



## Output-only Modal Analysis of a Beam Via Frequency Domain Decomposition Method Using Noisy Data

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### PAPER INFO

#### Paper history:

Received 04 March 2019

Received in revised form 23 September 2019

Accepted 07 November 2019

#### Keywords:

Frequency Domain Decomposition

Modal Parameters

Noise Level

Output-only Modal Analysis

Simply Supported Beam

### ABSTRACT

The output data from a structure is the building block for output-only modal analysis. The structure response in the output data, however, is usually contaminated with noise. Naturally, the success of output-only methods in determining the modal parameters of a structure depends on noise level. In this paper, the possibility and accuracy of identifying the modal parameters of a simply supported beam in the presence of noise has been discussed. The output-only modal analysis method with frequency domain decomposition was used and output data with various noise levels were considered. Initially, finite element modal analysis was used to determine the modal parameters for the beam which were afterwards enforced as the reference modal parameters. Then, appropriate input was applied to the beam and the acceleration signals of different nodes were produced through finite element transient analysis. In order to simulate noisy data, noises with different power levels were generated and added to the signals. Finally, the modal parameters were obtained by frequency domain decomposition method. The results showed that the modal parameters corresponding to the first vibration mode could only be identified with acceptable validity at low to moderate noise levels, whereas for higher modes, the modal parameters can be correctly obtained even at high noise levels.

doi: 10.5829/ije.2019.32.12c.08

## 1. INTRODUCTION

Modal parameters of structures, such as natural frequencies, damping ratios and mode shapes, have been used vastly in the past for numerous civil engineering applications. In order to obtain these parameters experimentally, there are two groups of modal analysis methods: input-output methods and output-only methods. Output-only modal analysis methods are mostly used due to their numerous advantages in comparison with other methods [1, 2]. One of the most common methods of output-only modal analysis is Frequency Domain Decomposition (FDD) which was proposed by Brincker [3]. In FDD method, matrices of Power Spectral Density (PSD) are formed with output data, after that the mode shapes and natural frequencies are extracted via Singular Value Decomposition (SVD) [4-6].

The measured output data, however, contains some noise [7]. Noise level in output data of a structure must be adequately low so that the output-only methods can identify the modal parameters. To keep noise at adequately low level, it is usually attempted to reduce noise power. For this purpose, suitable measurement techniques must be used to minimize the noise level [8, 9]. After a clean data acquisition, techniques such as filtering and averaging are used to reduce noise effects [10-12]. Bonness and Jenkins [13] presented a noise elimination technique wherein it is possible to remove a limitless number of undesirable correlated noise from a collection of experimental data by modifying the statistical correlation relations and spectral functions. Focusing on the study of the Gaussian noise elimination, Feng and Lin [14] introduced numerous noise removal procedures based on wavelet thresholding including global, Maxmin and BayesShrink. Khoshnood et al. [15]

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employed a dynamic noise remover on the basis of the online wavelet concept for ignoring unwanted noise effects.

In spite of all these efforts, because of uncertainties in quality and quantity of noise in reality, it is not possible to entirely eliminate noise [9, 16]. Therefore, output-only methods rely on contaminated data with unknown noise level to identify modal parameters of structures. Although the effect of low levels of noise (up to 60%) were investigated in previous work [17, 18]. There was no study -to the best of our knowledge- in finding the maximum tolerable noise level; or equivalently minimum required value of signal to noise ratio (SNR) to identify the modal parameters via output-only methods. In this research, we will undertake this task. Moreover, we will evaluate the performance of the output-only method of FDD for various SNRs. For this purpose, a simply supported beam was chosen. An appropriate load was applied on the beam and acceleration signals of the beam were obtained using dynamic transient analysis. To generate noisy data, noises with different powers than signal power were added to the signals. Using these noisy data, the modal parameters of beams were obtained at various noise levels using FDD method. These parameters were compared with reference modal parameters of the beam that had been obtained by finite element modal analysis.

