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A Comprehensive Mathematical Model for a Location-routing-inventory Problem under Uncertain Demand: a Numerical Illustration in Cash-in-transit Sector

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ABSTRACT

The purpose of this article is to model and solve an integrated location, routing and inventory problem (LRIP) in cash-in-transit (CIT) sector. In real operation of cash transportation, to decrease total cost and to reduce risk of robbery of such high-value commodity. There must be substantial variation, making problem difficult to formulate. In this paper, to better fit real life applications and to make the problem more practical, a bi-objective multiple periods, capacitated facilities with time windows under uncertain demand (BO-PCLRIP-TW-FD) in the LRIP, motivated by the replenishment of automated teller machines, is proposed. Then, using the chance constrained fuzzy programming to deal with uncertain parameters, the comprehensive model is formulated as a crisp mixed-integer linear programming. At last, to validate the mathematical formulation and to solve the problem, the latest version of ε-constraint method (i.e., AUGMECON2) is used. The proposed solution approach is tested on a realistic instance in CIT sector. Numerical results demonstrate the suitability of the model and the formulation. The ability of the model to be useful references for security carriers in real-world cases.

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NOMENCLATURE

Sets	
K	Set of vehicle type
M_K	Set of number of vehicles with type k
L	Set of candidate Facility centers
C	Set of customers
T	Set of time periods
N	Set of candidate facility centers and customers
Parameters	
ON_l	Opening cost of facility center l (million toman)
C_{ij}	Cost of travelling from node i to node j (million toman)
FV^{mk}	Fixed cost of vehicle k (million toman)
OP_t	Rate of opportunity cost in time period t
Q^k	Capacity of vehicle k
tt_{ijt}	Travel time from node i to node j in period t (minutes)
st_{it}	Service time for each node i in time period t (minutes)
d_{ct}	Demand of customer c in time period t
$[es_{it}, ls_{it}]$	Time window constraint for node i in time period t
L_{max}	Maximum number of opened facility centers
η	The minimum amount of commodity delivered
Decision Variables	
$h_{ijt}^{m_k k}$	Binary decision variable, equal to 1 if use the vehicle m_k of type k between arc (i,j) in time period t
$g_{ct}^{m_k k}$	Binary decision variable, equal to 1 if use the vehicle m_k of type k visiting customer c in time period t
y_l	Binary decision variable, 1 if use logistics center l
γ_{lct}	Binary decision variable, equal to 1 if use the vehicle m_k of type k visiting customer c in time period t
$\beta_{ct}^{m_k k}$	The amount of commodity delivered to customer c by vehicle m_k of type of k in time period t
I_{ct}	Inventory level of customer c at the end of period t
$ts_{it}^{m_k k}$	Arrival time of vehicle m_k of type k to node i in period t

1. INTRODUCTION

In the modern distribution management, integrated logistics systems have become a key necessity for business managers to optimize their logistics network and boost their position in the market. To do so, different decision levels should be simultaneously considered, which averts sub-optimality resulting from separated design and significantly decreases total cost reduces the traffic congestion [1]. Location routing problem (LRP) is

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a major concern in integrated logistics systems, which comprehensively considers the relationship between logistics centers, distribution centers and determines a suitable location for facilities and appropriate routing of vehicles. However, the integrated location routing inventory problem (LRIP) contains three combinatorial optimization problem of location routing problem, inventory routing problem (IRP) and location inventory problem (LIP) in which deal with combination of strategic, tactical and operational decisions concurrently.

Several LRP variants have appeared in the literature (e.g., capacity of vehicles, time-windows, multi-period, multi-objective) by different researchers. One realistic variants that seldom studied in LRP is multi-period time horizon arising uncertainty in parameters and inventory decision by considering the inventory balance of the previous period as well as demand and amount of delivered commodity of the current period. However, recently, extensive variation has been considering in models to better fit real life applications and make the models more practical. This approach, though, makes such rich problems difficult to formulate and solve [2-5].

Among all different kinds of commodities, cash money plays a huge role in everyday life and widespread automated teller machines (ATM) has made withdrawing money easy at any time. Despite the dematerialization of financial transactions and high penetration factor of e-payment mechanism, banknote and coins are still the most widely used payment method and it is expected to keep its dominance in the near future [6]. Cash-in-transit (CIT) companies and commercial banks carry out transportation of such commodity between cash deposits or banks' vaults and customers (e.g., ATMS, hypermarkets). However, because of the nature of the transported high-value products, CIT carriers are exposed to risk of attack by criminals and, consequently, minimizing the total costs while ensuring safe cash conveyance are the main challenges for the authorities. In real operations, to deal with these conflict objectives, there must be significant variations in model. To do so and to make the problem more applicable, we attempt to present a mathematical formulation for a rich location-routing-inventory problem in CIT sector arising in the replenishment of ATMs. The model contains various real-world variants of multi-objective, multi-periods, multi-vehicle, capacitated facilities, time windows under uncertain parameter. Afterwards, the chance constrained fuzzy programming (CCFP) is used to deal with uncertain parameter of demand, and a mathematical mixed integer linear programming (MMILP) formulation is presented. Finally, the AUGMECON2 is used to solve a numerical illustration.

