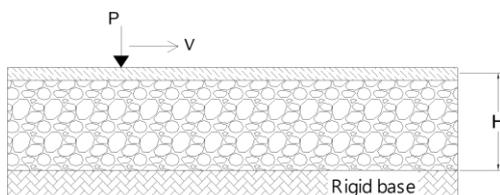


Figure 1. The infinite beam on soil under moving load

Furthermore with $\chi = \sqrt[4]{\bar{k}_{st}/4EI}$, the moving coordinate changes to dimensionless coordinate $\xi = \chi s$, and dividing all terms by the static displacement $w_{st} = \frac{P\chi}{2\bar{k}_{st}}$, to attain dimensionless \hat{u}_r and \hat{w} , provides Equation (6):

$$\left(1 - \left(\frac{\vartheta_s}{\alpha}\right)^2\right) \frac{\partial^2 \hat{u}_r}{\partial \xi^2} - \frac{\eta_f}{\alpha \mu^2} \frac{\partial \hat{u}_r}{\partial \xi} - \frac{1}{\alpha^2 \mu^2} \frac{\partial^2 \hat{u}_r}{\partial \xi^2} = - \left(1 - \left(\frac{\vartheta_s}{\alpha}\right)^2\right) (1 - \zeta) \frac{\partial^2 \hat{w}}{\partial \xi^2} \tag{6}$$

where, $\vartheta_s = \frac{v_s}{v_{cr}}$ shows the shear coefficient. Also, the term v_s is the velocity of the shear waves, $\alpha = \frac{v}{v_{cr}}$ is the velocity ratio with $v_{cr} = \frac{\sqrt[4]{4\bar{k}_{st}EI}}{\sqrt{m^2}}$, μ is the mass ratio that explained as $\mu = \sqrt{\bar{\rho}H/m}$, and $\eta_f = \frac{c_f H}{\sqrt{\bar{k}_{st}m}}$.

According to the homogeneous conditions, the following relation. i.e. Equation (7) can be presumed as follows:

$$\hat{u}_r = \sum_{j=r}^{\infty} U_j \sin(j\pi\zeta) \tag{7}$$

Thereafter, multiplication with one mode shape, substitution and integration from 0 to 1 depth, and using Fourier transform yields Equation (8) as follows:

$$U_j^* = \frac{\omega^2 \frac{2}{j\pi} \left(1 - \left(\frac{\vartheta_s}{\alpha}\right)^2\right)}{-\omega^2 \left(1 - \left(\frac{\vartheta_s}{\alpha}\right)^2\right) - i\omega \frac{\eta_f}{\alpha \mu^2} + \left(\frac{j\pi}{\alpha \mu}\right)^2} W^* \tag{8}$$

According to the foundation pressure in Equation (9) [23]:

$$P_s = -(1 - i\eta_h) \bar{k}_{st} \left(\sum_{j=1}^{\infty} j\pi u_j - w\right) \tag{9}$$

where η_h illustrates the coefficient of the hysteretic damping and $u_j = U_j w_{st}$. Hence, getting back to Equation (1), one attains Equation (10) as follows:

$$(EI - c_i v) \frac{\partial^4 w}{\partial s^4} + (N + mv^2) \frac{\partial^2 w}{\partial s^2} - v c_b \frac{\partial w}{\partial s} - (1 - i\eta_h) \bar{k}_{st} \left(\sum_{j=1}^{\infty} j\pi u_j - w\right) = P\delta(s) \tag{10}$$

Changes to dimensionless values, Equation (11) is obtained. Here moreover $\eta_i = 2\alpha\chi c_i/2\sqrt{mEI}$, $\eta_b = c_b/2\sqrt{m\bar{k}_{st}}$ and $\eta_N = N/2\sqrt{\bar{k}_{st}(EI - c_i v)}$ are presented.