## 2. FREQUENCY DOMAIN DECOMPOSITION METHOD

Among the nonparametric methods in the frequency domain, FDD has been used widely in output-only modal analysis because of its simplicity, dependability, and efficiency [3, 19]. However, FDD always requires the prior-selected natural frequencies as well as its strict hypotheses of uncorrelated white noise excitations and low structural damping. Output PSD matrix can be decomposed via fast-decayed decomposition methods such as SVD. This way, the spectral density reveals contributions of various modes of the structure. By analyzing the singular values, the auto PSD functions conforming to individual modes of a structure can be identified. In this method, the approximated mode shapes are obtained as the singular vectors at maximum location of any auto PSD function conforming to a mode. In what follows, the theoretical basis of the FDD method is summarized [20].

The pole-residue formula of the PSD matrix of output can be written as:

$$[G_{yy}(\omega)] = \sum_{k=1}^r \left( \frac{[A_k]}{j\omega - \lambda_k} + \frac{[A_k]^*}{j\omega - \lambda_k^*} + \frac{[B_k]}{-j\omega - \lambda_k} + \frac{[B_k]^*}{-j\omega - \lambda_k^*} \right) \quad (1)$$

where  $[G_{yy}(\omega)]$  is the output PSD square matrix of dimension  $n$  in where  $n$  is the number of output data;  $r$ ,

$j$  and  $\omega$  are number of desired modes, imaginary unit and natural frequency, respectively.  $[A_k]$  and  $\lambda_k$  are the  $k^{\text{th}}$  residue output PSD matrix which is an  $n \times n$  Hermitian matrix and pole of the  $k^{\text{th}}$  mode, respectively (Superscript \* defines complex conjugate).  $[A_k]$  and  $\lambda_k$  are given by:

$$[A_k] = [R_k][C] \sum_{s=1}^r \left( \frac{[R_s]^H}{-\lambda_k - \lambda_s^*} + \frac{[R_s]^T}{-\lambda_k - \lambda_s} \right) \quad (2)$$

$$\lambda_k = -\sigma_k + j\omega_{dk} \quad (3)$$

where  $[R_k]$  is the residue of  $k^{\text{th}}$  mode and equals to product of modal participation vector  $\{\gamma_k\}$  and that of mode shape  $\{\phi_k\}$ .  $[C]$  is PSD matrix of input which is constant;  $\omega_{dk}$  and  $\sigma_k$  are natural frequency corresponding to the  $k^{\text{th}}$  mode of damped system and the modal damping ratio, respectively (Superscripts  $H$  and  $T$  denote complex conjugate and transpose, respectively). Considering only the  $k^{\text{th}}$  mode, Equation (2) turns into:

$$[A_k] = \frac{[R_k][C][R_k]^H}{2\sigma_k} \quad (4)$$

If the modal damping ratio is low, Equation (4) is dominant and the residue proportional to the mode shape vector can be achieved by the following equation:

$$[A_k] \propto [R_k][C][R_k]^H = \{z_k\{\phi_k\}\{\gamma_k\}^T [C] \{\gamma_k\}\{\phi_k\}^T = z_k \{\phi_k\}\{\phi_k\}^T \quad (5)$$

where  $z_k$  is the  $k^{\text{th}}$  mode scaling factor. For a system with low damping in which at a certain frequency, finite number of modes participate in the structure response, the output PSD matrix can be stated as:

$$[G_{yy}(\omega)] = \sum_{k \in \text{Sub}(\omega)} \left( \frac{z_k \{\phi_k\}\{\phi_k\}^T}{j\omega - \lambda_k} + \frac{z_k^* \{\phi_k\}^* \{\phi_k\}^{*T}}{j\omega - \lambda_k^*} \right) \quad (6)$$

where  $k \in \text{Sub}(\omega)$  is the collection of participating modes at desired natural frequency. The singular value decomposition of the PSD matrix of output at natural frequency  $\omega = \omega_i$  leads the following equation:

$$[\hat{G}_{yy}(j\omega_i)] = [V]_i [N]_i [V]_i^H \quad (7)$$

In Equation (7) the matrices  $[V]_i$  and  $[N]_i$  are unitary matrix containing the singular vector  $\{v_{ij}\}$  and diagonal matrix of singular values  $n_{ij}$ , respectively. Each mode corresponds to a local maximum in the spectrum. When there is only one dominant mode, one term in Equation (6) remains, and the power spectral density matrix approximates to a matrix with rank equal to one:

$$\hat{G}_{yy}(j\omega_i) = s_i \{v_{i1}\}\{v_{i1}\}^H \omega_i \rightarrow \omega_k \quad (8)$$

