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Dynamic Response of Multi-cracked Beams Resting on Elastic Foundation

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ABSTRACT

Cracks cause to change dynamic response of beams and make discontinuity in slope of the deflection of the beams. The dynamic analysis of the Euler-Bernoulli cracked beam on the elastic foundation subjected to the concentrated load is presented in this paper. The stiffness of the elastic foundation and elastic supports influence on vibrational characteristics of the cracked beam. The Dynamic Green Function is applied to solve the governing equation. Thus, the dynamic response of the cracked beam is determined by Laplace Transform method. The effects of depth and location of the crack on natural frequency and deflection of the cracked beam on an elastic foundation are evaluated. In order to demonstrate the capability of the present approach, several numerical examples are worked out and the results are discussed.

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NOMENCLATURE

EI	Flexural rigidity
K_w	Stiffness of elastic foundation
$q(x,t)$	The applied load
$W(x,t)$	The deflection of beam
$H(.)$	The Heaviside function
x_{io}	The exerted load location
$\sum_{i=1}^{4}\frac{\partial^{4-i}W(0)}{\partial x^{4-i}}$	Boundary condition
$K_{LT};K_{RT}$	The transfer spring stiffness at the left and right support
$K_{LR};K_{RR}$	The rotational spring stiffnessat the left and right support
Δy	Slope in deflection of beam
M	Bending moment that transmitted by the cracked section
$D_{h'}$	The non-dimensional constant spring model
k	The stiffness of massless torsional spring
L	Length of the beam
h	Height of the beam
b	Width of the beam
h'	Depth of the crack $\left(h' = h_c/h\right)$

Greek Symbols

μ	Mass per unit length of the beam
$\delta(.)$	Dirac delta function
ρ	Material density

1. INTRODUCTION

The foundation plays the important role in most of the engineering structures. Various researchers are investigated the dynamic response of the cracked beam. Bovsunovsky and Matveev [1] assessed the dynamic behavior of the beam with a closing crack. The effect of crack factors on natural frequencies and mode shapes of a cantilever Euler-Bernoulli beam is obtained by the analytical method. The effects of cracks on beam with different boundary conditions were investigated by Khiem and Lien [2]. They assessed the effects of number of cracks and the boundary condition on natural frequency based on the transfer matrix approach. Qiang et al. [3] computed the electromagnetic area in a simulation of conflicting beams using Green function. An approach for detecting location and size of cracks

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according to changes in modal energy and natural frequencies was provided by Kim and Stubbs [4].

In order to obtain the nonlinear vibration of the cracked beam on the elastic foundation, a discrete physical model is presented by Khnaijar and Benamar [5]. The cracked section and soil stiffness are modeled by the spiral and the vertical spring, respectively. The influences of soil parameters and large amplitude vibration are assessed. Chang and Chen [6] investigated the cracked beam by the spatial wavelet analysis to detect the location and depth of cracks. They used modal shape to anticipate locations of cracks, and natural frequencies for various depth of the cracks. Xiang et al. [7] worked on the cracked and non-cracked beams to predict the location and size of the crack. Hence, the B-spline wavelet approach is used to gain the first three frequencies for analysis of cracked beams [7]. The identification of the dynamiccharacteristics of the double cracked beam is analyzed numerically and experimentally by Yoon et al. [8]. They modeled the damaged section of the cracked beam by a local flexibility matrix, also they recognized the dynamic behavior of the cracked beam using Hamilton's principle. Sekhar [9] briefed the different assessments on the cracked beam and assessed the methods effects. Xiaoqing et al. [10] investigated the natural frequencies of various boundary condition cracked beam based on the bending vibration theory. Deokar and Wakchaure [11] researched the location and crack depth using natural frequencies. They plotted the first three natural frequencies to find the intersection of them in order to identify the location and depth of crack [11].