$$\frac{\partial^4 \hat{w}}{\partial \xi^4} + \frac{4}{(1 - \eta_i)} (\alpha^2 + \eta_N) \frac{\partial^2 \hat{w}}{\partial \xi^2} - \frac{8}{(1 - \eta_i)} \eta_b \alpha \frac{\partial \hat{w}}{\partial \xi} + \frac{4}{(1 - \eta_i)} (1 - i\eta_h) (\hat{w} - \sum_{j=1}^{\infty} j\pi U_j) = \frac{8\delta(\xi)}{(1 - \eta_i)} \tag{11}$$

By the Fourier transform, we have:

$$W^* = \frac{8}{\beta + 4(1 - i\eta_h)S} \tag{12}$$

where

$$\beta = (1 - \eta_i)\omega^4 - 4\omega^2(\alpha^2 + \eta_N) - 8i\omega\eta_b\alpha + 4(1 - i\eta_h) \tag{13}$$

$$S = 1 + \sum_j \frac{2(\omega\alpha\mu)^2 \left(1 - \left(\frac{\vartheta_s}{\alpha}\right)^2\right)}{(\omega\alpha\mu)^2 \left(1 - \left(\frac{\vartheta_s}{\alpha}\right)^2\right) + i\omega\alpha\eta_f - (j\pi)^2}$$

Figure 2 presents a flowchart of the main steps of the final algorithm for calculate the displacement of the uniform beam.

3. NUMERICAL EXAMPLES

The Euler–Bernoulli beam under a moving load is considered for the purpose of verification. The beam is expressed with the following features: beam bending stiffness ($EI = 6.4 \text{ MN m}^2$), beam mass per unit length ($m = 60 \text{ kg/m}$), beam damping ($\eta_b = 0.02$), Soil Young's modulus ($\bar{E}_s = 10 \text{ MN m}^{-1}$), Soil Poisson's ratio ($\nu = 0.2$), Soil density ($\bar{\rho} = 185 \text{ kg/m}^2$), active depth ($H = 1.3 \text{ m}$), Foundation damping ($\eta_f = 0.629$), force ($P = 100 \text{ kN}$), and critical velocities ($v_{cr} = 497,286,325 \text{ m s}^{-1}$).

Therefore, by assuming $\eta_i = 0$, in Equation (12), the governing equation for the Euler–Bernoulli beam gets as follows [8]:

$$W^* = \frac{8}{\omega^4 - 4\omega^2(\alpha^2 + \eta_N) - 8i\omega\eta_b\alpha + 4(1 - i\eta_h)S} \quad (14)$$

Regarding published results by other researchers, considering only vertical displacement, like reported in literature [32], the vertical displacement for various values of mass ratio is obtained in Figure 3. In order to compare and justify various theoretical models with each other, such as classical Winkler foundation, the model without and with shear contribution, and classical Pasternak and visco-elastic foundations, deflection shapes for mentioned cases are investigated and illustrated in Figures 4 and 5. By using the presented values, Equation (12) is obtained. Also, by introducing $\vartheta_s = 0$ and $\mu = 0$, solution for classical Winkler's foundation; $\vartheta_s = 0$ model without shear contribution, $\mu = 0$ solution for classical Pasternak foundation, and $\eta_h = 0.529$ model for visco-elastic foundation are attained.

According to the graphs, it can be seen, that the occupied large area with superior displacement behind the load, stands for the solution without shear contribution. That is because of the vibration that is not interacted soil columns.

The classical solution for Pasternak, Winkler and visco-elastic foundations provided very low displacement, because the applied velocity is approximately far from the critical one. On the other hand, the determination of the critical velocity by pulling out the maximum downward and upward displacements as functions of velocity is shown in Figures 6 and 7. Numerical input data are summarized in this section. The graphs in Figures 6 and 7 depict that there is rarely any displacement directed upward and downward under the critical velocity. Both of the displacements over the critical velocity, for classical Winkler foundation, the model without and with shear contribution, classical Pasternak foundation, and visco-elastic foundation are compared.

