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Effect of Fuzzy Boundaries on the Bearing Capacity of Footings on Two-Layered Clay

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ABSTRACT

In this study, fuzzy logic was implemented to formulate the fuzziness of layer boundaries for a two-layered clay soil. A field of two-layered clay with fuzzy boundaries between layers was generated, and then the bearing capacity of strip footing on this field was calculated by the assumption of plane-strain conditions. The Mohr-Coulomb failure criterion was used and bearing capacity calculations were based on finite difference method. The effect of fuzziness in layer boundaries was investigated for the case of strong-over-weak clay. It is concluded that the analyses by applying fuzzy boundaries yielded more conservative results than classical two-layered bearing capacity calculations when the ratio of thickness of upper layer to the width of footing exceeds 1. When the ratio of thickness of upper layer to the width of footing is lower than 1, the bearing capacity of footing on two-layered clay is higher by considering fuzzy boundaries.

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NOMENCLATURE

N_c	Cohesion factor	c_2	Cohesion of lower layer(kN/m^2)
L_1	fuzzy number for the upper layer	B	Width of footing
L_2	fuzzy number for the lower layer	h_{12}	Fuzzy depth between layers(m)
H_1	The thickness of upper layer(m)	R_{hf}	Fuzzy depth index
H_2	The thickness of lower layer(m)	R_C	Cohesion ratio
c_1	Cohesion of upper layer(kN/m^2)	R_{qf}	Fuzzy bearing capacity ratio

1. INTRODUCTION

Soil deposits are formed in a long-time complex process. Therefore, uncertainty in the mechanical behavior of soil deposits is expected. One of the sources of uncertainty in the mechanical behavior of soil deposits is the spatial variability of the soil. The effect of this kind of uncertainty on the mechanical response of soil has been studied in previous work [1-3]. In order to deal with the uncertainty in geotechnical engineering, soft computation methodologies, especially fuzzy logic and neural networks have been implemented [4-7].

The calculation of bearing capacity of soil is a common geotechnical problem which usually solved by the assumption of footing on homogeneous soil [8-11]. Bearing capacity of foundations on two-layered clay has been determined in previous studies by assuming deterministic boundaries for layers [12-16]. However, in natural geotechnical deposits, no exact boundary exists between layer interfaces due to gradual formation and sedimentation. Layer interfaces are in fact a fuzzy transition zone where the properties of one layer gradually change to the properties of another layer. The fuzziness of layer boundaries in geotechnical deposits has been clearly demonstrated in in-situ investigation tests (e.g., standard penetration tests (SPTs), cone penetration tests (CPTs), and geophysical soundings) [17-19].

In previous studies, layer boundaries for geotechnical deposits were assumed deterministic; however, in real field problems, this is not the case and layer boundaries

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especially in clays can be considered as fuzzy boundaries (i.e. the boundaries are not obviously separated and they are mixture of two adjacent layers). In the present paper, Fuzzy Logic (FL) was implemented for generating a field of two-layered clay with considering fuzziness in the layer boundaries. Then bearing capacity of footings on this field was calculated by using the numerical finite difference method. This will produce design guidelines for considering fuzziness in layer boundaries for estimation of bearing capacity of footings resting on two-layered clays.

2. MATERIALS AND METHODS

2. 1. Fuzzy Inference Systems FL was introduced for dealing with the partial truth principles. This theory is formally originated by Zadeh in his prominent paper in 1965 [20]. He introduced the fuzzy set theory by membership functions, which belong to the interval [0, 1].

Fuzzy inference system (FIS) use FL concept for addressing complex, real-world problems. The major components of FIS are fuzzification, fuzzy inference engine, fuzzy rule base, and defuzzification. The general form of an FIS is illustrated in Figure 1 [21].