Attar et al. [12] studied the cracked Timoshenko beam that supported by elastic foundation. A lattice spring model is used to represent the transverse shear deformation and the rotary inertia. The LSM is applied to discrete beam into the one-dimensional segment. A fluid conveying micro-beams is used to analyze dynamic behavior of the micro-structure resting on the foundation by Kural and Ozkaya [13]. The Hamilton's principle is applied to solve the nonlinear equations of motion. Batihan and Kadioğlu [14] investigated the cracked beam on a Pasternak and generalized elastic foundations. Both the Euler-Bernoulli and the Timoshenko theories were used to analyze the cracked beam and determine the natural frequency in different location and depth of the crack. Cracked nano-beam is investigated by TadiBeni et al. [15]. The effect of crack position on natural frequency is studied in this paper. They found that the effect of this characteristics is significant. Frequency analysis is employed to obtain the natural frequency of the stepped and cracked Timoshenko beam by Sadeghian and Ekhteraei [16]. The various boundary condition and the elastic foundation are considered and the dynamic response of cracked beam is determined. Analysis of the non-uniform cracked beam using employing optimization method is presented by Ranjbaran et al. [17]. Free vibration of the cracked beam based on functionally graded materials and using Euler-Bernoulli, Rayleigh and Timoshenko theory is studied by Sherafatnia et al. [18]. In this study, the influences of material properties are evaluated. The modeling of the cracked section as beam element is introduced by Nakhaei et al. [19]. A stepped cross-section is used to take in account characteristics of crack. Also, Zhao et al. [20] studied the damped multi-cracked beam using Green function method. The results are compared with the finite element method and it is observed that there are insignificant errors.

In most studies, the effect of elastic support, and in particular the use of the Green Function method for solving the governing equations of the elastic foundation of the cracked beam, is not considered. The influences of elastic foundation and boundary condition are studied and the dynamic response of cracked beam is obtained, in this paper.

2. MODELING OF MULTI-CRACKED EULER-BERNOULLI BEAM BY GREEN FUNCTION

The governing equation of the Euler-Bernoulli beam supported on elastic foundation is defined as follows [21]:

$$EI\frac{\partial^4 W(x,t)}{\partial x^4}+\mu\frac{\partial^2 W(x,t)}{\partial t^2}+K_W-q(x,t)=0 \tag{1}$$

Now, let us assume that q(x,t) and $w(x,t)$ are a harmonic function. The terms of load and deflection are written as follows:

$$q(x,t)=q(x)T(t)$$
$$W(x,t)=W(x)T(t) \tag{2}$$

where $T(t)$ is defined as $T(t)=Exp(i\Omega t)$. By substituting Equation (2) in Equation (1), the following relation obtained:

$$\frac{\partial^4 W(x)}{\partial x^4}+\frac{\mu\Omega^2+K_W}{EI}W(x)=\frac{q(x)}{EI} \tag{3}$$

In the above equation, on taking the Green's function method the resulting equation may be obtained as follows [22]:

$$\frac{\partial^4 G(x)}{\partial x^4}+\frac{\mu\Omega^2+K_W}{EI}G(x)=\frac{\delta(x-x_0)}{EI} \tag{4}$$

Now, by applying the Laplace Transform on Equation (4), the differential equation is converted to algebraic equation as follows [23]:

$$G(x;x_0) = -\frac{H(x-x_0)}{4EI\beta^3}\sum_{n=1}^{4}\alpha_n Exp\,(\alpha_n \beta(x-x_0)) + \frac{1}{4}\sum_{i=1}^{4}\left(\sum_{n=1}^{4}\alpha_n^{\,i}\beta^{\,i-4} Exp(\alpha_n \beta x)\right)\frac{\partial^{4-i}W\,(0)}{\partial x^{4-i}} \tag{5}$$