The remainder of this paper has been organized as follows. Section 2 reviews the literature related to LRP under certain/uncertain conditions. In section 3, the proposed problem has been thoroughly described and modeled. Section 4 explains the research methodology including model formulation, uncertainty modeling, solution method, and application. The results are reported in section 5. Finally, section 6 concludes the paper presenting also some suggestions for future research.

2. LITERATURE REVIEW

To systematically compare the related literature, in this section the studied LRP modelling extensions such as capacity of facilities, time windows, multiple periods, inventory levels, uncertain parameters and risk-related constraints is investigated.

The location-routing problems deal with both strategic decision (e.g., facility location) and tactical decision (e.g., inventory control and vehicle routing) to prevent sub-optimality resulted from separated consideration. Numerous LRP variants and characteristics have appeared in the literature and a conventional LRP has been developing on various forms. The most commonly used extension in LRP is capacity constraint of vehicles and facility centers on holding/storing commodities. Many papers proposed such variant and presented capacitated LRP (e.g.,[7, 8]). Attracting more attention by researchers in recent years, time-window constraint is another real-life variant in LRP so that either a vehicle is allowed to depart from or return to a specific node within a pre-defined time window, or loading/unloading of commodities can be accomplished within an allowable time window (e.g.,[8, 9]).

One other complex real-life variant in LRP is multiple periods (PLRP) in which the planning horizon is divided into multiple periods and delivery of the goods should be made within the planning horizon. This variant is hardly used, because it significantly increase size and computational time of the problem. However, PLRP has very newly attracted the interests of researchers thanks to its practicality [8, 10, 11]. This variant arises two other characters of inventory and uncertain parameters. Regarding the former, recently some studies have assumed the integrated LRIP by considering all three-decision levels simultaneously in their models. The first attempt to deal with a complex LRIP was proposed by Javid and Azad [12]. However, neither did they consider much variants in their model, nor proposed a MMILP. They formulated it as a mixed integer nonlinear programming (MINLP) and then applied a meta-heuristic algorithms of Tabu Search and Simulated Annealing to solve it. Tang and Jiang [13] presented a LRIP for a bi-objective two-echelon supply chain aimed at minimizing cost and greenhouse gasses emission. Tavakkoli and Raziei [14] proposed a new location routing problem with fuzzy demand and two conflict objective functions including minimization of total cost as well as the shortage of products for each customer. Recently, Karakostas et al. [15] proposed a MMIL formulation for optimization of a LRIP with distribution Outsourcing with the purpose of minimizing a single objective of total cost.

Normally by considering a periodic time horizon in the routing problem, there is some uncertainty about some parameters of the model and it typically occurs in demand. There are some papers in the literature that applied a fuzzy chance constrained programming to deal with uncertain parameters (e.g., [7, 14, 16-18]). One of the first papers in this area was proposed by Mehrjerdi and Nadizadeh [7]. They presented a dynamic capacitated LRP with fuzzy demands for a single objective problem to minimize total cost. They applied a FCCP and then presented a hybrid heuristic algorithm with four phases to solve the problem. Tavakkoli and Raziei [14] used a fuzzy system to formulate and solve a LRIP. One other newly published paper was conducted by Sun et al. [17]. They established a bi-objective fuzzy mixed integer nonlinear programming model and applied a fuzzy credibilistic chance-constrained programming approach for the hazardous materials road-rail multimodal routing problem. Similarly, Fazayeli et al. [18] formulated a LRP in multimodal transportation network with time windows and represented fuzzy numbers for customers' demand enabling the problem formulation to make it close to the real-world situation and then used two-part genetic algorithm to solve the large-sized problem.

The above-reviewed articles and most published papers in LRP are largely related to shipment of hazardous materials, parcel or food. However, there are few papers applied LRP for dispensing items of value (i.e., cash money, gold). Such commodity are highly at risk of robbery and, thus, practitioners and academics deal with this issue to increase the security of transportation. There are different approaches to reduce the risk of routes and transportation in the literature. These include: a peripatetic routing problem through serving customers by a vehicle more than once within a planning horizon and using a different road segment (e.g., [19]); unpredictable routes by generating several solutions through defining specific time-window with a (min, max) time lag between two consecutive nodes (e.g., [20, 21]); a global exposure to risk and then minimize the index (e.g., [22]); and a route risk based on road characters (e.g., [23]).