Various FISs have been proposed to apply in engineering. Two of the most commonly used FISs are Mamdani and Takagi–Sugeno–Kang (TSK) fuzzy models. This two FISs have been applied in many engineering problems. Mamdani and TSK FISs are similar in almost all components except in the output of the consequent statement in fuzzy rules. The TSK fuzzy model was used in this study [12].

A typical form of fuzzy *i*th rule in TSK fuzzy model is:

$$\text{IF } x \text{ is A and } y \text{ is B, then } z = f(x,y) \quad (1)$$

In this rule, x and y are input variables, A and B are fuzzy sets, and $z = f(x,y)$ is a crisp function. Usually $f(x,y)$ is a polynomial in the input variables x and y, but it can be any other functions that can properly define the output of the system. In this study, this function was assumed to be a constant parameter.

The algorithm for an example of two-rule, one-input TSK fuzzy systems, with similar application, used in this study is illustrated in Figure 2.

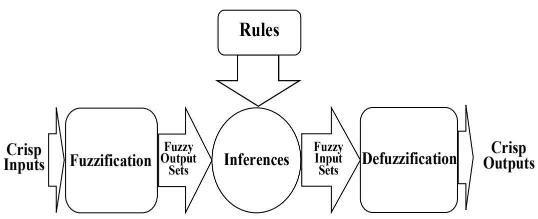


Figure 1. Major components and processes in a FRBS

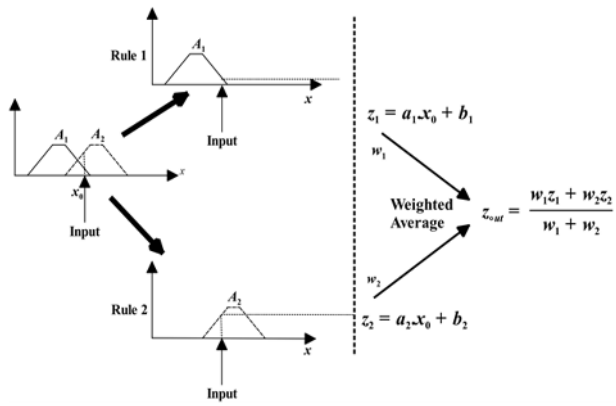


Figure 2. TSK FRBS for a two-rule one-input system

2. 2. Application of Fuzzy Logic in Cohesion Distribution In this study, layer boundaries were considered as fuzzy transition zones and the cohesion values were distributed accordingly. The assumption was that cohesion of a layer change gradually to another adjacent layer. For formulating this assumption the TSK FIS was used. The fuzzy if-then rules for the two-layered clay are:

$$\begin{aligned}&\text{Rule 1 : If } z \text{ is } L_1 \text{, then cohesion is } c_1\\ &\text{Rule 2 : If } z \text{ is } L_2 \text{, then cohesion is } c_2\end{aligned} \quad (2)$$

where, z is the depth from ground surface, L_1 and L_2 are two trapezoidal fuzzy numbers corresponding to the depth and c_1, c_2 are the crisp numbers for cohesion of upper and lower layer, respectively. Figure 3(a) schematically illustrates a two-layered soil with the assumed coordinate axis (z) in downward direction. Two fuzzy numbers L_1 and L_2 are depicted in this figure. In this graph, the horizontal and vertical axes are respectively the depth (in z direction) and membership degree of the layers. In Figure 3(b), the hypothetical interface between the layers is depicted as a dashed line. In classical geotechnical analyses, for simplification, this hypothetical interface has been regarded as deterministic boundary between layers. Thus, in this study, the term "classical bearing capacity analyses" means the bearing capacity analyses, which assume deterministic boundaries for layers. In Figure 3(b), two assumed trapezoidal Fuzzy numbers L_1 and L_2 in the hypothetical boundary zone have joint domain of h_{12}, which is called fuzzy depth. This depth indicates a zone, in which by increasing the depth, the membership grade of upper layer gradually diminishes and on the contrary, the membership grade of lower layer increases proportionally. In Figure 3(b), H_1 and H_2 are the hypothetical depths for upper and lower layers, respectively. Here h_1 and h_2 are termed the deterministic depths of the upper and lower layers, in which the properties of the layers are known with certainty.