Thus, the singular vector related to the first mode  $\{v_{i1}\}$  gives a good estimate of mode shape as follows:

$$\{\hat{\phi}\} = \{v_{i1}\} \quad (9)$$

In addition, the associated singular value is related to the auto PSD function of the SDOF system of the desired mode. Comparing the approximated mode shape  $\{\hat{\phi}\}$

with the singular vectors related to the frequency lines near the maximum, the auto PSD function of the equivalent SDOF system is recognized near the maximum of the singular value diagram. Every line distinguished by a singular vector which offers a Modal Assurance Criterion (MAC) value greater than a suitable MAC rejection level, is related to the PSD function of SDOF system. The corresponding PSD function of SDOF system is utilized to obtain estimates of the damping ratios and natural frequencies [21].

### 3. MATHEMATICAL MODEL OF NOISE

Output-only modal analysis is performed using only the measured response of structure ( $NR(t)$ ). This response contains the structure response ( $R(t)$ ) and the output noise ( $N(t)$ ). The output noise consists of any undesired signals and physically may arise from measurement devices, sensors, etc. [22]. The simplest mathematical model for noise in data is the additive noise model [23]:

$$NR(t) = R(t) + N(t) \quad (10)$$

In this model, the output noise is demonstrated as a white Gaussian process with zero average [23] as shown in Equation (11) [25, 26]:

$$N(t) = RMS(R(t)) \times NL \times W(t) \quad (11)$$

where  $RMS(R(t))$  is the root mean square of signals,  $W(t)$  is a random function with the same dimension as  $R(t)$ , and  $NL$  is a constant that determines the noise level. The noise level is defined as the ratio of the root mean square (RMS) of the noise process to the RMS of the noise-free data. Noise level can also be represented by Signal to Noise Ratio (SNR):

$$SNR = \frac{1}{(NL)^2} \quad (12)$$

### 4. FINITE ELEMENT MODELING

As shown in Figure 1, a beam with simply supported condition was considered to study the noise level effect on output-only modal analysis. Finite element modeling was done in ANSYS<sup>1</sup> software using BEAM 188 element which has six degrees of freedom at any nodes. A beam with a square cross section of 10 mm × 10 mm was modeled in 2-D space with 10 elements, each 100 mm long. The properties of material, namely elastic Young modulus, Poisson's ratio and mass density of the beam were considered  $2 \times 10^{11}$  N/m<sup>2</sup>, 0.3 and 7850 (N.s<sup>2</sup>/m)/m<sup>3</sup>, respectively according to typical structural steel. Geometrical properties of the beam were selected so that the vibration modes of the beam were sufficiently separated.

**4. 1. Finite Element Modal Analysis** The beam's modal parameters were obtained by finite element modal analysis and were considered as its reference modal parameters. In this type of analysis, the following Eigenvalue problem is solved [27]:

$$[K - \omega^2 M]\{\varphi\} = 0 \quad (13)$$

In Equation (13) matrices  $K$  and  $M$  are stiffness and mass, respectively;  $\omega$  and  $\varphi$  are Eigen-frequency and Eigen-vector of structure, respectively. The reference modal parameters (natural frequencies and mode shapes) of the beam for the modes from 1<sup>st</sup> to 5<sup>th</sup> were calculated and have been demonstrated in Tables 1 and 2.

**4. 2. Applying Load and Transient Analysis** To obtain the modal parameters corresponding to the first to fifth modes of this beam using FDD method, the frequency content of the applied load should include all frequencies of these modes. For this purpose, the load shown in Figure 2(a) is considered in which the load begins from zero at the time  $t = 0$  and linearly reaches 0.3 N at 0.0006 s. It then returns to zero at 0.0012 s and then stays at zero until  $t = 1$  s. Fast Fourier Transform (FFT) of the load can be observed in Figure 2(b) where due to symmetry, only one-half is displayed. In order to assure excitation of the first five modes of the beam, the first frequency with zero FFT value must be greater than the values of natural frequency of the fifth mode. Since this frequency is reversely proportional to time step in transient analysis, the time step should be sufficiently small. Time step of 0.0001 s was considered that corresponds to first zero FFT frequency of 1667 Hz, which is greater than the natural frequency related to the fifth mode of this beam, i.e. 773.8 Hz (Table 1).