The terms β and α_n are defined as the following expression:

$$\beta = \left(\frac{\mu\Omega^2 + K_W}{EI}\right)^{0.25} \tag{6}$$

$$\alpha_n = \{1,-1,+i,-i\} \tag{7}$$

In this paper, the multi-cracked Euler-Bernoulli beam on elastic foundation with elastic boundary condition is modeled as Figure 1. The cracks divide beam into $n+1$ parts. For each part, the Equation (5) is expanded. According to Figure 1, for the first and last segments of multi-cracked beam, the Equation (5) is presented as follows:

$$G_i(x_i;x_{i0}) = \frac{H(x_i - x_{i0})}{-4EI\beta^3}\sum_{n=1}^{4}\alpha_n Exp\,(\alpha_n \beta(x_i - x_{i0})) + \frac{1}{4}\sum_{n=1}^{4}[EI\,(\frac{-\alpha_n^4}{k_{LT}}\frac{\partial^3 w_i(0)}{\partial x^3} + \frac{\alpha_n^3}{\beta k_{LR}}\frac{\partial^2 w_i(0)}{\partial x^2}) + (\frac{\alpha_n^2}{\beta^2}\frac{\partial^2 w_i(0)}{\partial x^2} + \frac{\alpha_n}{\beta^3}\frac{\partial^3 w_i(0)}{\partial x^3})]Exp\,(\alpha_n \beta x_i) \tag{8a}$$

$$G_n(x_n;x_{n0}) = \frac{H(x_n - x_{n0})}{-4EI\beta^3}\sum_{n=1}^{4}\alpha_n Exp\,(\alpha_n \beta(x_n - x_{n0})) + \frac{1}{4}\sum_{n=1}^{4}[EI\,(\frac{-\alpha_n^4}{k_{LT}}\frac{\partial^3 w_n(0)}{\partial x^3} + \frac{\alpha_n^3}{\beta k_{LR}}\frac{\partial^2 w_n(0)}{\partial x^2}) + (\frac{\alpha_n^2}{\beta^2}\frac{\partial^2 w_n(0)}{\partial x^2} + \frac{\alpha_n}{\beta^3}\frac{\partial^3 w_n(0)}{\partial x^3})]Exp\,(\alpha_n \beta x_n) \tag{8b}$$

In addition, the following equation for the middle segments of cracked beam can be expressed as follows:

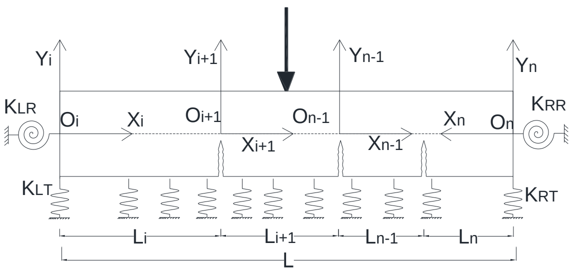


Figure 1. Multi-cracked Euler-Bernoulli beam on elastic foundation

$$G_{i+1}(x_{i+1},x_{(i+1)0}) = \frac{H(x_{i+1} - x_{(i+1)0})}{-4EI\beta^3}\sum_{n=1}^{4}\alpha_n Exp\,(\alpha_n \beta(x_{i+1} - x_{(i+1)0})) + \frac{1}{4}\sum_{n=1}^{4}[\alpha_n^4 w_{i+1}(0) + \frac{\alpha_n^3}{\beta}\frac{\partial w_{i+1}(0)}{\partial x} + \frac{\alpha_n^2}{\beta^2}\frac{\partial^2 w_{i+1}(0)}{\partial x^2} + \frac{\alpha_n}{\beta^3}\frac{\partial^3 w_{i+1}(0)}{\partial x^3}]Exp\,(\alpha_n \beta x_i) \tag{9}$$

The continuity of the cracked beam must be satisfied. For this purpose, the continuity relation for i-th segment $(1<i<n-1)$ may be written as follows:

$$\begin{aligned}
&G_i\big|_{x_i=L_i} - G_{i+1}\big|_{x_{i+1}=0} = 0\\
&\left.\frac{\partial^2 G_i}{\partial x_i^2}\right|_{x_i=L_i} - \left.\frac{\partial^2 G_{i+1}}{\partial x_{i+1}^2}\right|_{x_{i+1}=0} = 0\\
&\left.\frac{\partial^3 G_i}{\partial x_i^3}\right|_{x_i=L_i} - \left.\frac{\partial^3 G_{i+1}}{\partial x_{i+1}^3}\right|_{x_{i+1}=0} = 0\\
&\left.\frac{\partial G_i}{\partial x_i}\right|_{x_i=L_i} + \frac{1}{k}\left.\frac{\partial^2 G_{i+1}}{\partial x_{i+1}^2}\right|_{x_{i+1}=0} - \left.\frac{\partial G_{i+1}}{\partial x_{i+1}}\right|_{x_{i+1}=0} = 0
\end{aligned} \tag{10}$$