Furthermore, as mentioned earlier and according to literature [2-5], most routing problems have considered a single objective function in their models, which is generally monetary objective. However, in real-world cases, DMs usually have to deal with various conflict objectives concurrently. For instance, the goal in CIT sector is to minimize transportation cost while boost safe and efficient routes. However, to the best of our knowledge and according to the published scientific works, the only multi-objective papers in CIT sectors can found only a few articles [8, 23-25], and there is not any multi-objective problem in LRIP in CIT sector.

According to the exhaustive literature review papers [2-5, 10, 26], a substantial trend in papers considering several real-world variants of LRP and proposing integrated problems existed. Despite making formulation hard, this approach is used thanks to making models more practical and close to real situations. Taking this trend into account, the main contribution of this paper is to propose a comprehensive mathematical model for a LRIP in CIT sectors motivated by the replenishment of ATMs. The real-life variants and constraints of this rich LRP model contain multi-objective, multi-period, multi-vehicle, capacitated facilities and vehicles, time windows and uncertain demand.

3. PROBLEM DEFINITION

Proposed network consists of some facility centers (also called distribution/logistics centers) and a number of customers. In CIT sector, vaults and ATMs are considered as facility centers and customers, respectively. Thus, vaults supply the ATMs' required cash by armored vehicles. This operation performs in each time period with various variants and constraints, and typically concerns cost and risk as the main objectives.

To take the advantage of formulation of such realistic case and to make it more practical, primarily a comprehensive mathematical modeling that concurrently considers as much variants and constraints as possible is needed. Thus, in this paper we present a mathematical mixed-integer linear programming for a rich integrated LRP having bi-objective multi-period, capacitated inventory-location-routing problem with time windows (BO-PCLRIP-TW-FD). The main purposes are facility location (number and location of distribution centers), vehicle routing and optimization of amount of cash on board of vehicles and dispensed to ATMs so that simultaneously minimize total cost and risk. Figure 1 shows the schematic of the studied LRP.

Some assumptions are considered in this study as follows:

- Demands must be fulfilled;

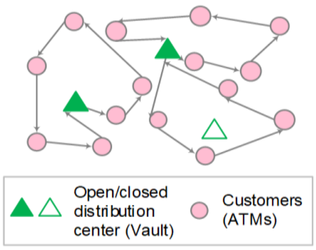


Figure 1. The general setting of LRP in CIT sector

- Location of facility centers (i.e., vaults) and customers (i.e., ATMs) are fixed.
- The begin/end node of each route is a same open distribution center;
- Fleet of vehicles are heterogeneous with limited capacities;
- A stretch of each period is considered daily over a five-day planning horizon;
- All nodes have time window constraint;
- Distribution centers have limited capacity of cash handling;
- The average travel time and cost between each arc (*i*, *j*) are known.

4. RESEARCH METHODOLOGY

In this section, model formulation, uncertainty modeling, solution method, and application is described.

4. 1. Mathematical Formulation Considering a graph G=(N, A), the BO-PCLRIP-TW-FD is defined in which N is set of nodes and A is set of arcs. The node set N contains candidate facility centers (l) and ATMs as customers (c). Each arc has a nonnegative cost c_{ij} based on the real distance between i and j.

Thus, the bi-objective periodic capacitated location-inventory-routing problem with time-window aimed at concurrently minimize total cost and reduce total risk is formulated as follows:

$$Min\,Z_1 = \sum_{l\in L} ON_l\, y_l + \sum_{t\in T}\Big(\sum_{m_k\in M_K}\sum_{k\in K}\sum_{i,j\in N} c_{ij}\, h_{ijt}^{m_k k} + \sum_{l\in L}\sum_{c\in C}\sum_{m_k\in M_K}\sum_{k\in K} FV^{m_k k} h_{lct}^{m_k k} + OP_t \sum_{c\in C} I_{ct}\Big) \quad (1)$$

$$Min\,Z_2 = \sum_{t\in T}\sum_{k\in K}\sum_{m_k\in M_K}\sum_{i,j\in N} r_{ij} h_{ijt}^{m_k k} \quad (2)$$

subject to

$$\sum_{i\in N} h_{ict}^{m_k k} = \sum_{j\in N} h_{cjt}^{m_k k} = g_{ct}^{m_k k}\,; c\in C,\, m_k\in M_K,\, k\in K,\, t\in T \quad (3)$$

$$\sum_{k\in K}\sum_{m_k\in M_K}\sum_{\substack{i\in N\\ i\neq c}} h_{ict}^{m_k k} \le 1 \qquad ;\forall c\in C,\, t\in T \quad (4)$$

$$\sum_{i\in L}\sum_{j\in C} h_{ijt}^{m_k k} \le 1 \quad ;\forall m_k\in M_K,\, k\in K,\, t\in T \quad (5)$$