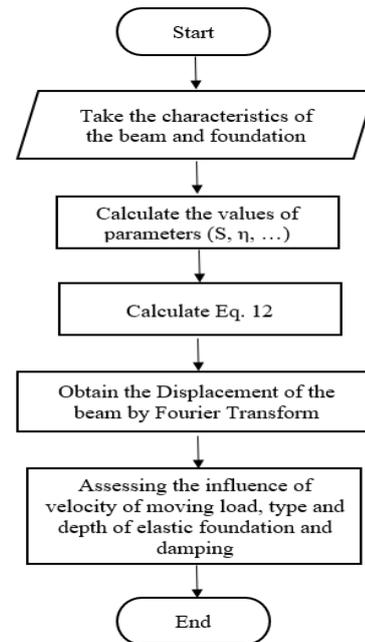


Figure 2. flowchart of Analysis process

4. THE EFFECT OF SOIL DEPTH AND DAMPING

In this section analysis of the soil depth and various types of damping are provided. The depth of the soil is really effective on the dynamic behavior of the beam. In hence, displacement shapes for different values of the velocity ratio and active depth ($H = 2, 4, 8, 12 \text{ m}$) are obtained in Figure 8. From Figure 8 it is observed that by increasing the soil depth, the displacement of the beam is decreased. On the other hand, by increasing the velocity ratio and getting closer to critical velocity the displacement of the beam is also increased.

In most of the situations, the estimating of the damping values for recognizing the comparable levels is necessary. The relationships between damping are a little bit complicated in this model, but in some cases their practical characteristics can be obtained. Firstly, the influence of the soil depth on the foundation damping with various values is investigated in Figure 9. Then another similar analysis is taken in Figure 10 for assessing the relation and the effect of the beam damping and internal damping, by introducing the following values: $\mu = 4, \vartheta_s = 0.4, \eta_f = 0, 0.5, 1, 2, \eta_h = 0, \eta_i = 0, \eta_N = 0$ and $\alpha = 0.25, 0.55$.

From Figure 9 can be seen that by increasing the active depth of soil, the effect of foundation damping on the behavior of the beam is also increased. Furthermore, from Figure 10 it is seen that the influence of damping for $\alpha = 0.25$ is not more significant because the velocity is far from the critical velocity, but by soaring of the velocity ratio, i.e. getting closer to the critical velocity, the influence of the damping on the dynamic behavior of the beam is increased.