In addition to appropriate frequency content for the applied load, the load position must not be located on the nodal points of the considered modes [28]. As can be observed in Table 2, appropriate locations to excite are nodes 2, 4, 8 and 10. The load was applied to this beam on node 4 in vertical direction. Through standard transient analysis in ANSYS, the vertical acceleration of nodes 2 to 10 were obtained.



Figure 1. Finite element model of beam with node numbers

TABLE 1. Natural frequencies of the beam extracted by modal analysis of finite element model

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
Natural frequency (Hz)	23.2	95.9	228.7	442.5	773.8

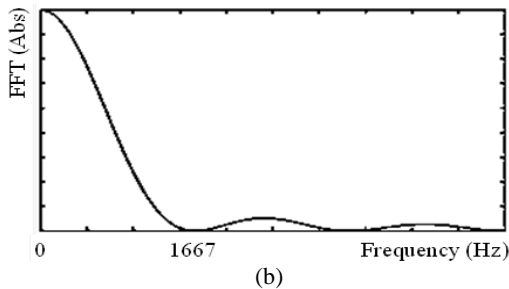
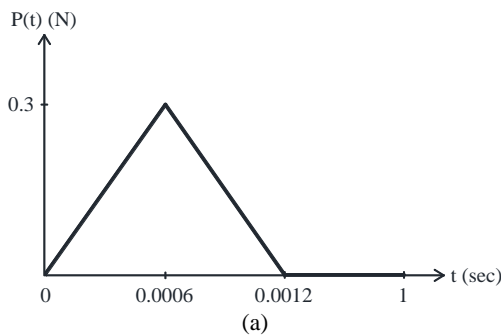
<sup>1</sup> www.ansys.com

**TABLE 2.** Mode shape values of the beam obtained by finite element modal analysis

	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
1	0.000	0.000	0.000	0.000	0.000
2	0.309	0.618	-0.809	-1.000	-1.000
3	0.588	1.000	-0.951	-0.618	0.000
4	0.809	1.000	-0.309	0.618	1.000
5	0.951	0.618	0.588	1.000	0.000
6	1.000	0.000	1.000	0.000	-1.000
7	0.951	-0.618	0.588	-1.000	0.000
8	0.809	-1.000	-0.309	-0.618	1.000
9	0.588	-1.000	-0.951	0.618	0.000
10	0.309	-0.618	-0.809	1.000	-1.000
11	0.000	0.000	0.000	0.000	0.000

Mode shape drawing					
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**Figure 2.** (a) Excitation function and (b) its FFT (Not to scale)

**5. OUTPUT-ONLY MODAL ANALYSIS**

As mentioned in Section 4, the acceleration signals ( $R(t)$ ) were obtained from the transient analysis of the beam. These signals are free of noise (zero noise level). Using these signals, the noise  $N(t)$  was generated with different noise levels ( $NL$ ) according to Equation (11).

By adding  $N(t)$  to  $R(t)$  according to Equation (10), the noisy data ( $NR(t)$ ) was generated which was used for output-only modal analysis by FDD method. The considered noise levels and the corresponding signal to noise ratios are shown in Table 3. Practically, the amount of noise in recorded response affects the accuracy of the estimated modal parameters [29, 30].

In the noise generation process, each ensemble is statistically independent from others. Moreover, in order to capture the effect of randomness and average out the results, the process should be repeated [24]. At each noise level, 100 noise ensembles  $N(t)$  and consequently 100 noisy data  $NR(t)$  were generated. Output-only modal analysis by FDD method was performed using each of the  $NR(t)$ s, and the average of the results obtained for each noise level is presented in the following. The process of noise generation, noisy data generation and Output-only modal analysis by FDD method were all performed in MATLAB<sup>1</sup>.

