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RESEARCH NOTE

Numerical Simulation of Seepage Flow through Dam Foundation Using Smooth Particle Hydrodynamics Method

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ABSTRACT

In this paper, a mesh-free approach called smooth particle hydrodynamics (SPH) is proposed to analyze the seepage problem in porous media. In this method, computational domain is discredited by some nodes, and there is no need for background mesh; therefore, it is a truly meshless method. The method was applied to analyze seepage flow through a concrete dam foundation. Using the SPH method, the computational boundary being coincident with the physical boundary, was numerically acquired by solving seepage flow govern in equations. The numerical results of the presented method were compared with ones calculated by the Geostudio-SEEP\W (finite element based software). The water head values were calculated through the dam foundation, and there was a good agreement between results. Moreover, results showed that the SPH method is efficient and capable of analyzing seepage flow particularly in complex geometry problems.

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NOMENCLATURE

$f(r)$	typical field variable	m_j	mass of particle j
H	smoothing length	r	position vector
h	water head	r_{ij}	distance between particles i and j
kx	hydraulic conductivities horizontal direction	W	smoothing kernel function
kz	hydraulic conductivities in vertical direction	ρ_j	density of particle j
m_i	mass of particle i	η	0.1H

1. INTRODUCTION

In various engineering projects including environmental, hydraulic and civil engineering, the water flow through a porous media is discussed. The analysis of seepage flow plays a remarkable role in design of hydraulic structures such as dams and embankments, so that the problems of slope stability and seepage failure are crucially affected by seepage [1, 2].

Because of the complexity of analytical solution of seepage flow governing equations, numerical methods are usually pursued to deal with problems. The finite element method (FEM) has been widely accepted due to its capability to get along with the inhomogeneous and anisotropy materials and complicated boundaries [3-5]. The finite difference method (FDM) [6, 7] and finite volume method (FVM) [8, 9] have been successfully used for seepage analysis through dams. In these methods, the grid system in computational domain is numerically obtained by solving Laplace equation. Jie et al. [10] proposed a meshless based method called the natural element method (NEM) for the solution of free surface seepage problem. Results showed that the presented method is more suitable for the seepage analysis with a free surface compared with the finite element method. Shahrbanozadeh et al. [11] successfully simulated seepage flow through dam foundation using isogeometric-method. There was a good agreement between the numerical results and the experimental measurements. Toufigh [12] investigated the variability

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of the permeability coefficients in Laplace equation. Results showed that the effect of a variable permeability coefficient is not significant on small dam. However, it can be more considerable in large dams. Marandi et al. [13] numerically calculated seepage flow rate earth dams considering effects of horizontal filter blanket. Javanmard et al. [14] investigated the utilization of a plastic concrete cutoff-wall in seepage control through alluvial foundation of earth dams.

As mentioned , in most studies on seepage problem, the application of traditional element based methods has been considered. These methods have limitations including mesh generation, computation time, difficulties in modeling complex geometries and etc.; therefore, mesh-free methods can be a good approach to deal with this problem. In this paper, the smooth particle hydrodynamics (SPH) method is proposed to analyze the seepage flow through a concrete dam foundation. It is truly a mesh-free approach, and capable of solving partial differential equations (PDEs) governing seepage flow.

2. NUMERICAL MODEL

Smoothed particle hydrodynamics (SPH) is a meshfree Lagrangian approach utilized to obtain approximate numerical solutions of the fluid dynamics equations by replacing the fluid with a set of particles. The method was introduced by Gingold and Moraghan [15] to deal with astrophysical simulations.

SPH is a significantly simple and versatile mesh free approach for numerical fluid mechanics, which there is no need for generating or adapting any mesh. It can easily deal with large regions of space and complicated geometric settings that are completely devoid of particles. However, it may have higher computational cost for prescribing solid boundary conditions [16].