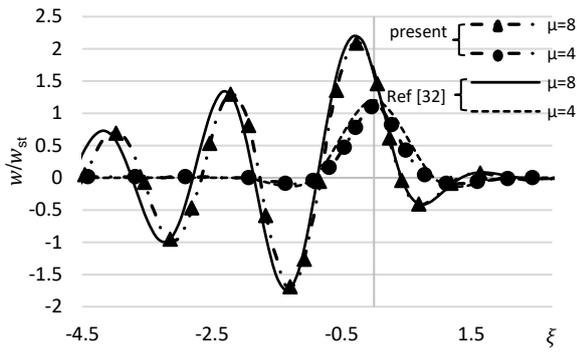


Figure 3. Displacement shapes for different values of the mass ratio

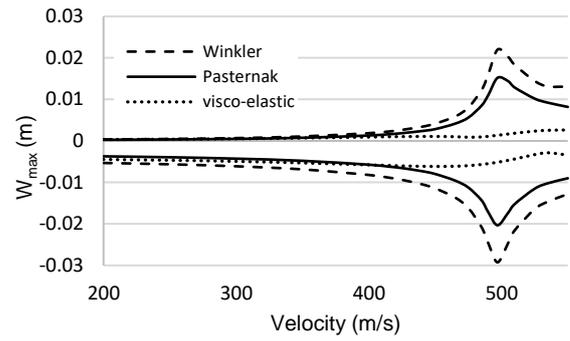


Figure 7. Maximum displacements directed downward and upward

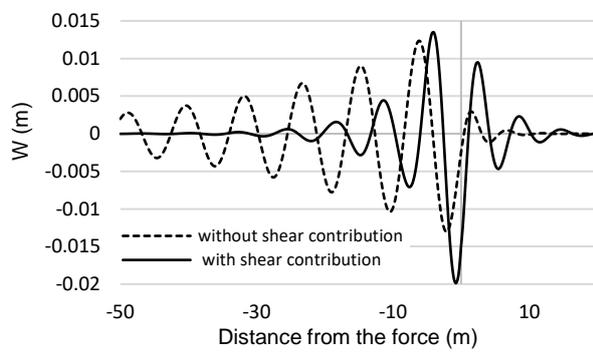


Figure 4. Deflection shapes comparison for presented values

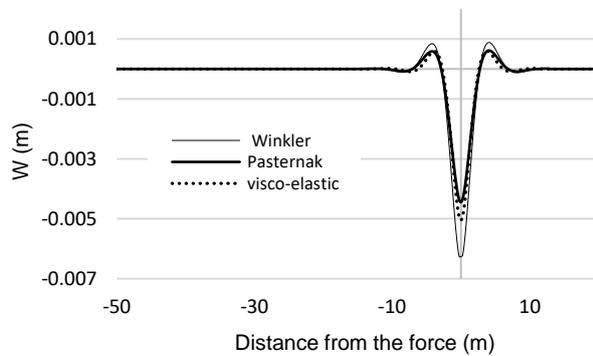


Figure 5. Deflection shapes comparison for presented values

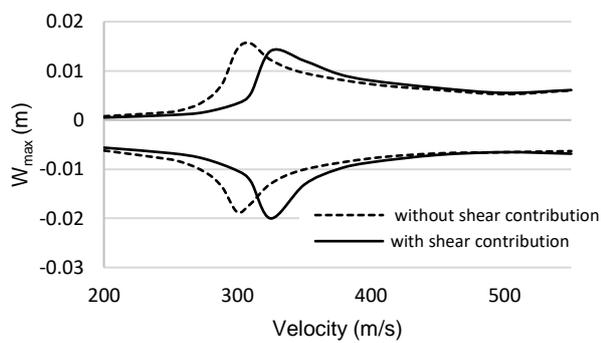
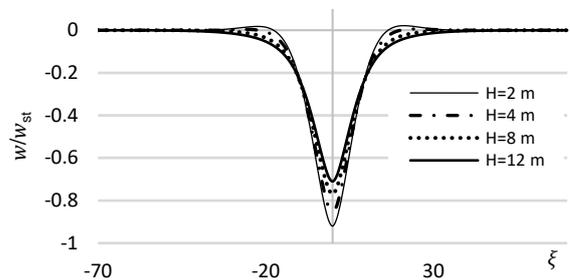
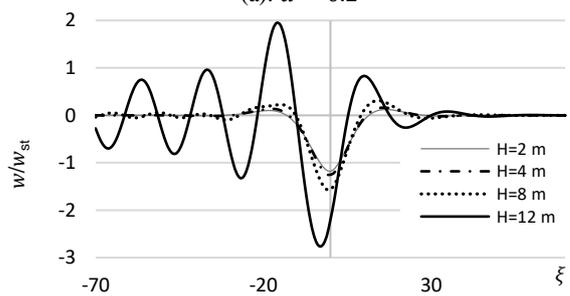


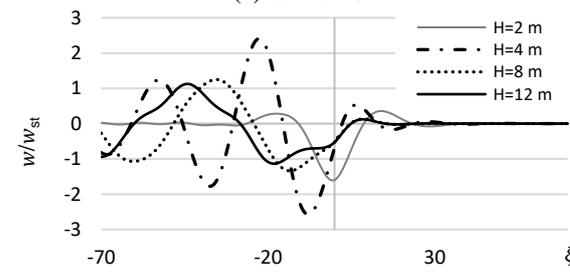
Figure 6. Maximum displacements directed downward and upward