$$\sum_{c\in C}\sum_{k\in K}\sum_{m_k\in M_K} h_{lct}^{m_k k} \le y_l\, M \quad ;\forall l\in L,\, t\in T \quad (6)$$

$$\sum_{c\in C}\sum_{k\in K}\sum_{m_k\in M_K} h_{lct}^{m_k k} \ge 1-(1-y_l)\,M \quad ;\forall l\in L,\, t\in T \quad (7)$$

$$h_{clt}^{m_k k} \le \gamma_{lct}\,;\forall l\in L,\, c\in C,\, m_k\in M_K, k\in K,\, t\in T \quad (8)$$

$$h_{lct}^{m_k k} \le \gamma_{lct}\,;\forall l\in L,\, c\in C, m_k\in M_K, k\in K,\, t\in T \quad (9)$$

$$h_{ijt}^{m_k k} + \gamma_{lit} + \sum_{\substack{l'\in L\\ l'\neq l}} \gamma_{l'jt} \le 2 \;;\forall l,l'\in L,\, i,j\in C, i\neq j,\; m_k\in M_K,\; k\in K,\; t\in T \quad (10)$$

$$\sum_{l} \gamma_{lct} = 1 \quad ;\forall c\in C,\; t\in T \quad (11)$$

$$\sum_{i\in C'}\sum_{j\in C'} h_{ijt}^{m_k k} \le |C'|-1 \;;\forall |C'|\subseteq C,\, |C'|\ge 2, m_k\in M_K, k\in K, t\in T \quad (12)$$

$$\beta_{ct}^{m_k k} \le g_{ct}^{m_k k}\times(CAP_c - I_{ct}) \;;c\in C,\; m_k\in M_K,\; k\in K,\; t\in T \quad (13)$$

$$\beta_{ct}^{m_k k} \ge \eta - (1-g_{ct}^{m_k k})\,M \;;c\in C,\; m_k\in M_K, k\in K,\; t\in T \quad (14)$$

$$\sum_{c\in C} \beta_{ct}^{m_k k} \le Q^k \quad ;m_k\in M_K,\; k\in K,\; t\in T \quad (15)$$

$$\sum_{c\in C} d_{ct}\gamma_{lct} \le HC_l\, y_l \qquad ;l\in L,\; t\in T \quad (16)$$

$$\sum_{k\in K}\sum_{m_k\in M_K} g_{lt}^{m_k k} \le PC_l \qquad ;\forall l\in L,\; t\in T \quad (17)$$

$$I_{ct} \le CAP_c \qquad ;\forall c\in C,\; t\in T \quad (18)$$

$$I_{ct} = I_{ct-1} + \sum_{k\in K}\sum_{m_k\in M_K} \beta_{ct}^{m_k k} - d_{ct}\,;\forall c\in C,\, t\in T \quad (19)$$

$$ts_{it}^{m_k k} + s_{it} + tt_{ijt} - M(1-h_{ijt}^{m_k k}) \le ts_{jt}^{m_k k} \;;\forall i\in C,\, j\in N, m_k\in M_K, k\in K, t\in T \quad (20)$$

$$tt_{ijt} - M(1-h_{ijt}^{m_k k}) \le ts_{jt}^{m_k k} \;;\forall i\in L,\, j\in N,\; m_k\in M_K,\; k\in K,\; t\in T \quad (21)$$

$$g_{it}^{m_k k}\, es_{it} \le ts_{it}^{m_k k} \le g_{it}^{m_k k}\, ls_{it} \;;\; \forall i\in N,\; m_k\in M_K,\; k\in K,\; t\in T \quad (22)$$

$$\sum_{l\in L} y_l \le L_{\max} \quad (23)$$

$$h_{ijt}^{m_k k},\, g_{ct}^{m_k k}, \gamma_{lct} \in \{0,1\};\forall i,j\in N, c\in C, l\in L, k\in K, t\in T \quad (24)$$

$$\beta_{ct}^{m_k k},\, I_{ct},\, ts_{it}^{m_k k} \ge 0;\forall c\in C, i\in N, m\in M, k\in K, t\in T \quad (25)$$

This problem has two conflict objective functions; the objective function (1) is related to minimization of total cost and the second one (2) is related to minimization of route risk. The total cost in objective function (1) consists of three main components, which are associated to location, routing and opportunity cost, respectively. The objective function (2) minimizes the total transportation risk.