The last segment continuity relation has the following expression [24]:

$$\begin{aligned}
&G_{n-1}\big|_{x_{n-1}=L_{n-1}} - G_n\big|_{x_n=L_n} = 0\\
&\left.\frac{\partial^2 G_{n-1}}{\partial x_{n-1}^2}\right|_{x_{n-1}=L_{n-1}} - \left.\frac{\partial^2 G_n}{\partial x_n^2}\right|_{x_n=L_n} = 0\\
&\left.\frac{\partial^3 G_{n-1}}{\partial x_{n-1}^3}\right|_{x_{n-1}=L_{n1-}} + \left.\frac{\partial^3 G_n}{\partial x_n^3}\right|_{x_n=L_n} = 0\\
&\left.\frac{\partial G_i}{\partial x_i}\right|_{x_i=L_i} + \frac{1}{k}\left.\frac{\partial^2 G_{i+1}}{\partial x_{i+1}^2}\right|_{x_{i+1}=0} + \left.\frac{\partial G_{i+1}}{\partial x_{i+1}}\right|_{x_{i+1}=0} = 0
\end{aligned} \tag{11}$$

By substituting the Equations (8) and (9) in continuity relation, the matrix form equation of the multi-cracked beam is obtained and the natural frequency is determined by computing the determinant of matrix.

Existence crack in beam make local flexibility in crack location. The massless torsional spring is used to cracked section. Crack during length of the beam discretes beam and creates discontinuity. The stiffness of massless torsional spring is given as follows:

$$k = \frac{EI}{(h)(D_{h'})} \tag{12}$$

From mechanics fracture, the non-dimensional constant $(D_{h'})$ can be calculated. After evaluating various theories of $(D_{h'})$, Bilello [25] proposed the fallowing relation for $(D_{h'})$ based on the depth of the crack ratio:

$$D_{h'} = \frac{h'(2-h')}{0.9(1-h')^2} \tag{13}$$

3. NUMERICAL EXAMPLES

3. 1. Free Vibration of Clamped Cracked Beamon Elastic Foundation

The natural frequency of clamped cracked beam on elastic foundation, based on Batihan and Kadioğlu data's [14] presented in Table 1. The crack located in $L/8$ and the crack ratio varies from 0.02 to 0.5. In order to modeling the clamped beam the value of k_{LT}, k_{RT}, k_{LR} and k_{RR} is considered a large number. As illustrated in Table 1, there are insignificant error in the results.

3. 2. The Natural Frequencies of Elastic-elastic Cracked Beam on Elastic Foundation

The elastic– elastic cracked beam under concentrated load at $L/2$ has the following properties (see Figure 1):

$L = 3m$, $E = 2\times10^9 Pa$, $\rho = 7850 \, kg/m^3$

$b = h = 0.2m$, $h' = 0.5$

$K_w = 6.71\times10^4 \, N/m^2$

$k_{LT} = k_{RT} = k_1$, $k_{LR} = k_{RR} = k_2$

The first three natural frequencies of elastic-elastic beam on elastic foundation with a crack at $L/2$ are provided in Table 2. According to Table 2, increasing the stiffness of the springs lead to increasing natural frequencies. In the case which $k_1 = 0$, second and third natural frequency changes are more evident. In the case $k_1 = k_2 = 1000$, the action of cracked beam is as same as clamped cracked beam and also, in case which $k_1 = 1000$ and $k_2 = 0$ **,** the cracked beam behave such as a simply supported cracked beam.