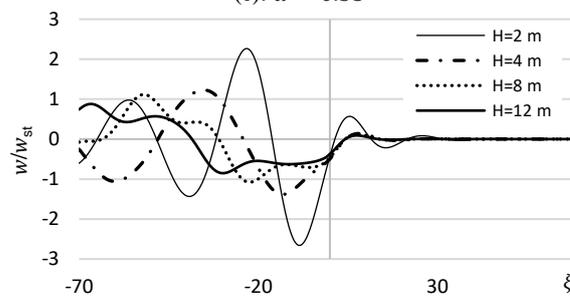
(a): $\alpha = 0.2$



(b): $\alpha = 0.45$



(c): $\alpha = 0.55$



(d): $\alpha = 0.65$

Figure 8. Displacement shapes for different values of the active depth and $\alpha = \frac{v}{v_{cr}}$

Finally, in order to present the influence of the foundation on the dynamic behavior of the beam, the desired beam once on the Winkler foundation, once on the soil with the shear contribution, and once without shear contribution is assumed as shown in Figure 11. Using the Equation (12) and the presented values the deflection shapes of the Euler-Bernoulli beam for different velocities and various depths of foundation are attained. As a result, by increasing depth of soil, the displacement of the beam is decreased.

In the following, the deflection ratio $\gamma = \frac{W_D}{W}$ for various foundations based on beam internal damping coefficient is presented in Table 1 The corresponding values are: $\eta_i =$

0.05,0.1. From Table 1 it is seen, that the deflection ratio for solution with shear contribution is lower than the other and by increasing the beam internal damping coefficient the deflection of the beam decreases.

TABLE 1. The deflection ratio for various foundations

η_i	Winkler Foundation	With shear contribution	Pasternak Foundation	Visco-Elastic Foundation
0.05	0.9750	0.8812	0.9783	0.9806
0.1	0.9548	0.7836	0.9543	0.9635