**5.1. Natural Frequencies**

Table 4 presents natural frequencies of the beam predicted using FDD method for different noise levels from  $NL=0$  (noise free case) to  $NL=200\%$ . The prediction error with respect to reference frequencies (Table 1) are shown in the last row of Table 4. As can be seen, FDD method predicted most natural frequencies of the beam accurately. Maximum error is -5.2% which belongs to mode 1. Moreover, natural frequency of mode 1 could not be identified at high noise levels; i.e.  $NL=100\%$  and  $NL=200\%$ . The error of most of the estimated natural frequencies is negative, which shows that they are underestimated.

As shown in Table 4, by increasing the noise level, the values of the natural frequencies identified using FDD method do not change. The results corroborate with those reported by Gkoktsi et al. [31]. In the frequency domain methods such as FDD, the natural frequency of a structure is estimated from the peak values of PSD

**TABLE 3.** Noise levels and corresponding signal to noise ratios

Noise Level (NL)	5%	10%	20%	50%	75%	100%	200%
Nominal Signal to Noise Ratio (SNR)	400	100	25	4	1.78	1	0.25

**TABLE 4.** Natural frequencies (Hz) of the beam estimated via FDD method

Noise Level (%)	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
0, 5,10,20,50,75	22.0	95.2	229.5	439.5	759.3
100, 200	-	95.2	229.5	439.5	759.3
Error (%)	-5.2	-0.7	+0.3	-0.7	-1.9

<sup>1</sup> [www.mathworks.com/products/matlab.html](http://www.mathworks.com/products/matlab.html)

diagram. Despite the fact that different noise levels lead to different PSD diagrams, the peaks location or the values of natural frequencies of the structure do not change. This phenomenon is shown in Figure 3.

**5. 2. Mode Shapes** The first five mode shapes of the beam were obtained by FDD at various noise levels. For instance, mode shape values of the beam obtained at noise level 5% are shown in Table 5. In order to check accuracy of the obtained mode shapes, they will be compared with reference mode shapes obtained by finite element modal analysis (Table 2).

For the purpose of comparison, the MAC criterion is utilized. The MAC as a statistical index, shows a high sensitivity to large variations between two mode shape vectors, but it has relatively no sensitivity to small variations. This statistic indicator produces a good measure and degree of compatibility between two mode shapes. The MAC formulation is [32]:

$$MAC(r, q) = \frac{|{\varphi_A}_r^T {\varphi_X}_q|^2}{({\varphi_A}_r^T {\varphi_A}_r)({\varphi_X}_q^T {\varphi_X}_q)} \quad (14)$$

where  ${\varphi_A}_r$  is mode shape number  $r$  from source  $A$ ,  ${\varphi_X}_q$  is mode shape number  $q$  from source  $X$ , and the superscript  $T$  denotes transpose. The MAC values vary between zero and one; the one represents full compliance of two mode shapes [33].

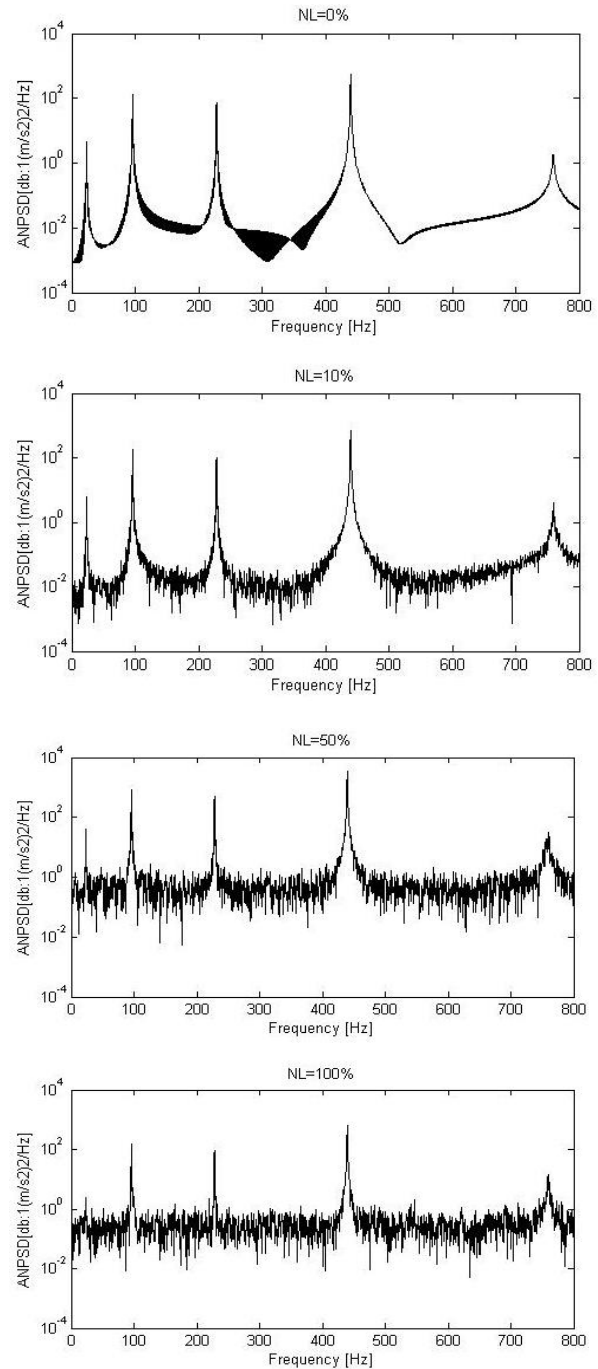
Using Equation (14), the values of MAC between the five mode shapes of the beam obtained by FDD method and five reference mode shapes were calculated at various noise levels. For instance, these MAC values at noise level of 5% are presented in Figure 4. (At noise level of 5%, mode shape values in Table 5 and Table 2 were used for calculation of MAC values.) This figure shows that the values of MAC between the same mode shape numbers are equal to one and between different

