The Object Detection Efficiency in Synthetic Aperture Radar Systems

O. V. Chernoyarov\textsuperscript{a,b,c}, B. Dobrucky\textsuperscript{4}, V. A. Ivanov\textsuperscript{5}, A. N. Faulgaber\textsuperscript{c}

\textsuperscript{a}International Laboratory of Statistics of Stochastic Processes and Quantitative Finance, National Research Tomsk State University, Tomsk, Russia
\textsuperscript{b}Department of Higher Mathematics and System Analysis, Faculty of Electrical Engineering, Maikop State Technological University, Maikop, Russia
\textsuperscript{c}Department of Electronics and Nanoelectronics, Faculty of Electrical Engineering, National Research University "MPEI", Moscow, Russia
\textsuperscript{4}Department of Mechatronics and Electronics, Faculty of Electrical Engineering and Information Technology, University of Zilina, Slovak Republic
\textsuperscript{5}Department of Scientific Production Center "Design Engineering Bureau of Radio Engineering Devices and Systems", Faculty of Radio Engineering, National Research University "MPEI", Moscow, Russia

\begin{abstract}

The main purpose of this paper is to develop the method of characteristic functions for calculating the detection characteristics in the case of the object surrounded by rough surfaces. This method is to be implemented in synthetic aperture radar (SAR) systems using optimal resolution algorithms. By applying the specified technique, the expressions have been obtained for the false alarm and correct detection probabilities. In order to illustrate the effective application of the introduced approach in the case of the generic SAR system, the results are presented of the calculation of the detection characteristics of the signal from the extended object surrounded by a rough surface. It is shown that the analysis allows us to substantiate the structure of the SAR signal processing channel and to obtain the improved relations for the radio observation characteristics in this case. The efficiency of the optimal signal processing in SAR systems can also be determined without the approximate calculations involved.

\end{abstract}

1. INTRODUCTION

In various fields of physics and engineering, processing should be implemented of the information signals observed against random interferences and in the conditions of various prior uncertainty [1–3]. The use of synthetic aperture radar (SAR) [4–6] makes it possible to rapidly detect the signal that arrives from the object surrounded by rough surfaces. The task of this paper is to theoretically study the efficiency of the optimal procedure [7] of inter period signal processing in the SAR when a target is detected against both reflections from the underlying surface (correlated interference) and the inherent noise of the receiver. It is intended to develop an accurate method for calculating the detection characteristics based on the calculation of the kernel of the characteristic function [8] and determine the conditions for achieving the specified values for both the false-alarm and the correct detection probabilities of the signals produced by the extended object situated within the surface element. Such calculations involve determining the statistical characteristics of SAR output data, and, consequently, the evaluation of the method used for processing the received signals. The problem is reduced to the formation of statistics providing the total power reflected by the element of the signal with the subsequent comparison of the found value with a certain selected threshold. Unlike previous studies [9–13], we consider the detection of an object as based not on the results of the secondary processing of its radar image, but on the comparison of the statistics generated at the SAR output with the selected threshold. Thus, the speed of making a decision on whether or not an object is present greatly increases.

2. PROCESSING ALGORITHM

It is assumed that the processing of the received signal can be intra-period and inter period. Intraperiod
processing is a generally accepted procedure of the complex signal compression followed by the formation of the samples during the period of repetition and with the interval corresponding to the resolution on the surface.

The inter period processing is based on the technique [7], according to which, on the basis of the laws of the phase change of the signals reflected by the resolution element points by the azimuth and from period to period, the column vector of the received signal samples \( y_k \) is being formed over the entire observation interval (with the compensation of the effect of "migration of the distance"). A correlation matrix is formed for the vector \( y_k \) as:

\[
R_k = y_k y_k^T
\]

where symbol \( \cdot^T \) means transposition with complex conjugation.

Under certain conditions, the power reflected by the resolution element number \( k \) is determined by the quadratic form

\[
P_k = \hat{y}_k^2 P_k \hat{y}_k
\]

where \( P_k = R_k^r R_k \) presents the processing matrix.

The Relationship (2) reflects the final result of the received oscillation processing when the evaluation is involved in the intensity of the signal reflected by the surface element number \( k \). The specific type of matrix \( P_k \) depends on the model of a priori distribution of the reflected signal intensities \( \sigma^2 \).