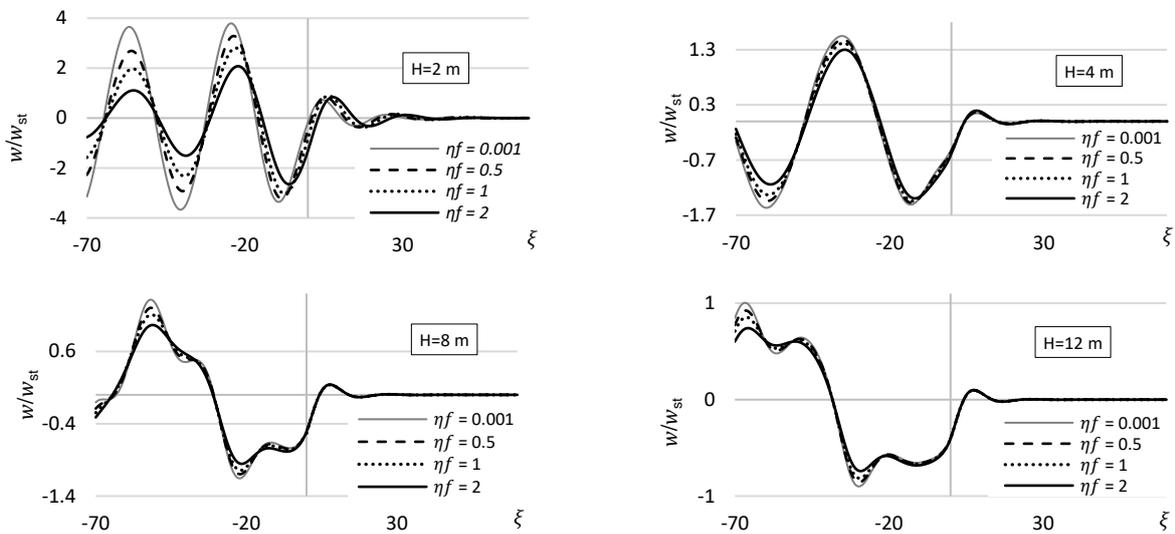
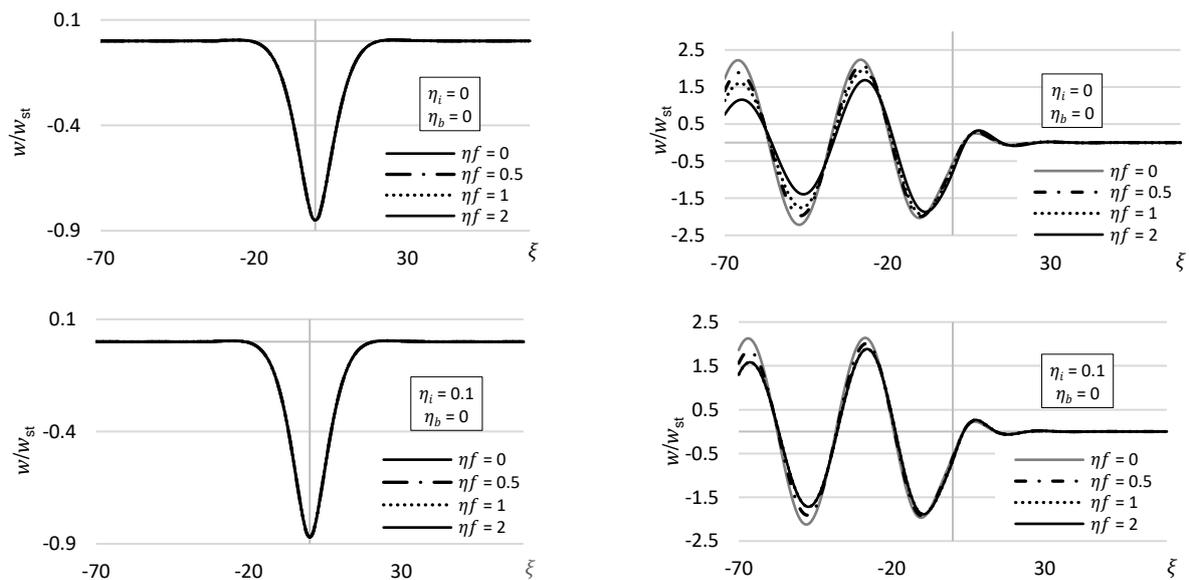


Figure 9. Displacement shapes for different values of the active depth and foundation damping



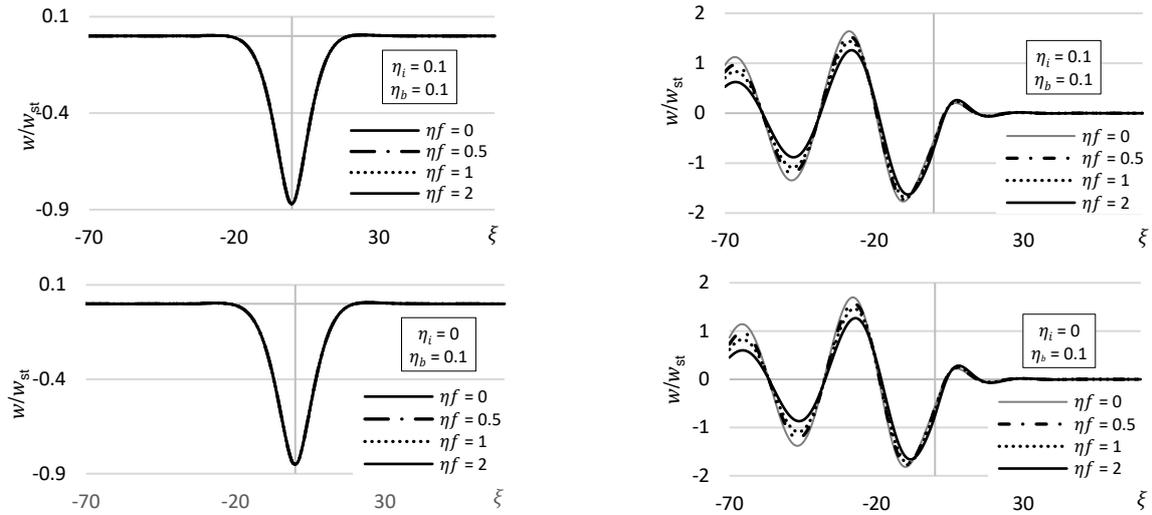


Figure 10. Dimensionless deflection shapes for two velocity ratio ($\alpha = 0.25, 0.55$) left and right column respectively, and damping values