The SPH method relies on a full Lagrangian view of problems in which particles are given characteristic properties such as density, mass, internal energy, velocity and a characteristic smoothing length [16]. Fundamental to SPH is the theory of integral representation of functions. In SPH approach, the function integral representation is named *kernel approximation* [17].

The smoothing kernel function characterizes the contribution of a typical field variable, *f(r)*, at position, *r*, in space. The kernel estimate of *f(r)* is defined as follows [17, 18]:

$$f(r) = \int f(R)\,\bar{w}(r-R,H)\,dR \tag{1}$$

where *r* and *R* are the position vectors at different points, *H* is the smoothing length representing the effective width of the smoothing kernel function. *W(r-R, H)* is termed the smoothing kernel function in SPH or the weight function in general has the following properties:

$$\begin{aligned} &\int f(R)\,\bar{w}(r-R,H)\,dR = 1 \\ &\lim_{h\to 0} \bar{w}(r-R,H) = \delta(r-R) \end{aligned} \tag{2}$$

If r_{ij} is the distance between particles *i* and *j*, then:

$$\bar{W}_{i,j} = \bar{W}(x_i - x_j, H) = \bar{W}(|x_i - x_j|, H) \tag{3}$$

$$\nabla_i \bar{W}_{i,j} = \frac{x_i - x_j}{r_{i,j}} \frac{\partial \bar{W}_{i,j}}{\partial r_{i,j}} = \frac{x_{i,j}}{r_{i,j}} \frac{\partial \bar{W}_{i,j}}{\partial r_{i,j}} \tag{4}$$

Equation (1) can be discretized into a form of summation over all the nearest neighboring particles which are within the region controlled by the smoothing length for a given particle *i* at a certain time instant.

$$f_i = \sum_{j=1}^{n} \left(\frac{m_j}{\rho_j}\right) f_j \bar{W}_{i,j} \tag{5}$$

where m_j and ρ_j are the mass and density of particle *j*. The approximation of spatial derivatives of the function can be obtained as follows:

$$\nabla_i f_i = \sum_{j=1}^{n} \left(\frac{m_j}{\rho_j}\right) f_j \nabla \bar{W}_{i,j} \tag{6}$$

Using Equations (1) to (6), the numerical value of a function *f* and its spatial derivatives can be obtained by SPH kernel and particle approximation over a collection of smoothing particles.

3. NUMERICAL MODEL IMPLEMENTATION

The two-dimensional governing equation describing the seepage flow through a porous media in steady state and obeying the Darcy's law can be written as follows:

$$\begin{aligned} &\frac{\partial(-k_x.\frac{\partial h}{\partial x})}{\partial x} + \frac{\partial(-k_z.\frac{\partial h}{\partial z})}{\partial z} = 0 \\ &\left[\left(\frac{\partial k_x}{\partial x}.\frac{\partial h}{\partial x}\right) + k_x.\frac{\partial^2 h}{\partial x^2}\right] + \left[\left(\frac{\partial k_z}{\partial z}.\frac{\partial h}{\partial z}\right) + k_z.\frac{\partial^2 h}{\partial z^2}\right] = 0 \end{aligned} \tag{7}$$

where *kx* and *kz* are hydraulic conductivities in the horizontal and vertical directions and *h* is the water head. For homogeneous soil with respect to the hydraulic conductivity, Equation (7) simplifies to the Laplace equation.

$$\begin{aligned} &\frac{\partial^2(kh)}{\partial x^2} + \frac{\partial^2(kh)}{\partial z^2} = 0 \\ &\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2} = 0 \\ &\nabla^2\phi = 0 \end{aligned} \tag{8}$$