Procedure (2) can be represented as a multichannel scheme that performs the transformation [14]:

\[
P_k = \sum_{m=1}^{M} \lambda_m \left| \sum_{i=1}^{M} v_m^* y_i \right|^2
\]

where \( \lambda_m \) and \( v_m \) are the eigenvalues and the eigenvectors of the matrix \( P_k \); \( M \) is the number of samples within the observation interval (the area of the correlation matrix is \( M \times M \)).

From the Equation (3) it follows that the optimal processing of the signal \( y \) includes: linear filtering with weight functions \( v_m \) (\( M \) channels), determining the power output of the linear filters and the subsequent summation of the results with weight coefficients \( \lambda_m \). The number of the considered values \( \lambda_m \) depends on the ratio between the azimuth resolution and the vector \( y \) length. It determines the number of non-coherent accumulations during the aperture synthesis, as each of the independent channels provides independent processing results.

The specific form of the processing matrix \( P_k \) depends on the accepted model of the prior intensity distribution \( \sigma_k \) of the signal reflected from the \( k \)th surface element. When using the model \( \sigma_k = \text{const.} \), the processing matrix \( P_k \) has the form \( P_k = R_k^r R_k \). The processing matrix \( P_k \) is presented in Figure 1.

The length of the resolution element along the flight path leads to the expansion of the Doppler signal spectrum \( \Delta f \). If, as a first approximation, the effect of range migration and the nonlinearity of the law of phase change of the reflected signal are not taken into account (it is presupposed that they are compensated), then the correlation matrix of inter period samples with the period \( T_r \) can be represented as:

\[
R_{pq} = \frac{\sin[\pi f_T (t_0 - p-q)]}{\pi \Delta f T_r} \exp[2\pi i f_T (t_0 - p-q)]
\]

Here \( f_T \) is the average Doppler frequency shift of the signal reflected by the \( k \)th surface element (within a fixed range band), \( p, q = 1...M \), while the total observation time is \( T = (M - 1)T_r \).

The number of the eigenvalues \( \mu_m \) of the matrix \( R_k \), which are substantially nonzero, is determined by the product \( \Delta f T \). If \( \Delta f T = 1 \), then the first four eigenvalues normalized to the maximum value are \( \mu_1 = 1.0, \mu_2 = 0.26, \mu_3 = 0.0145, \mu_4 = 0.00027 \). The eigenvalues \( \lambda_m \) of the matrix \( P_k \) are equal, \( \xi_m = \mu_m^2 \). The subsequent eigenvalues of the processing matrix become comparable with the first ones if the total signal observation time exceeds the effective synthesis interval (that is, \( \Delta f T > 1 \)). This is provided for obtaining the several independent looks.

Dividing the total observation time \( T \) into independent synthesis intervals (that is used in common processing systems [9–13, 15]) is substantiated with regard to a simpler processing implementation. However, in this case, multi-channel processing over the whole interval \( T \) is optimal such that it uses several weight functions \( v_m \) that correspond to the eigenvalues of the processing matrix \( P_k \) comparable to the maximum value.

In realistic observation conditions, the number of eigenvalues \( \lambda_m \) that are significantly different from zero is limited, and, thus, in this case, one may be confined to a small number of channels. Modern computing tools allow us to implement this processing, therefore, it can be used in promising SARs.
3. THE METHOD OF DETECTION CHARACTERISTICS CALCULATING

Two approaches are known to calculate the detection characteristics of inter period signal processing systems in SAR. They are based on the calculation of the probability density through the characteristic function. One of them (the trace method) uses the preliminary expansion of the probability density in an Edgeworth series or another expansion [8]. The second approach involves the calculation of the eigenvalues $\lambda_m$ of the determining matrix $W = PR$, where $P$ is the processing matrix, $R$ is the correlation matrix (1) of samples of the input signal $y$. Below the second approach is applied which allows us to obtain the closed expressions for calculating the detection characteristics.