TABLE 1. Natural frequency of clamped cracked beam on elastic foundation

Crack ratio	Ref. [14]	Present study	Error (%)
0.02	83.532	83.661	0.1841
0.04	83.523	83.626	0.1771
0.2	83.267	83.257	0.0144
0.4	82.420	82.438	0.0267
0.5	81.673	81.756	0.1248

TABLE 2. The first three natural frequencies of elastic-elastic cracked beam on elastic foundation

k_1	Natural frequencies	k_2				
		0	1	10	100	1000
0	FNF	14.618	14.618	14.618	14.618	14.618
	SNF	59.682	25.481	33.386	34.950	35.123
	TNF	200.231	80.780	103.851	109.401	110.035
1	FNF	23.360	25.451	26.769	26.997	27.021
	SNF	42.497	45.094	47.780	48.388	48.456
	TNF	76.115	91.065	109.512	114.181	114.719
10	FNF	29.236	38.337	49.756	52.829	53.190
	SNF	97.759	99.890	102.962	103.834	103.938
	TNF	153.353	153.988	155.062	155.409	155.451
100	FNF	30.212	41.390	58.290	63.642	64.300
	SNF	124.977	137.859	167.401	180.227	181.955
	TNF	236.525	246.541	273.862	287.678	289.635
1000	FNF	30.316	41.728	59.304	64.946	65.642
	SNF	128.293	143.210	179.429	195.903	198.147
	TNF	249.25	264.068	308.817	333.64	337.247

FNF, SNF and TNF demonstrate the first, second and third natural frequency

3. 3. Dynamic Response of Cracked Clamped-elastic Beam The cracked beam with clamped-elastic boundary conditions illustrated in Figure 2. The following properties are assumed for beam:

L=2.1m, b=0.25m, h=0.1m, h´=0.6

E=2.01×10^{9} *N/m*^{2}*, ρ=7800 kg/m*^{3}

The results of the cracked beam with variety of the stiffness values in support A and B are illustrated in Figure 3. The vertical dash line shows the crack location and the vertical axis denotes the beam displacement. Comparison of results show:

When there is no stiffness in support B, the reached displacement in first segment has more value (see Figures 3A and 3B). In addition, when support A has no stiffness, the effects of support A reaction on beam displacement in Figure 3C is evident. By increasing the stiffness value of support A, the cracked beam can absorb the applied load then the effect of support A reaction decrease (see Figure 3D).

3. 4. Force Vibration of Double Cracked Elastic-elastic Beam with Different Foundation Stiffness The double cracked beam according to the specifications of section 3.2 is presented. As showed in Figure 4, the elastic foundation is placed between two cracks. In this example, the concentrated load with $\Omega = 7 rad/s$ is applied at center point of the cracked beam.

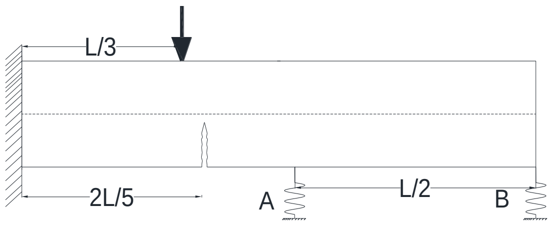
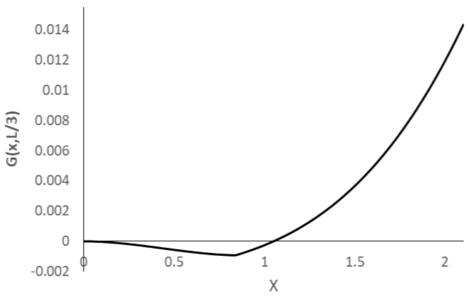
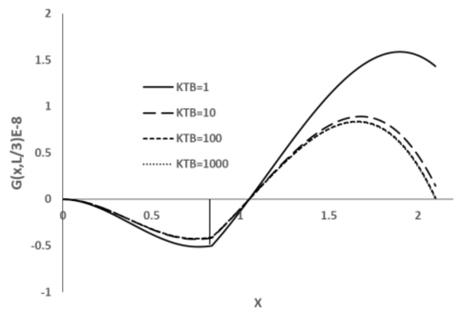