Constraint (3) imposes flow conservation in each time period. Inequality (4) expresses, in each period each customer receives commodity at most by one vehicle and constraint (5) ensures that each armored vehicle can move toward customers only from one facility center. Inequalities (6) and (7) model that in case a candidate facility center is selected, at least one vehicle is transferred from that facility center to a customer. The following constraints (8)-(10) prohibit illegal routes and oblige vehicles return to the origin facility center. Equation (11) expresses each customer is allocated to one of facility centers. Constraint (12) considers sub-tour elimination. Constraints (13) and (14) denote the maximum and minimum allowed amount of commodity that can be delivered to customers only if the corresponding binary decision variable is equal to 1.

Vehicle capacity and cash handling capacity in facility centers are expressed in constraint (15) and (16). Constraint (17) guarantees total number of vehicles in each facility center should respect the capacity of parking lots. Constraint (18) states that inventory held in each ATM is not allowed to exceed the holding capacity. Equation (19) model the inventory conservation condition over successive periods. Inequalities (20) and (21) model the arrival time of vehicles to each node. Time window constraint and maximum number of selected facility centers are shown in constraints (22) and (23), respectively. Finally, constraints (24) and (25) define binary and non-negativity conditions on the variables. The formulation is nonlinear due to constraint (13). However, to transform the model from non-linear to linear programming, such constraint should be rewritten using a set of linear constraints as follows:

$$\beta_{ct}^{m_k k} \le (g_{ct}^{m_k k} \times CAP_c) - \zeta_{ct}^{m_k k} \quad ;c \in C,\ m_k \in M_K,\ k \in K,\ t \in T \tag{26}$$

$$\zeta_{ct}^{m_k k} \le M\, g_{ct}^{m_k k};c \in C, m_k \in M_K, k \in K, t \in T \tag{27}$$

$$\zeta_{ct}^{m_k k} \le I_{ct};c \in C,\ m_k \in M_K,\ k \in K,\ t \in T \tag{28}$$

$$\zeta_{ct}^{m_k k} \ge I_{ct} - M(1 - g_{ct}^{m_k k});c \in C, m_k \in M_K, k \in K, t \in T \tag{29}$$

$$\zeta_{ct}^{m_k k} \ge 0\ ;c \in C,\ m_k \in M_K,\ k \in K,\ t \in T \tag{30}$$

4. 2. Uncertainty Modeling There are some fuzzy models to deal with uncertainty in parameters. In the fuzzy chance constraint programming (FCCP), at least one of parameters (in objective functions and/or constraints) is a fuzzy random variable. In this paper to deal with uncertainty in demand, the FCCP is applied to make the basic model and transform the fuzzy model into equivalent crisp one. The FCCP approach relies on insightful mathematical concepts by considering the expected value of a fuzzy number, and the necessity (Nec), possibility (Pos) and credibility (Cr) measures. Since satisfying the demand is of a great priority, we suppose DMs have pessimistic approach to the problem. Thus, we apply the necessity approach. Besides, we used triangular fuzzy distribution for modeling when demand can be defined by three sensitive points (i.e., $\tilde{d} = \tilde{d}_1, \tilde{d}_2, \tilde{d}_3$). The constraints having uncertain parameters must be molded with a satisfaction level of at least α_i, meaning that we do not allow the constraint to be violated. Then, the equivalent auxiliary crisp model, by using the Nec measure and considering triangular fuzzy distribution for the uncertain parameter of demand in constraints (16) and (18) can be formulated as follows (for more information, we refer the readers to literature [27]):

$$\sum_{c \in C} ((1-\alpha) d_{(2)ct} + \alpha\, d_{(3)ct})\, \gamma_{lct} \le HC_l\, y_l \quad ;l \in L,\ t \in T \tag{31}$$

$$I_{ct} = I_{ct-1} + \sum_{k \in K} \sum_{m_k \in M_K} \beta_{ct}^{m_k k} - ((1-\alpha) d_{(2)ct} + \alpha\, d_{(3)ct}) \quad ;\forall c \in C,\ t \in T \tag{32}$$

$$0.5 \le \alpha \le 1 \tag{33}$$

4. 3. Solution Method: the AUGMECON2 Method

In multi-objective optimization problems (MOOPs), there are more than one objective function and it is impossible to boost the value of one objective function without deteriorating value of at least one of other objective functions. In MOOPs, DMs ought to interfere and select the “most preferred” solution among dominated solutions (also referred as Pareto optimal or efficient solution.

There are several approaches for solving MOOPs in literature (e.g., Weighted Sum Method, Goal Programming, ε-constraint method). Among the exact methods, ε-constraint method outperforms specifically in the problems with discrete variables in pure integer or mixed integer problems [28]. Other merits are as follows:

- It is possible to acquire diverse optimum solutions with a change in ε.
- There is not necessary to scale the objective functions having strong impact in the obtained results.

The general form of the ε-constraint is that one of the objective function with the highest priority (total cost) will be chosen as objective function and the other objective functions (risk, in this study) will be converted to constraints.