The task is to find out the probability density function of the measurement results for the case of observations focused on a particular surface reflection element or on the mixture of this reflection and the object reflection. When the Neyman-Pearson criterion is applied, then the detection threshold is derived from the condition ensuring the specified false alarm probability $F$. Since the receiver's internal noise is present in all the cases, an appropriate selection of the probing signal power is required in order to achieve the desired correct detection probability $D$.

The probability density $w(p)$ of a random variable is conveniently determined in terms of its characteristic function $\theta(x)$. In this case, the probability density is associated with its relation

$$w(p) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \theta(x) \exp(-jpx) \, dx$$

(4)

For the random variable presented as a quadratic form

$$z = y^* P y$$

(5)
of the normal vectors $y$ with the specified correlation matrix $R_y$, the method for calculating the characteristic functions is introduced in [8]. In accordance with this technique, the result of the coherent processing of the samples of the input signal $y$ observed at the output of each of the channel is presented as a quadratic form of these samples

$$z = y^* Q^* Q y$$

(6)

where $Q$ is the matrix of linear processing.

If we assume that the samples of the total reflected signal $y$, when they are combined with a radiation interval $T_y$, are almost uncorrelated, then, in accordance with [7], the following equalities should be adopted: $Q = R_k, P_k = R_k^* R_k$.

The probability density function of the quadratic form (6) is determined by the characteristic function [8]

$$\theta(\xi) = \prod_{m=1}^{M}(1 - j\xi \lambda_m)^{-1}$$

(7)

where $\lambda_m$ are eigenvalues of the definition matrix $W = PR_y, R_y = yy^*$ is the covariance matrix of the input signal $y, M$ is the number of samples (the vector $y$ length).

Noncoherent summation of $N$ independent results of the linear analysis corresponds to the multiplication of the characteristic functions of the distributions of each of the channels:

$$\theta_N(\xi) = \prod_{n=1}^{N} \prod_{m=1}^{M}(1 - j\xi \lambda_{nm})^{-1}$$

(8)

here $\lambda_{nm}$ is an eigenvalue of the number $m$ of the matrix $W$ in the number $n$ channel.

The probability density function for the cumulative result $z_N = \sum_{n=1}^{N} z_i$ is determined by the relation

$$w(z_N) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \theta_N(\xi) \exp(-j\xi z_N) \, d\xi$$

(9)

If all the poles in (8) are simple, then, by introducing the relation

$$\prod_{n=1}^{N} \prod_{m=1}^{M}(1 - j\xi \lambda_{nm})^{-1} = \prod_{n=1}^{N} \prod_{i=1}^{K^+}(1 - j\lambda_i)$$

(10)

we can get

$$w(z_N) = \sum_{k=1}^{K^+} \frac{\exp(-z_N/\lambda_k)}{\lambda_k} \prod_{i=k}^{K^+} \left(1 - \frac{\lambda_i}{\lambda_k}\right)^{-1},$$

(11)

$z_N \geq 0$

where $K^+$ is the number of positive eigenvalues $\lambda_k$.

If there are multiple poles in (11), then the exact expression $w(z_N)$ can be presented as [16]:

$$w(z_N) = \sum_{k=1}^{K^+} \frac{1}{\lambda_k} \prod_{i=1}^{I_k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)^{-1} \lambda_k^{-1} \times$$

(12)

$$\times \exp(-z_N) \prod_{i=k}^{K^+} \left(1 - j\lambda_i \alpha_i \right)^{-p_i}, \quad z_N \geq 0$$

where $\alpha_k = (2l_k)\lambda_{k}^{-1}$; $I_k$ is the multiplicity of the number $\lambda_k$; $p_i$ is the multiplicity of number $\lambda_i$; $K_0^+$ is the number of different eigenvalues.