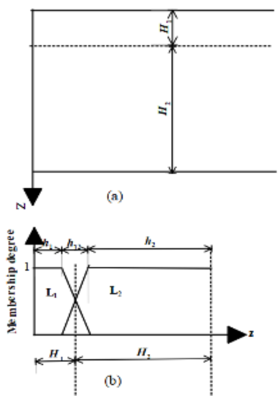


Figure 3. a) two-layered clay b) Trapezoidal Fuzzy Numbers for layers

The fuzzy depth (h_{12}) could be estimated by analysing the results of field tests (like CPT, SPT or geophysical sounding). An example for estimation of h_{12} from the CPT is illustrated in Figure 4. In this figure h_{12} is the transitional depth, in which the cone tip resistance changes from higher values in the top layer to lower values in the bottom layer.

In this study, the dimensionless ratio in Equation (4) was used to represent the level of fuzziness in the interface of layer boundaries. This ratio is termed the fuzzy depth index(R_{hf}):

$$R_{hf} = \frac{h_{12}}{2H_1} \tag{4}$$

When two layers are distinct and there is no fuzzy depth (h_{12}=0) between the layers, the R_{hf} tends to be zero and when the entire depth of the upper layer is merged into the lower layer ($h_{12}=2H_1$), then this index is 1. However, in real world cases, the situation is usually between these two extremes. Figure 5 illustrates the variation of R_{hf} with the associated fuzzy numbers for a two-layered clay soil. It must be noted that the thickenss of bottom layer (H_2) did not have any influence on the results of current study because it was far apart from the footing.

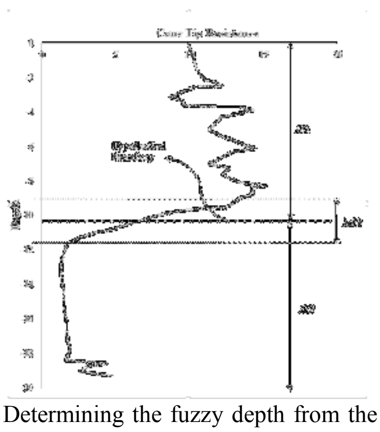


Figure 4. Determining the fuzzy depth from the results of CPT

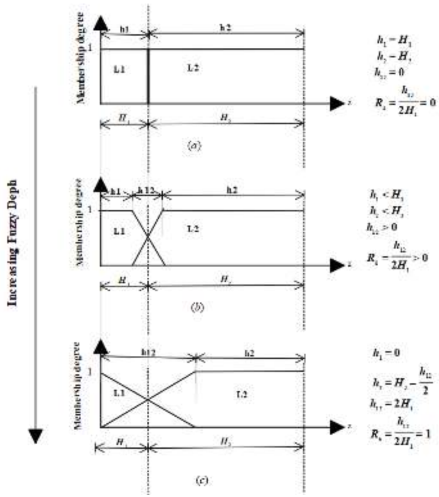


Figure 5. The Trapezoidal Fuzzy numbers for layers by changing R_{hf} ratio from 0 to 1

2. 3. Numerical Modelling As noted in this study the numerical finite difference method was implemented for calculating the bearing capacity of footing [13]. Figure 6 illustrates the geometry and boundary conditions of the model to calculate the bearing capacity of footing. As shown in this figure, the analysed model was two-dimensional. The assumption of plane strain condition was considered in this study and the strip footing was regarded for the analysis. Since the soil is completely cohesive in the present study, its friction angle was considered negligible. As noted previously, the cohesion of upper and lower layer (or top and bottom layer) are denoted by c_1 and c_2, respectively.