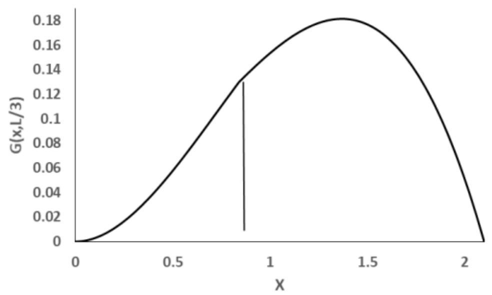
Figure 2. Cracked beam with fixed-elastic conditions



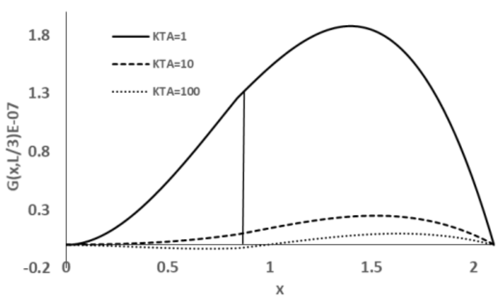
(a) $K_{TA} = 1000EI$, $K_{TB} = 0$



(b) $K_{TA} = 1000EI$, $K_{TB} = 0$



(c) $K_{TA} = 0$, $K_{TB} = 100EI$



(d) $K_{TA} = 0$, $K_{TB} = 1000EI$

Figure 3. The deflection of fixed-elastic cracked beam in various stiffness

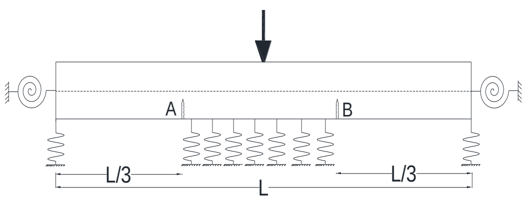


Figure 4. Double-cracked beam with partial elastic foundation

First, it is assumed that k_{LT}, k_{RT}, k_{LR} and k_{RR} are in the largest amount of their own. Subsequently, it is considered that $k_{LR} = k_{RR} = 0$ as well as k_{LT} and k_{RT} are in the largest value. Figures 5 and 6 are represented the deflection of the cracked beam with $h' = 0.5$. Conforming to Figures 5 and 6, by increasing the stiffness of the elastic foundation leads to decrease the deflection, especially variations of $k_w = 0$ are more evident than $k_w = 100EI$ .It is note that the simply supported beam is more sensitive against the stiffness of elastic foundation than clamped beam.

3. 5. The Effects of Stiffness of Elastic Foundation on First Three Natural Frequencies

The double simply supported cracked beam on elastic foundation with $h' = 0.7$ under concentrated load is exhibited in Figure 7. The beam properties are same as section 3.2. The first three natural frequency in various stiffness of elastic foundation are provided in Table 3.

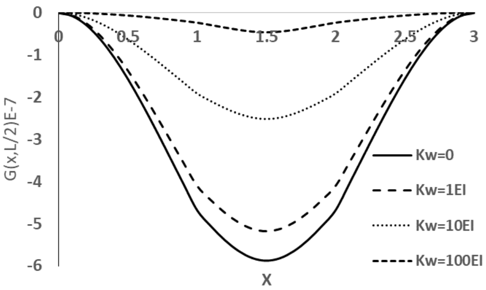


Figure 5. The deflection of clamped-clamped cracked beam on elastic foundation with various stiffness

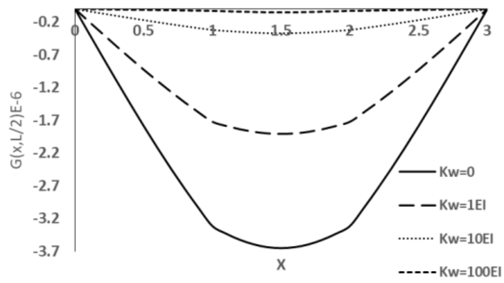


Figure 6. The deflection of simply supported cracked beam on elastic foundation with various stiffness