The probability of the value of $z_N$ exceeding the set threshold $z_0$ is

$$p(z_0) = \int_{z_0}^{\infty} w(z_N) \, dz_N$$

(13)

In the case of simple poles in (11), it is possible to obtain

$$p(z_0) = \sum_{k=1}^{K^+} \exp \left(-\frac{z_0}{\lambda_k}\right) \prod_{i=k}^{K^+} \left(1 - \frac{\lambda_i}{\lambda_k}\right)^{-1}$$

(14)

While in the presence of multiple poles [7] we get

$$p(z_0) = \sum_{k=1}^{K^+} \frac{1}{\lambda_k} \prod_{i=1}^{I_k} \left(1 - \frac{\lambda_i}{\lambda_k}\right)^{-1} \lambda_k^{-1} \times$$

(15)

$$\exp \left(-\frac{z_0}{\lambda_k}\right) \prod_{i=k}^{K^+} \left(1 - \frac{\lambda_i}{\lambda_k}\right)^{-p_i}.$$

In fact, the exact calculation of $p(z_0)$ according to the Formula (15) under a high multiplicity of the poles
requires cumbersome computations indeed. In this case, it is preferable to approximate calculations by the trace method [8] or by numerical integration of (9), (13), by means of MATLAB software [17].

4. Calculation of the False Alarm and Correct Detection Probabilities

The output result of the estimation of the power of the signal reflected from the surface element is a quadratic form (2) found at each band of the range, and it is equal to the range resolution in use. Therefore, the above technique is quite applicable.

The calculation sequence is as follows:

a) In case the object is absent
   - calculating the defining matrix \( W_0 = P_k(R_{sk} + P_kI) \), where \( R_{sk} \) is the matrix \( R_k \), normalized by the power of the signal received from one surface element having the area \( L \times L \) (\( k \) is the conditional element number); \( P_k = R_k^*R_k \) is the processing matrix; \( I \) is the unit matrix; \( P_n \) is the noise power;
   - calculating \( N \) eigenvalues \( \lambda_n \) of the matrix \( W_0 \);
   - calculating the characteristic function \( \theta_0(x) \) according to the Formula (8);
   - calculating the probability density \( w_0(p) \) according to the Formulas (11) and (12) or by numerical integration \( \theta_0(x) \) according to the formula (9);
   - calculating the probability \( p(z_0) \) according to the Formulas (14) and (15) or by numerical integration of \( w_0(p) \) according to the Formula (13), subsequently determining the threshold \( z_0 \), at which the required value of the false alarm probability \( F \) is achieved.

b) In case the object is present
   - calculating the definition matrix \( W_1 = P_k(R_{sk} + P_kI + R_k) \), where \( R_k \) is the correlation matrix of the object signal (depends on the area of the object, its orientation and position within the resolution element);
   - calculating \( N \) eigenvalues \( \lambda_n \) of the matrix \( W_1 \);
   - calculating the characteristic function \( \theta_1(x) \) according to the Formula (8);
   - calculating the probability density \( w_1(p) \) according to the Formulas (11) and (12) or by numerical integration of \( \theta_1(x) \) according to the Formula (9);
   - calculating the correct detection probability \( D \) according to the Formulas (14) and (15) or by numerical integration of \( w_1(p) \) according to the Formula (13).

The eigenvalues of the defining matrices \( W_0 \) and \( W_1 \) are shown in Figures 2(a) and (b).

5. Numerical Results

As an example, we now consider the case of the detection of the object having the area \( S_e = 28 m \times 98m \) and with a specific normalized radar cross-section \( \sigma_0 = -3 \, dB \). The area of the element surface where the object is located in \( S \), and in our case \( S = 253 \, m \times 252m \) For the specific normalized radar cross-section stands \( \sigma_p \), its value here is \( \sigma_p = -25 \, dB \).

The object is oriented along the velocity vector of the satellite, the orbital altitude is 500 km. Downrange for the middle section of the swath is designated as \( R \), and we take \( R = 800 \, km \). It is presupposed that the slip angle under irradiation \( \psi \) is equal to 39 deg.

Characteristics of SAR are: the first is wavelength \( \lambda \), here we take \( \lambda = 9 \, cm \); then the distance resolution that is 14 m; the number of radiation periods over the observation interval is \( M \), and its value here \( M = 375 \); antenna gain is designated as \( G \), and here we take \( G = 40 \, dB \); the pulse-compression ratio (that describes increasing SAR energy potential due to the chirped pulse compression) is \( B \), and \( B = 1000 \); the bandwidth and the noise temperature of the receiver are \( \Delta F_n \) and \( T \) correspondingly, and the values are: \( \Delta F_n = 22 \, MHz \) and \( T = 300 \, K \); for pulse power stands \( P_p \), and here \( P_p = 18 \, W \).
Processing of the surface band having the area $S_1$, its value being $S_1 = 252 \text{m} \times 14 \text{m}$, is carried out according to the algorithm (3) with the subsequent summation of 18 bands. Processing of the object band having the area $S_{11}$, its value being $S_{11} = 14 \text{m} \times 98 \text{m}$, is carried out according to the algorithm (3) with the subsequent summation of 2 bands.