Since the problem was symmetric, half of the model was analysed with proper boundary conditions. The displacement vectors were applied in consecutive steps in downward vertical direction on the nodes beneath the footing. Since the footing was assumed rigid, the relative movements of the nodes beneath the footing were restricted in x and y directions. In order to simulate the boundary conditions in the field for the bearing capacity problem, the displacements in the lateral boundaries were restricted in the x-direction whereas the displacements in the bottom of the model were restricted in the x and y directions [2]. The finite difference grid which consists of 10000 of 20cm×20cm square zones was used for the numerical model.

For minimizing the effect of boundaries on bearing capacity analyses, the boundary surfaces were assumed far enough from the footing (i.e, 10B for bottom boundary and 5B for the lateral boundaries, where B is the footing width). Since the effect of the soil specific weight for the current study was trivial, then it was set to be constant and equal to 17 kN/m^3. Values of the elastic parameters also have insignificant effects on the results. However, for approaching the best shape of stressdisplacement curve, these parameters were chosen as in the literature [2]. The footing width (B) was also ineffective on the results of the current study. Thus, this parameter was also set as a constant value of 2 m.

It must also be noted that since the cohesion of upper layer (c_1) is assumed as a constant value for pure cohesive clay with no friction coefficient thus in this research the value of N_c is equivalent to bearing capacity [12].

3. RESULTS AND DISCUSSIONS

3. 1. Parametric Study A parametric study was carried out by considering the effect of fuzziness in layer boundaries. The parameters considered for parametric study were: the cohesion ratio ($R_c = \frac{c_1}{c_2}$), ratio of the top-layer thickness to the footing width ($\frac{H_1}{B}$), and R_{hf}. The values of the parameters used in the parametric analyses are reported in Table 1. The range and numerical values of the parameters were chosen as to cover all possible cases which would be encountered in real field problems only for strong-over-weak clay($R_c > 1$).

The results of this study are presented by charts, each with curves having constant ratio of ($\frac{H_1}{B}$) and various R_c ratios. In these charts, the horizontal and vertical axes are R_{hf}, and the fuzzy bearing capacity ratio (R_{qf}), respectively. Here R_{qf} was calculated by following expression:

$$R_{qf}(\%) = \frac{N_c(R_{hf}) - N_c}{N_c} \times 100 \quad (5)$$

In this equation, $N_c(R_{hf})$ is the cohesion factor (N_c) by applying the fuzzy depth between the layers. N_c is obtained from classical bearing capacity analyses. R_{qf} is an indication of variation of $N_c(R_{hf})$ with respect to N_c by assuming deterministic boundary conditions. Indeed, R_{qf} indicates the effect of fuzzy depth on the bearing capacity with respect to classical bearing capacity analyses for the footing located on two-layered clay. By incorporating the effect of fuzzy depth between the layers and determining R_{qf} from the charts of the current study, the N_c factor from classical bearing capacity analyses can be modified using following equation:

$$N_{cmf} = (1 + \frac{1}{100} R_{qf}) \times N_c \quad (6)$$

In this equation N_{cmf} is the modified version of classical cohesion factor (N_c) by consideration of fuzzy depth between the layers.

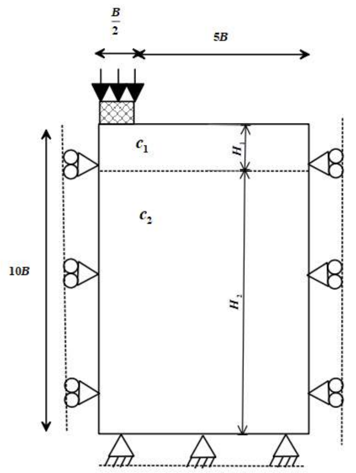


Figure 6. Schematic of geometry and boundary conditions for numerical model [19]