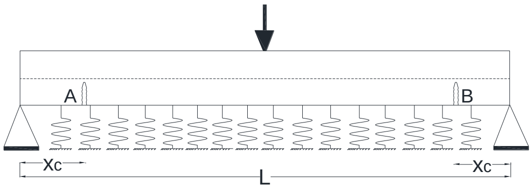


Figure 7. Double-cracked beam with different cracks location

According to Table 3, by increasing stiffness of elastic foundation (k_w), natural frequencies enhance. In high value of the stiffness of elastic foundation, the characteristics of simply supported cracked beam are changed.

TABLE 3. The first three natural frequencies of simply supported beam in various stiffness of elastic foundation

Frequency	x_c	Stiffness of Elastic Foundation (k_w)								
		0	1EI	5EI	10EI	20EI	50EI	100EI	200EI	500EI
FNF		28.060	40.454	70.948	96.332	133.313	207.967	292.768	413.084	652.239
SNF	0.1L	87.258	91.994	108.904	126.911	156.84	270.383	304.203	447.767	674.739
TNF		175.057	177.464	186.790	197.83	218.241	381.995	339.956	522.789	726.695
FNF		22.170	36.616	68.831	94.784	132.199	207.254	292.262	412.726	652.012
SNF	0.2L	64.470	70.750	91.666	112.468	145.401	292.249	298.466	417.142	683.795
TNF		207.237	209.275	217.239	226.802	244.810	433.900	357.592	461.300	755.271
FNF		18.394	34.461	67.709	93.972	131.619	206.884	292.000	412.541	651.895
SNF	0.3L	60.009	66.710	88.585	109.971	143.479	214.625	297.535	416.476	654.392
TNF		466.777	467.677	471.295	475.778	484.621	510.231	550.271	622.675	801.561
FNF		16.440	33.459	67.205	93.610	131.360	206.720	291.884	412.458	651.843
SNF	0.4L	64.906	71.148	91.973	112.718	145.595	216.046	298.561	490.941	654.860
TNF		266.784	268.367	274.623	282.248	296.912	337.097	395.092	993.257	704.131

4. CONCLUSION

The force and free vibration of multi-cracked beam with elastic support and elastic foundation is investigated based on Dynamic Green Foundation method in this paper. The natural frequencies of clamped beam in various depth of the crack obtained and compared with Batihan and Kadioğlu investigation the result showed that there are insignificant errors. The deflection and natural frequencies of various boundary condition as well as the various stiffness cracked beam supports studied. As denoted by increasing the stiffness of supports, the natural frequency ascended. The effects of elastic foundation on natural frequencies and deflection studied and the results of simply supported and clamped cracked beam is presented. The results demonstrated that the simply supported double cracked beam is more sensitive than clamped double cracked beam.

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Dynamic Response of Multi-cracked Beams Resting on Elastic Foundation

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چکیده

وجود ترک در طول تیر باعث به وجود آمدن ناپیوستگی در شیب تغییر مکان تیر می‌شود. تحلیل دینامیکی تیر با ترک‌های متعدد بر روی بستر الاستیک با استفاده از روش تابع گرین دراین مقاله مطالعه شده است. تیر ترک دار با شرایط تکیه گاهی الاستیک و همچنین بستر الاستیک در این مطالعه بررسی می‌شود.تابع گرین برای تیر با شرایط تکیه گاهی مختلف حل شده است و تاثیر شرایط مرزی مختلف بر روی تغییر مکان و همچنین فرکانس‌های طبیعی تیر ترک‌دار مطالعه شده است. علاوه بر این، اثر ترک در موقعیتو عمق مختلف تیر بررسی می‌شود. تاثیر سختی بستر و تکیه گاه‌ها بر روی پاسخ دینامیکی تیر ترک‌دار نیز ارزیابی شده است و برای مقایسه بهتر، مثال‌های متعددی ارائه شده است.

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