**TABLE 5.** Mode shape values of the beam obtained by FDD method at noise level of 5%

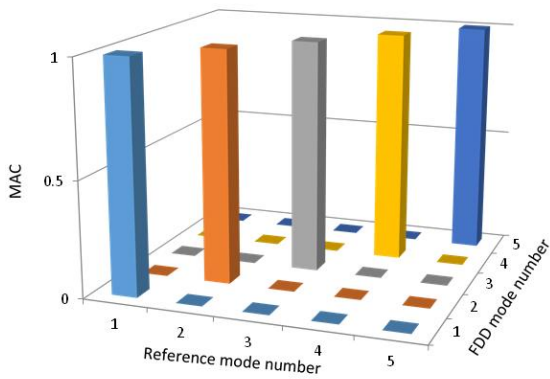
	Mode 1	Mode 2	Mode 3	Mode 4	Mode 5
1	0.000	0.000	0.000	0.000	0.000
2	0.312	0.620	0.813	1.000	0.999
3	0.585	0.998	0.950	0.618	-0.001
4	0.819	1.000	0.311	-0.618	-0.996
5	0.956	0.618	-0.588	-0.999	-0.001
6	1.000	0.003	-1.000	-0.001	1.000
7	0.960	-0.619	-0.586	0.999	-0.001
8	0.823	-0.998	0.307	0.618	-0.999
9	0.593	-1.000	0.951	-0.618	0.000
10	0.311	-0.618	0.809	-0.999	0.998
11	0.000	0.000	0.000	0.000	0.000

mode shape numbers are equal to zero. This means that the FDD uniquely identified all of the five mode shapes and there is no correlation between the mode shapes with different mode number. This result was also obtained for other noise levels.

Table 6 shows the values of MAC between the mode shapes obtained by FDD and the corresponding reference mode shapes at various noise levels (same mode



**Figure 3.** The Average Normalized PSD diagram of the beam for various noise levels



**Figure 4.** MAC values between all FDD mode shapes (at noise level of 5%) and all FEM mode shapes

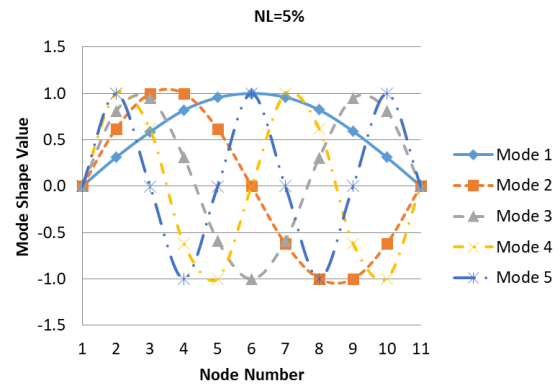
number). The MAC values equal or close to one indicate that the FDD method appropriately extracted five mode shape vectors of the beam in the presence of noise. As shown in the table, as the noise level increases, the MAC values either decrease or remain constant. The lower values of the MAC for the first mode shape of the beam show that this mode of the beam was not extracted flawlessly in high noise levels. However, for the second to fifth mode shapes, the MAC value remains equal to one even at high noise levels which shows that these mode shapes were extracted with high accuracy.