TABLE 1. Parameters and their numerical values for analyses

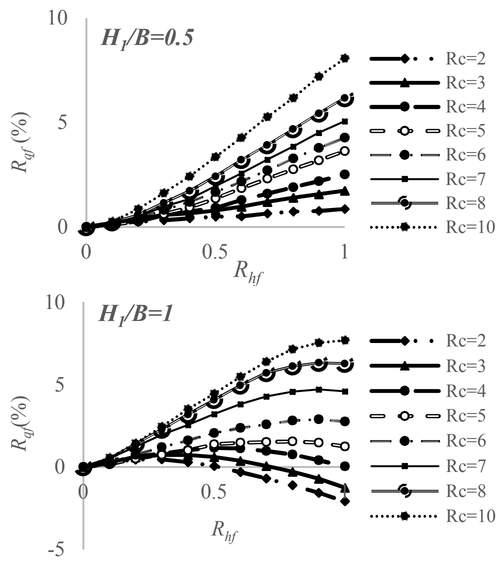
Parameter	Numerical Values
R_C	2 , 3 , ... , 10
H_1/B	0.2 , 0.75 , 1 , 2.5
R_{hf}	0 , 0.1 , ... , 1

3. 2. Results The results of parametric study are presented in Figure 7. This figure shows that only for $\frac{H_1}{B} = 0.5$, R_{qf} all values are positive and have ascending trend with increasing R_{hf}; however, the R_{qf} values are still positive in some cases when $\frac{H_1}{B} = 0.5, 1$ and 1.5. When R_{qf} is positive, the bearing capacity of two-layered clay with fuzzy boundary between layers is higher than that in classical bearing capacity analyses. This is because the majority of failure surface occurs in the lower weaker layer and by applying fuzzy depth between the layers, the upper layer merges into the the lower layer and as a result, the strength of lower layer and subsequently general strength of the soil increases. Hence, by increasing the general strength of the soil, the bearing capacity of the footing also increases and thus the R_{qf} values increases by R_{hf}. It can be also inferred from Figure 7 that by raising the R_c ratio, when $\frac{H_1}{B} < 1.0$, the R_{qf} values increase more steeply with increasing R_{hf} (i.e., when R_{hf} is constant R_{qf} increases by R_c). This is because when the difference between strength of layers increases, by applying the fuzzy depth between the layers, the upper layer with higher cohesion affects the strength of lower layer and subsequently the bearing capacity of the footing on two-layered clay more steeply.

It must be also noted that the general trend of R_{qf} is increasing with increasing the $\frac{H_1}{B}$ ratio when $\frac{H_1}{B} < 1$. When $\frac{H_1}{B}$ increases, the larger portion of failure wedge extends into fuzzy zone and therefore, the bearing capacity of the footing on two-layered clay increases as fuzzy depth is applied.

Generally, for $\frac{H_1}{B}$ ratios lower than 1, the bearing capacity of two-layered clay increases by applying fuzzy depth. This means that in these cases, classical bearing capacity solutions with considering deterministic boundary for two-layered clay are conservative and the results from these analyses can be modified economically to consider the increasing effect of applying fuzzy depth for the design purposes. The maximum value of R_{qf} for $\frac{H_1}{B} < 1$ is 8% which occurs at $\frac{H_1}{B} = 0.5$ and $R_c = 10$. when the $\frac{H_1}{B}$ ratio exceeds 1.0, the larger portion of failure surface occurs in the upper clay layer. Hence, the R_{qf} ratio decreases. For $\frac{H_1}{B} \geq 1$ and in lower R_c ratios, R_{qf} obtain negative values. For example, for $\frac{H_1}{B} = 1, R_c = 2\,, 3$ and $R_{hf} > 0.5$, the R_{qf} is negative. The absolute of negative values of R_{qf} increases by increasing the $\frac{H_1}{B}$ ratio, until $\frac{H_1}{B}$ equals 2.5. When $\frac{H_1}{B} = 2.5$ nearly the entire failure surface occurs in the top layer and by applying fuzzy depth, the general shear strength of the soil decreases. Subsequently, the absolute value of R_{qf} decreases.