The latest improved version of the ε-constraint, proposed by Mavrotas et al. [28], is augmented ε-constraint 2 (called AUGMECON2). This version has addressed the substantial weak points of conventional method: (1) guarantee of efficiency of the found solution by appropriately tuning its parameters, (2) make the payoff table with efficient solutions using the lexicographic optimization, and (3) improve computational time by integrating acceleration issues. The authors claimed that not only is AUGMECON2 one of the best available exact methods for solving the MOOPs, but also it is competitive with meta-heuristics algorithms in producing adequate approximations of the Pareto set, especially in small-medium sized problems.
The following equations show the reformed problem by using the AUGMECON2 (for more information, we refer the readers to literature [29]:

$$\begin{aligned} &\min f_j(x) - eps\left(\sum_{i=2}^{p} 10^{-(i-2)} \times \frac{S_i}{r_i}\right) \\ &f_i(x) + S_i = e_i \quad \forall i \neq j \\ &x \in S,\ S_i \in R^+ \end{aligned} \tag{34}$$

In this formulation, eps is in the range of $[10^{-6}, 10^{-2}]$; the slack variables of the respective ε-constraints are $s_2,\ldots, s_p$ and the parameters $r_2,\ldots, r_p$ are the ranges of the respective objective functions and $e_2,\ldots, e_p$ are the parameters for the right hand side for the specific iteration on the grid points of the objective functions 2, 3, …, p.

4. 4. Application To validate the proposed comprehensive MMILP formulation and solution method and to show its practicability, a numerical illustration is presented. The proposed model is related to CIT sector, and therefore we used real data from one of Iranian banks for cash dispensing in ATMs for a part of its network.

5. COMPUTATIONAL RESULTS

The sizes of the numerical illustration and values of the problem parameters are shown in Tables 1 and 2. Effort has been made, as much as possible, to collect values nearby the real ones. For instance, the parameter c_{ij} and tt_{ijt} are calculated based on the real data and using Google Map API. Time window constraints for dispensing cash by vehicles to ATMs and returning to the logistic centers are in accordance with the Central Bank's policy. For instance, time window constraint of es_{ct} = 5 means that the earliest possible time for delivering cash to customer c in period t is in minute of 15, and considering the start time at 7:00 am, that is 7:15 am. To produce the triangular fuzzy number, the three points for ill-known parameter of demand is estimated based on Lai and Hwang's report [28]. In this way, using the uniform distribution, first, the most likely value ($d_{(2)}$) is stochastically (randomly) generated (e.g., $d_{(2)}{\sim}U[30, 60]$). Afterwards, two random numbers (r_1 and r_2) are produced based on uniform distribution between 0.2 and 0.8, and subsequently optimistic ($d_{(3)}$) and pessimistic ($d_{(1)}$) values for demand are respectively calculated as follows:

$$\begin{aligned} d_3 &= (1+r_1)\, d_2 \\ d_1 &= (1-r_2)\, d_2 \end{aligned} \tag{34}$$

Finally, for calculating the risk of route between each arc (i,j) and forming the risk matrix between nodes, the presented approach in literature [23] is applied. According to this approach, risk of transportation between each two consecutive nodes are calculated based on experts' opinion on weighted factors of road characteristics such as type of road segment (e.g., high way, street, alley), number of lanes, light and traffic condition. Then, by normalizing the numbers, risk index for each arc is calculated that is a number between 0-1 in which the lowest and the highest are associated to no risk and high risk, respectively.

To Code the mathematical model, GAMS 24.1 and a personal computer with Intel core i5-3337U, 1.8 GHz processor, 6 GB RAM has been used.

The network and position of facility centers and customers is represented in Figure 2. Moreover, payoff table and and Pareto solutions for different values of objective functions using the AUGMECON2 are given in Table 3 and Figure 3, respectively. As shown, since the

TABLE 1. Size of the numerical illustration

No. of facility centers (vaults)	No. of customers (ATMs)	No. of periods (days)	No. vehicle types	No. vehicles in each type
3	10	5	2	2

TABLE 2. Value for the different parameters

Parameter	Parameter range	Parameter	Parameter range
η	10	$L_{\max}$	2
α	0.8	ON_l	[10, 15]
CAP_c	130	FV^k	[0.3, 0.5]
d_{ct}	[10, 150]	st_{it}	[10, 30]
Q^k	[1500, 2000]	$[es_{lt}, ls_{lt}]$	[0, 540]
PC_l	[4, 12]	$[es_{ct}, ls_{ct}]$	[15, 510]