The identified mode shapes of this beam via FDD method for the various noise levels have been drawn in Figures 5 to 8. As can be observed in Figures 5 and 6, that correspond to noise levels of 5% and 20% respectively, all of the five mode shapes were identified almost accurately. According to Table 5, the MAC values for these mode shapes at the same noise levels are equal to one. At noise levels of 75% and 200% in Figures 7 and 8, respectively, the second to fifth mode shapes are nearly perfect, but the first mode shape is clearly distorted. In Table 5, the values of MAC of second to fifth mode shape for all of the considered noise levels are equal to one. However, for the first mode shape, the MAC value decreases from 1.00 at noise level of 0, 5, 10 and 20% to 0.95 at noise level of 200%. Therefore, for the case being studied, MAC equal to or less than 0.99 means that the obtained mode shape is not reliable.

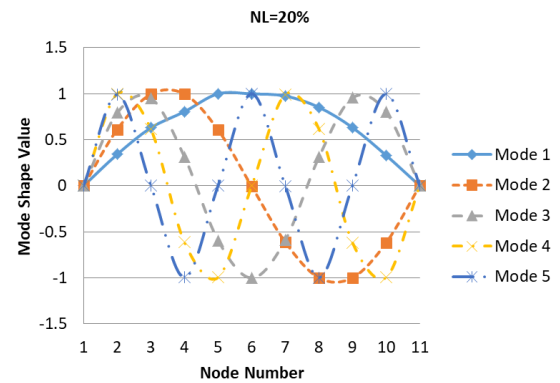
**TABLE 6.** values of MAC between FDD mode shapes and corresponding FEM mode shapes

Noise Level (%)	Mode 1	Modes 2 to 5
0, 5,10,20	1.00	1.00
50, 75	0.99	1.00
100	0.97	1.00
200	0.95	1.00

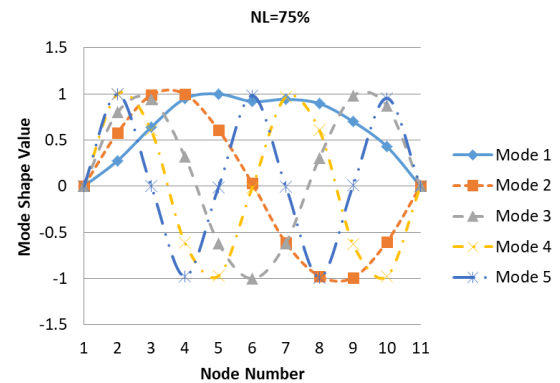
As can be seen in Tables 4 and 6, the natural frequency of the first mode was obtained up to noise level of 75%, but its mode shape was only identified with good accuracy up to noise level of 20%. Therefore, identification of natural frequency of a mode does not necessarily guarantee that its mode shape will be identified correctly by FDD method.



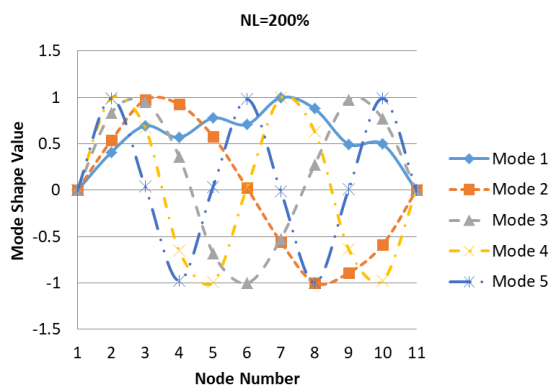
**Figure 5.** The beam's mode shapes obtained via FDD at noise level of 5%