Using SPH method, Equation (7) can be discretized as follows:

$$\frac{\partial k_x}{\partial x} = \sum_j \frac{m_j}{\rho_j}((k_x)_j - (k_x)_i)\frac{x_j - x_i}{|r_{ij}| + \eta}\frac{\partial \widehat{W}_{ij}}{\partial r_{ij}}$$
$$\frac{\partial k_z}{\partial x} = \sum_j \frac{m_j}{\rho_j}((k_z)_j - (k_z)_i)\frac{z_j - z_i}{|r_{ij}| + \eta}\frac{\partial \widehat{W}_{ij}}{\partial r_{ij}} \quad (9)$$

$$\frac{\partial h}{\partial x} = \sum_j \frac{m_j}{\rho_j}((h)_j - (h)_i)\frac{x_j - x_i}{|r_{ij}| + \eta}\frac{\partial \widehat{W}_{ij}}{\partial r_{ij}}$$
$$\frac{\partial h}{\partial z} = \sum_j \frac{m_j}{\rho_j}((h)_j - (h)_i)\frac{z_j - z_i}{|r_{ij}| + \eta}\frac{\partial \widehat{W}_{ij}}{\partial r_{ij}} \quad (10)$$

$$\frac{\partial^2 h}{\partial x^2} = \sum_j \frac{m_j}{\rho_j}\left(\frac{4\Delta x^2}{|r_{ij}|^2} - 1\right)\left(\frac{(h)_j - (h)_i}{|r_{ij}|^2 + \eta}\right)\nabla_i \widehat{W}_{ij}.r_{ij}$$
$$\frac{\partial^2 h}{\partial z^2} = \sum_j \frac{m_j}{\rho_j}\left(\frac{4\Delta z^2}{|r_{ij}|^2} - 1\right)\left(\frac{(h)_j - (h)_i}{|r_{ij}|^2 + \eta}\right)\nabla_i \widehat{W}_{ij}.r_{ij} \quad (11)$$

For homogeneous soil where $kx = kz$, the SPH discretized form of Equation (8) can be written as follows:

$$\nabla^2 \phi = \sum_j \frac{2m_j}{\rho_j}\left(\frac{\phi_i - \phi_j}{|r_{ij}|^2 + \eta^2}\right)\nabla_i \widehat{W}_{ij}.\vec{r}_{ij} = 0 \quad (12)$$

where $\eta=0.1H$, and it is applied to avoid dividing by zero during calculations.

3. 1. Solution Algorithm At first the solution domain, number of particles, initial distance between particles and also particles position is determined. Then according to particles density which equals water density, the mass of all particles is calculated using Equation (13):

$$m_i = m_j = \frac{\rho_i = \rho_w}{\sum_j \widehat{W}(|r_i - r_j|, h)} \quad (13)$$

The initial particles water head at boundary layers is specified respect to initial conditions at upstream and downstream. The particles density is calculated using Equation (14).

$$\rho_i = \sum_{j=1}^{n} m_j W(|r_i - \bar{r}_j|, H) \quad (14)$$

where m_i and ρ_i are the mass and density of particle i, m_j and ρ_j are the mass and density of neighbor particles, and h is the smoothing length. Finally, by solving governing equations, the values of water head for each particle and other parameters including seepage velocity and discharge can be calculated respectively. Figure 1 shows the SPH method algorithm for seepage flow simulation in a porous media.

The governing boundary conditions in seepage problem can be specified as Dirichlet and Neumann boundary condition. Boundaries with constant water head for water free surface at upstream and downstream can be defined as Dirichlet boundary condition.