objectives are minimization of cost and risk, the plot obtained after consecutive iterations has a downward trend meaning that value of one objective cannot improve if and only if the value of one other objective function gets worse. This result demonstrates that objective functions are in conflict with each other. In Table 4, one of obtained solutions (i.e., decisions in period *t2*) in terms of routes of vehicles, arrival time to each node and amount of commodity delivered to customers with respect to the constraints and objective functions is given. As shown, the third candidated distribution center (y_{l3}) is selected as a facility center. Besides, arrival time of vehicle *k* to *c7* is in minute of 15 and the amount of commodity delivered to the customer is equal to 28; these mean the armored vehicle delivers 28 units of cash to ATM7 at 7:15 am. Then, the vehicle continues its route in the same manner and returns to the same facility center at 3:00 pm. Taking these results into account, value of other binary decision variables can be interpreted (e.g., $\gamma_{l_3c_1t_2}$ is equal to 1 because in the second period, customer 1 has been assigned to logistics center 3).

4. 5. Sensitivity Analysis Results illiustrate a proper allocation of facilities to the customers as well as continuity of the routes and legal tours considering constraints such as start/end point from/to the origin facility center and avoided sub-tours. In order to ensure about the mathematical formulation and associated coding, effects of some important parameters on values of objective functions is studied in this section. To delineate, three parameters including cost of opening facility center (ON_l), fixed cost of vehicles (FV^k) and risk of transportation (r_{ij}) have been selected and executed in GAMS, and their results are given in Figure 4.

To do so, value of such parameters are considered in four different levels so that level 1 has the default value

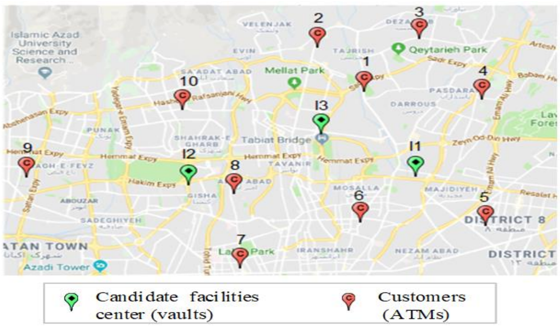


Figure 2. A schematic view of network

TABLE 3. Payoff table

The objective function value	Z_1	Z_2
Z_1	16.93	14.3
Z_2	16.97	13.2

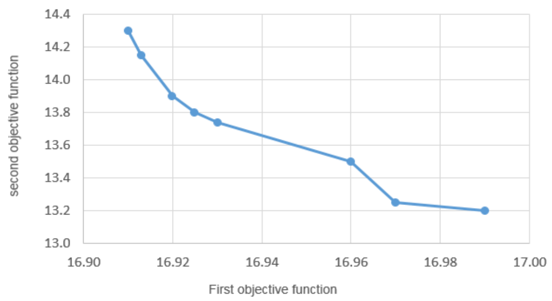
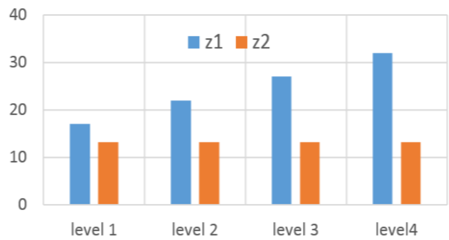


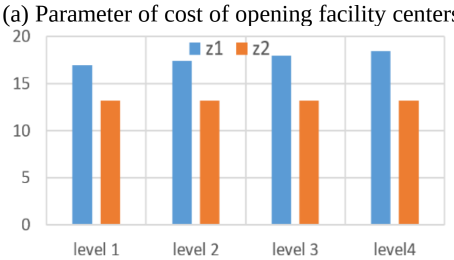
Figure 3. The Pareto frontier obtained by AUGMECON2 method

TABLE 4. The optimum route and amount of delivered cash to ATMs

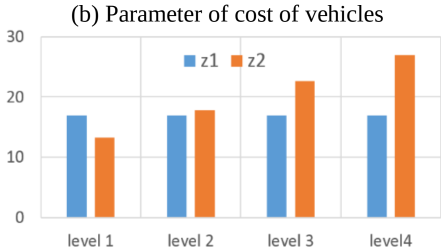
	C7	C9	C8	C10	C5	C4	C3	C1	C6	C2	L3
$ts_{it}^{m_k k}$	15	47	70	98	139	166	186	208	235	266	300
$\beta_{ct}^{m_k k}$	28	46	42	84	74	72	66	74	46	64	-



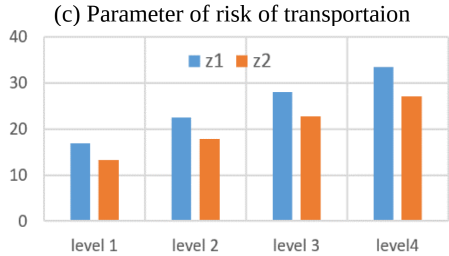
(a) Parameter of cost of opening facility centers