The power of the signal that is reflected from the surface of the band with the area $S_1 = 252 \text{m} \times 14 \text{m}$ while probing this surface by the single pulse with the power $P_r$, is, as in [15, 16]: $P_{rs} = P_p B G^2 \lambda^2 S_1 \sigma_p \sin \psi / (4 \pi)^3 R^4$.

The formula for calculating the power of the signal reflected from the object surface with the area $S_{11} = 14 \text{m} \times 98 \text{m}$ takes the form of $P_{rt} = P_p B G^2 \lambda^2 S_{11} \sigma_0 / (4 \pi)^3 R^4$.

The power of the reduced noise is $P_n = kT \Delta F_n$, where $k = 1.38 \cdot 10^{-23} \text{J/K}$ is the Boltzmann constant.

In that example we get: $P_{rs} \approx 0.4 \cdot 10^{-15} \text{W}$, $P_{rt} \approx 20 \cdot 10^{-15} \text{W}$, $P_n \approx 90 \cdot 10^{-15} \text{W}$.

The absolute values of the characteristic functions $\theta_0(x)$ and $\theta_1(x)$ are plotted in Figures 3(a) and (b), and the probability densities corresponding to them — $w_0(p)$ and $w_1(p)$ — are drawn in Figures 4(a) and (b).

Integration of the probability densities $w_0(p)$ and $w_1(p)$ in (13) shows that under the specified pulse power ($P_p = 18 \text{W}$), the false alarm and correct detection probabilities are $F = 10^{-7}$ and $D = 0.96$, respectively.

6. CONCLUSION

We have considered the method for the analysis of the detection characteristics of the inter period systems for processing the SAR signals reflected from the object surrounded with a rough surface, focusing on the class of the algorithms allowing presenting the output result of processing the sequence of the normal random samples of the additive mix of the clutter and the useful signal in a quadratic form. The method is based on the calculation of the kernel of the characteristic function and allows determining the efficiency of the detection of the objects by the SAR system without involving the approximate methods of calculation. The given example of the calculation of the SAR system detection characteristics presents optimal processing of the reflected signals and illustrates the possibilities that the considered method provides. The introduced technique for determining the SAR target detection efficiency can be used while
calculating other processing systems that are reduced to a quadratic form.

7. ACKNOWLEDGMENT

This study (Sections 1-4) was financially supported by the Russian Science Foundation (research project No. 14-49-00079). The numerical results (Section 5) were obtained with support from the Ministry of Education and Science of the Russian Federation (research project No. 2.3208.2017/4.6).

8. REFERENCES


The Object Detection Efficiency in Synthetic Aperture Radar Systems

O. V. Chernoyarov a,b,c, B. Dobrucky d, V. A. Ivanov e, A. N. Faulgaber c

a International Laboratory of Statistics of Stochastic Processes and Quantitative Finance, National Research Tomsk State University, Tomsk, Russia
b Department of Higher Mathematics and System Analysis, Faculty of Electrical Engineering, Maikop State Technological University, Maikop, Russia
c Department of Electronics and Nanoelectronics, Faculty of Electrical Engineering, National Research University “MPEI”, Moscow, Russia
d Department of Mechatronics and Electronics, Faculty of Electrical Engineering and Information Technology, University of Zilina, Slovak Republic
e Department of Scientific Production Center “Design Engineering Bureau of Radio Engineering Devices and Systems”, Faculty of Radio Engineering, National Research University “MPEI”, Moscow, Russia

Paper Info

Paper history:
Received 07 May 2019
Received in revised form 19 December 2019
Accepted 16 January 2020

Keywords:
Characteristic Function
Detection Characteristics
Extended Object
Probability Density
Synthetic Aperture Radar