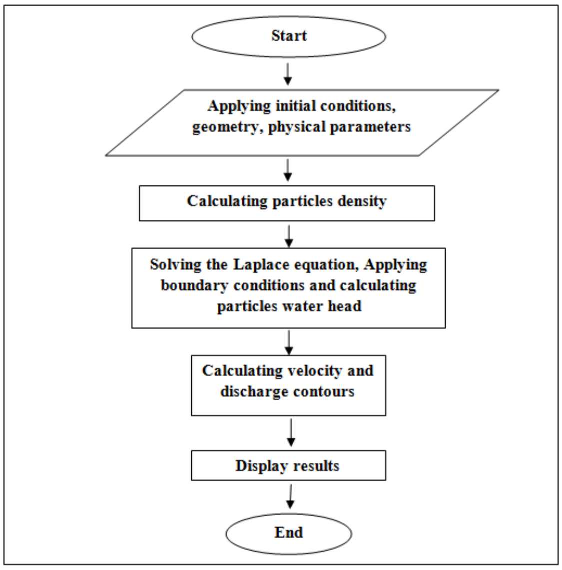


Figure 1. The SPH method algorithm for seepage flow simulation in a porous media

The Laplace equation is not solved for the particles with Dirichlet boundary condition. Neumann boundary condition can be set to boundaries with specified function derivative values such as impermeable boundaries.

4. RESULTS AND DISCUSSION

In this study, the SPH method has been implemented in seepage problem through concrete dam foundation with cut-off wall. To evaluate the effectiveness and practicability of the proposed method, a finite element based software namely Geostudio (SEEP\W) has been implemented under the same conditions. It is a powerful finite-element-based geotechnical program, capable of solving 2-D seepage problems with multiple soil layers [19].

The analysis of seepage flow through dam foundations is very important in stability analysis of these structures. Moreover, uplift force can be calculated by analyzing seepage through dam foundation. In this study, seepage flow through a concrete dam foundation with cut-off wall is investigated. The model features are presented in Table 1. Figure 2 shows the model geometry and geotechnical section introduced to SEEP\W.

Utilizing the SPH method, seepage through the concrete dam foundation with cut-off wall was simulated. The initial distance between particles was considered 0.2 m, and then the smoothing length was selected 0.24 ($h=1.2r$), respectively. The total number of fluid particles considered in simulation was 9134, and the walls were modeled by 526 particles. Moreover, the SEEP\W model of dam foundation is employed to verify the analytical results obtained by proposed method.

TABLE 1. The model features of concrete dam foundation

Material properties	**Foundation material**	**hydraulic conductivity (Foundation)**	**hydraulic conductivity (cut-off wall)**	**foundation dimensions (m)**	**Dam width (base) (m)**	**Upstream water level (m)**
Isotropic and homogeneous	Saturated clay	10-5 m/s	10-7 m/s	50 x 6	4	11

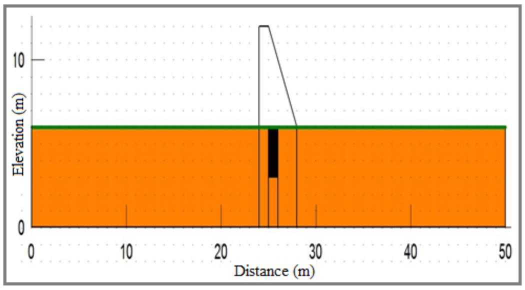
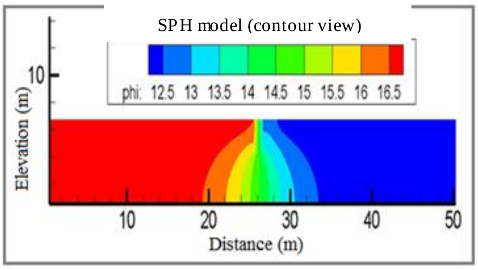


Figure 2. The model geometry and geotechnical section introduced to SEEP\W

Figure 3 shows the numerical results of water head values (contour and scatter view) obtained from the SPH modeling and SEEP/W software results. As it shown, the results agreed well. It can be seen that the process of water movement through concrete dam foundation with cut-off wall was modeled properly. In addition, the boundary conditions have been applied well in modeling which the equipotential lines are perpendicular to dam base and Impermeable layer. At the cross section below the cut-off wall, due to the narrowing of the flow cross section, the equipotential lines and also flow lines are compressed together. According to Figure 3, the formation of the flow lines along the concrete dam floor and the cut-off wall is well displayed. The seepage discharge calculated by SEEP/W software and SPH method are 1.4979 $\times 10^{-5}$ m^3/s and 1.3705$\times 10^{-5}$ m^3/s, respectively. The difference is about 8.5 percent that shows a reasonable agreement between results.