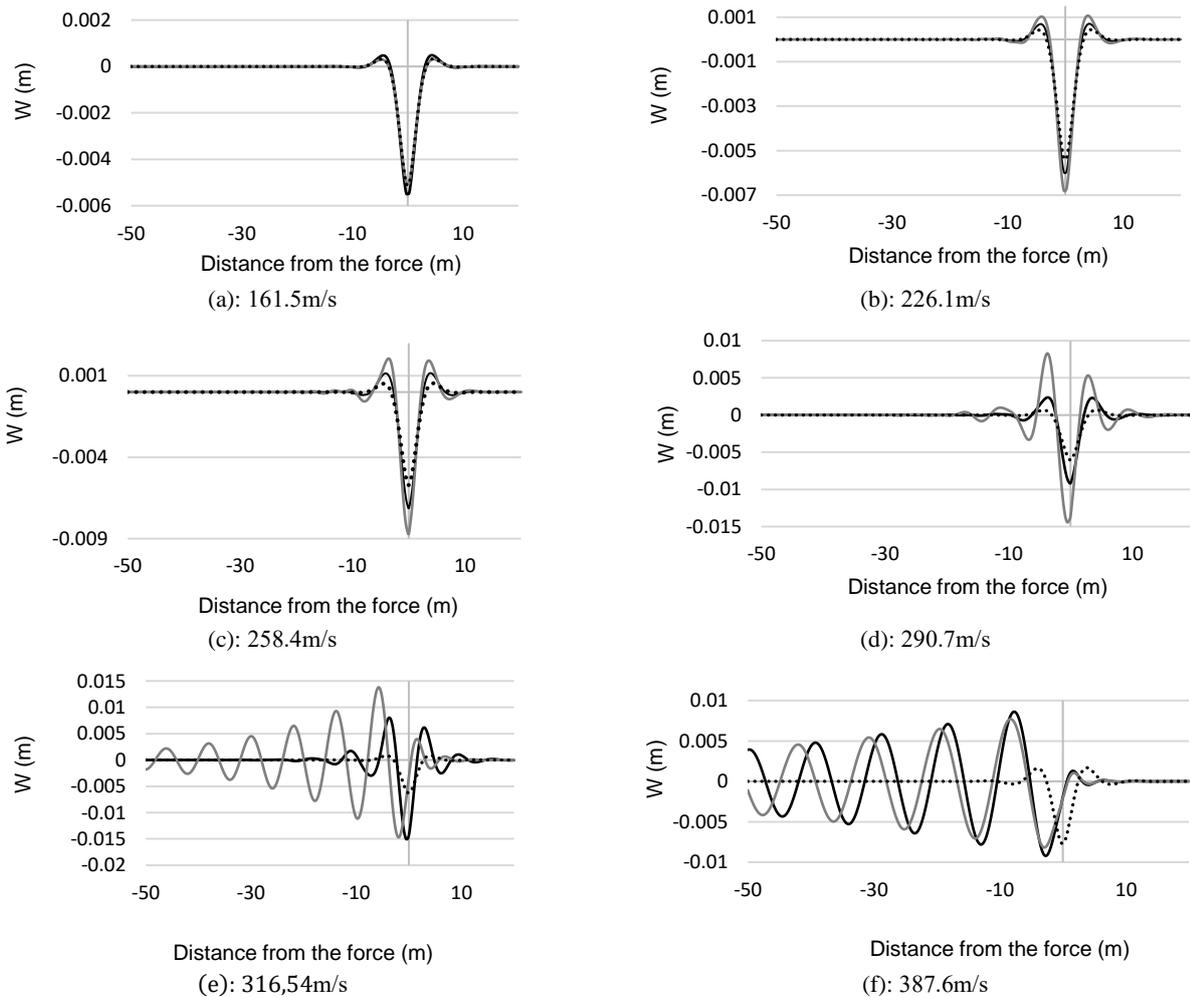


Figure 11. Deflection shapes for various velocities: classical Winkler's foundation (dotted line), solution with shear contribution (black line), solution without shear contribution (grey line)

5. CONCLUSION

In this paper, the Euler-Bernoulli beam was analyzed on the various depths of foundation under moving load and the displacement shapes for different values of active depth were compared. In the analysis, the effects of the foundation and beam damping have been incorporated. It was shown that the depth of soil is really effective on the dynamic behavior of the beam. By increasing depth of soil, the displacement of the beam is decreased. According to the obtained graphs, it was found that the occupied large area with superior displacement behind the load stands for the solution without shear contribution. That is because of the vibration of not interacted soil columns. The classical solution for Pasternak, Winkler and visco-elastic foundations provide very low displacement, because the applied velocity is approximately far from the critical one. The foundation influence has been illustrated to be very significant as long as it can decrease the critical velocity and also can intensify the response of the beam. It was shown, that without the shear contribution which strongly decreases the critical velocity. Also, the critical velocity depends on the mass ratio described as the square root of the fraction of the foundation mass to the beam mass. It was also shown, that by increasing the active depth of soil, the effect of foundation damping on the behavior of the beam is also increased. Then, the influence of damping for the velocity far from the critical velocity is not more significant, but by soaring of the velocity ratio, the influence of the damping on the dynamic response of the beam is increased.