**Figure 6.** The beam's mode shapes obtained via FDD at noise level of 20%



**Figure 7.** The beam's mode shapes obtained via FDD at noise level of 75%



**Figure 8.** The beam's mode shapes obtained via FDD at noise level of 200%

## 6. CONCLUSION

In the present work, a method of output-only modal analysis, namely FDD method, was utilized to identify modal parameters of a beam under noisy conditions. A finite element model was prepared for a simply supported beam. Applying a suitable dynamic load to the beam, the acceleration signals of the various points on the beam were obtained by transient analysis. Noise signals with different powers were added to the acceleration signals. Using these noisy acceleration signals, modal parameters of the beam were identified by FDD method. These identified modal parameters were then compared to reference ones that were obtained by finite element modal analysis method.

In noise free case, the FDD method could identify this beam's mode shapes and natural frequencies. The beam's natural frequency corresponding to the first mode was identified with more error in comparison to other modes. MAC values are equal to one in this case, which shows very good agreement between the identified mode shapes and the corresponding reference.

In the case of noisy signals, the beam's natural frequencies of all modes were obtained at all noise levels, except for the first natural frequency which was not identified for noise levels 100 and 200%. The first natural frequency could not be identified at very high noise levels. The natural frequencies of the beam, which were identified, show no change in comparison to the noise free case. The FDD was a powerful method for estimation of the beam's natural frequencies, because it identified the natural frequencies at the reasonably high noise levels. The inspection of the mode shapes obtained using FDD method under noisy conditions showed that all of the mode shapes were in very good agreement with reference mode shapes up to noise level of 20%. Up to this noise level, the calculated MAC values for all of the mode shapes are equal to one. For noise levels greater than 20%, accuracy of the first mode shape decreases

(MAC=0.99~0.95), but other mode shapes remain accurate (MAC=1.00). Therefore, the FDD method is more robust for prediction of natural frequencies in comparison to mode shapes. The amount of noise influenced properties of the first mode which is usually the most important vibration mode of a structure in practice. Because noise level is unknown in practice, the higher mode shapes identified by the FDD method are more reliable in comparison with the first mode shape.

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# Output-only Modal Analysis of a Beam Via Frequency Domain Decomposition Method Using Noisy Data

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## P A P E R I N F O

## چکیده

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### Paper history:

Received 04 March 2019

Received in revised form 23 September 2019

Accepted 07 November 2019

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### Keywords:

Frequency Domain Decomposition

Modal Parameters

Noise Level

Output-only Modal Analysis

Simply Supported Beam

آنالیز مودال خروجی-تنها با استفاده از داده‌های خروجی یک سازه انجام می‌شود. این داده‌ها معمولاً شامل پاسخ سازه به همراه مقداری نویز هستند. موفقیت روش‌های خروجی-تنها در تعیین پارامترهای مودال سازه بستگی به میزان نویز در داده‌های خروجی دارد. در این مقاله، امکان تعیین پارامترهای مودال یک تیر دو سر ساده و نیز دقت آنها تحت تأثیر وجود نویز مورد بررسی قرار گرفته است. از روش آنالیز مودال خروجی-تنهای تجزیه حوزه فرکانس استفاده شد و داده‌های خروجی با میزان نویز مختلف لحاظ گردید. در ابتدا پارامترهای مودال تیر از طریق آنالیز مودال اجزای محدود تعیین شده و به عنوان پارامترهای مودال مرجع مورد استفاده قرار گرفت. سپس ورودی مناسبی به تیر اعمال شد و سیگنال‌های شتاب در گره‌های مختلف از طریق تحلیل تاریخچه زمانی اجزای محدود بدست آمد. برای تولید داده‌های نویزی، نویزهای با توان مختلف تولید و به سیگنال‌ها اضافه شد. در نهایت پارامترهای مودال تیر از طریق روش تجزیه حوزه فرکانس محاسبه گردید. نتایج حاصله نشان دادند پارامترهای مودال اولین مود ارتعاشی، تنها در میزان نویز پایین تا متوسط با دقت قابل قبول شناسایی می‌شود، درحالی‌که برای مودهای بالاتر، پارامترهای مودال حتی در میزان نویز بالا به طور صحیح قابل حصول هستند.

**doi:** 10.5829/ije.2019.32.12c.08

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