The seepage velocity vectors through concrete dam foundation have been shown in Figure 4. According to Figure 4, it can be seen that flow vectors passed underneath the cut-off wall to downstream boundary of concrete dam.



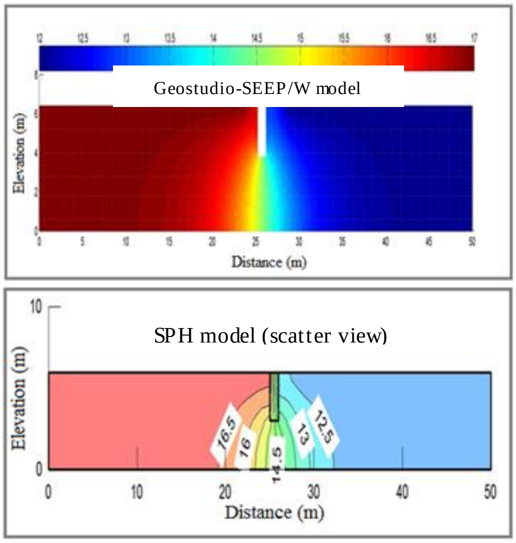


Figure 3. Seepage flow in concrete dam foundation simulated by SEEP\W and SPH method

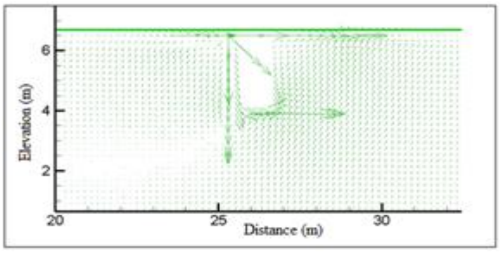


Figure 4. Seepage velocity vectors through dam foundation

5. CONCLUSIONS

In the present paper the capability of smoothed particle hydrodynamics (SPH) method for seepage flow analysis in a porous media has been investigated. The method was applied to deal with seepage problem through concrete dam foundation with cut-off wall. The method results in seepage analysis were verified by employing well-established seepage software (Geostudio-SEEP\W) in parallel. The results agreed well, and it was seen that the SPH method is capable of seepage problem analysis. Due to the constraints in mesh-based methods, the modeling of complex geometries such as cut-off wall edges is more difficult. While in SPH method, there is no limitation in mesh size, and it can easily model such geometries.

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چکیده

مقاله حاضر به بررسی روش بدون شبکه هیدرودینامیک ذرات هموار شده جهت آنالیز نشست آب در محیط های متخلل می پردازد. در این روش محدوده محاسباتی به وسیله تعدادی گره گسسته سازی شده و هیچ نیازی به شبکه بندی محیط حل در این روش نمی باشد. لذا این روش کاملا روشی بدون شبکه می باشد. از روش ارائه شده جهت شبیه سازی و انالیز نشت آب در فونداسیون سدهای بتنی استفاده شده است. با استفاده از روش (SPH)، محدوده محاسباتی که با محدوده فیزیکی منطبق بوده است، بصورت عددی با حل معادلات حاکم بر نشت آب حاصل شد. نتایج حاصل از روش ارائه شده با نتایج حاصل از مدل سازی توسط نرم افزار تحلیل نشت (Geostudio-SEEP\W) مورد مقایسه قرار گرفت. مقادیر هد آب در تمامی طول فونداسیون سد محاسبه گردید و نتایج از همخوانی و دقت قابل قبولی برخوردار می باشند. با بررسی روش هیدرودینامیک ذرات هموار شده مشخص گردید که این روش از توانایی قابل توجهی در آنالیز نشت به ویژه در محیط هایی با هندسه های پیچیده برخوردار می باشد.

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