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Sensitivity Analysis of Vibration Response of Railway Structures to Velocity of Moving Load and Various Depth of Elastic Foundation

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سازه‌های ریلی یکی از مهم‌ترین سازه‌ها در حمل و نقل می‌باشد. بنابراین، کمبود مطالعه دقیق رفتار دینامیکی آنها منجر به خسارات جبران ناپذیری می‌شود. فاکتورهای مهمی که برای تشخیص صحیح رفتار دینامیکی ریل‌ها کمک می‌کنند نوع بار و بستر به کار رفته در آنها است. در مطالعه حاضر، فرض شده است که تیر اولر-برنولی تحت اثر بار متحرک بر روی بستری با عمق محدود قرار گرفته است. با توجه به مشخصات تیرهای محدود، فقط تعادل دینامیکی در جهت قائم در نظر گرفته شده است. در این مقاله، با استفاده از معادلات تعادل و در نظر گرفتن اثر اندرکنش خاک و سازه، مسئله فیزیکی شبیه سازی شده و با استفاده از روش تبدیل فوریه معادلات دینامیکی حاکم تعیین شده است. سپس مدل ریاضی بر اساس مدل‌های پیشنهادی صحت سنجی و بسط داده شده است. با بررسی کارایی روش پیشنهادی، رفتار دینامیکی تیر تعیین شده است و نسبت تغییر مکان برای بسترهای مختلف ارائه شده است. آنالیز حساسیت برای مطالعه تأثیر پارامترهای مختلف مثل سرعت بار متحرک، عمق بستر الاستیک و میرایی به کار برده شده است. در نهایت، با در نظر گرفتن اثرات امواج برشی، سرعت بحرانی که به نسبت جرم بستگی دارد، و انواع مختلف میرایی، تغییر شکل‌های تیر برای سرعت‌های مختلف بار متحرک بدست آمده است، و اثرات عمق خاک بر روی رفتار دینامیکی تیر مورد نظر مورد بررسی قرار گرفته است. نشان داده شده است که اینرسی بستر منجر به کاهش چشمگیری در سرعت بحرانی شده و همچنین باعث تشدید پاسخ تیر مورد نظر شده است.

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