In addition, the absolute values of R_{qf} have increasing trend by increasing R_c and/or R_{hf}. From negativity of R_{hf} values it could be interpreted that when $\frac{H_1}{B} \geq 1$ the bearing capacity calculated by classical methods tend to be non-conservative in some cases. The highest negative value of R_{qf} ratio in the case of $\frac{H_1}{B} \geq 1$ is approximately -10% and occurs when $\frac{H_1}{B} = 2$ and $R_c = 10$.



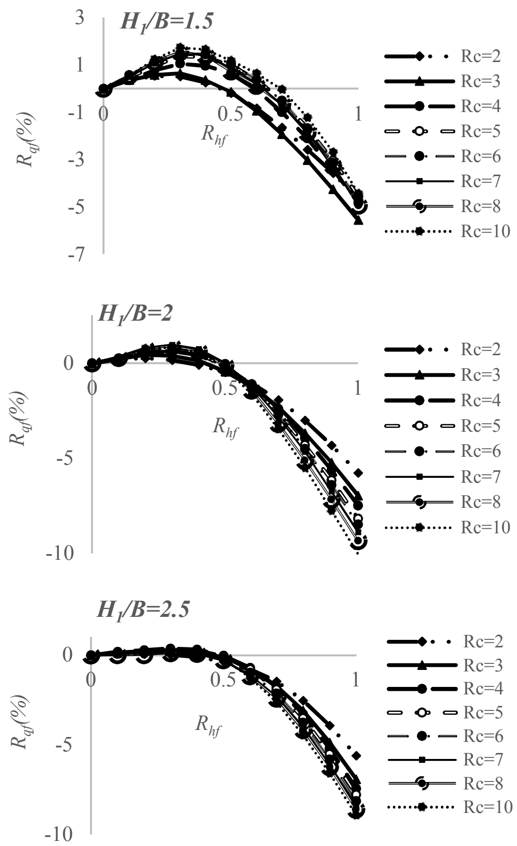


Figure 7. Variations of R_{qf} with R_{hf} for curves with different ratios of R_c

4. CONCLUSIONS

In this study, the bearing capacity of footings on two-layered clay soil was investigated by applying fuzzy boundary between the layers. Finite Difference Method was used for numerical calculation of the bearing capacity. The fuzziness in boundary of layers is considered by using fuzzy logic. From numerical analyses it can be concluded that in the case of strong over weak clay, when the thickness of upper layer is less than the width of footing, the design of footing could be more economical by considering the fuzzy boundaries between layers. However, when the thickness of upper layer is larger than the width of the footing by considering the fuzzy boundaries between layers the design of footing can be in some cases safer.

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چکیده

در این تحقیق منطق فازی جهت فرموله کردن ابهام در مرز لایه بندی برای خاک رسی دو لایه استفاده شده است. میدان خاکی با لحاظ مرزبندی فازی میان لایه‌ها ایجاد شده و سپس ظرفیت باربری پی نواری واقع بر این میدان توسط فرضیه کرنش صفحه‌ای محاسبه شده است. جهت محاسبه ظرفیت باربری مدل مور-کلمب با بهره گیری از روش تفاضل محدود استفاده شده است. اثر ابهام در مرز لایه‌بندی برای خاک دولایه رسی لایه سخت بر لایه ضعیف‌تر مورد بررسی قرار گرفته است. نتایج نشان‌دهنده این مطلب است که با اعمال لایه‌بندی فازی کاهش در ظرفیت باربری هنگامی که عمق لایه اول از عرض پی بیشتر است رخ می‌دهد. در حالتی که عمق لایه‌ی اول کمتر از عرض پی است ظرفیت باربری خاک دو لایه رسی با لحاظ مرزبندی فازی افزایش می‌یابد.

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