(b) Parameter of cost of vehicles



(c) Parameter of risk of transportaion



(d) All parameters in differenet levels

Figure 4. Sensitivity analysis

in which the problem has been initially solved with, and level 2, 3 and 4 have 133, 167 and 200% value of the level 1. For instanse, if the default value of parameters ON_{l1} is equal to 10, then this value for four different levels are as level 1=10, level 2=13.3, level 3=16.7 and level 4=20. As is shown in Figures 4(a) and 4(b), increased in the cost of opening facility centers (ON_l) and fixed cost of vehicles (FV^k) results in increasing the value of the first objective function (Z_1); and increased in the risk of transportation between each two consecutive nodes (r_{ij}) leads to growth in the value of the second objective (Z_2) as depicted in Figure 4(c). Moreover, the effect of simultaneosly changing all these parameters on both objective functions is given in Figure 4(d). As can be seen, by increasing parameters associated to cost and risk, total cost and total route risk increase. These results validate the formulation and coding of the presented model.

5. CONCLUSION AND FUTURE RESEARCH

Thanks to significant advances in technology and improvement in optimization algorithms, researchers can solve problems having much more variants. This helps researchers and practitioners to consider problems that are more complex to make them more close to actual cases. Such trend, in lieu of concentrating on stylish solution algorithm for problems modeled by limited variants, has been declared in the most in-depth literature. Taking this into consideration, this paper proposed the optimization of a new complex location-routing-inventory problem in CIT sector through integrating strategic, tactical, and operational level decisions. Hence, firstly a comprehensive mathematical model for a LRIP was formulated with the purpose of decreasing total cost (i.e., location, routing and opportunity cost) and reducing risk of robbery. T do so, various existing real-life variants and constraints containing multi-objective, multi-period, multi-vehicle, capacitated facilities and vehicles, heterogeneous fleet of vehicles, time windows and uncertain demand was considered and then the model was formulated as a MMILP. Afterwards, to validate the mathematical formulation and to solve the problem, the AUGMECON2 (i.e., the latest version of ε-constraint method) was tested on a realistic instance in CIT sector.

This work still has limitations offering research opportunities for further studies. Future works can be conducted in the following sides: (1) the proposed model can be extended with other real-life variants (e.g., integration of LRP with the revenue management, time-dependent networks, length restrictions, stochastic parameters, environmental issues). (2) Others parameter such as cost, time windows, travel time and capacity can be assumed in uncertainty. (3) Pickups and deliveries in LRIP are another possible direction, and instead of opening new pickup fa- cilities, distribution centers can be expanded in order to handle both delivery and pickup operations. (4) The ride sharing among CIT companies and banks in different networks and/or sharing of vehicles within one network (open LRP by relaxing limitations necessitating vehicles to return to origin facility center can also be studied. (5) Apply heuristic or metaheuristics algorithms for solving large sized problems.

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A Comprehensive Mathematical Model for a Location-routing-inventory Problem under Uncertain Demand: a Numerical Illustration in Cash-in-transit Sector

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چکید ه

هدف این مقاله مدلسازی و حل یک مساله یکپارچه مکان‌یابی، مسیریابی، موجودی (LRIP) در حوزه پول‌نقد در جریان (CIT) است. بمنظور کاهش هزینه کل و کاهش ریسک سرقت برای چنین کالای باارزشی در یک عملیات واقعی حمل‌و توزیع پول‌نقد، ویژگیهای زیادی بایستی در نظر گرفته شود که فرمول‌نویسی مساله را سخت می‌کند. در این مقاله، بمنظور مدلسازی دقیق‌تر و نزدیک‌تر کردن مدل به مسایل دنیای واقعی، یک مساله LRIP دوهدفه، چنددوره‌ای، همراه با محدودیت ظرفیت و پنجره زمانی و با تقاضای غیرقطعی (BO-PCLRIP-TW-FD) برای پول‌گذاری در دستگاه‌های خودپرداز ارایه می‌شود. سپس با استفاده از برنامه ریزی محدودیت فازی احتمالی برای مواجه با عدم قطعیت در پارامترها، مدل بصورت یک برنامه‌ریزی خطی عددصحیح مختلط قطعی فرمول‌نویسی می‌شود. در پایان، بمنظور اعتبارسنجی فرمول‌نویسی اریه شده و حل مساله، از اخرین نسخه روش محدودیت اپسیلون (AUGMECON2) استفاده می‌شود. رویکرد حل ارایه‌شده در یک مثال واقعی در حوزه CIT تست می‌شود. نتایج عددی مناسب بودن مدل و فرمول‌نویسی ارایه شده و قابلیت پیاده‌سازی مدل برای مسایل واقعی در حامل‌های امنیتی را تایید می‌کنند.

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