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Investigation of the Size Effect on the Nano-beam Type Piezoelectric Low Power Energy Harvesting

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A B S T R A C T

In this paper, size dependent nano-beam type peizoelectric energy hardvester is investigated. For this goal, first nonlinear formulation of isotropic piezoelectric Euler-Bernoulli nano-beam is developed based on the size-dependent piezoelectricity theory then special nano-beam type piezoelectric energy hardvester is probed for different parameters. Basic nonlinear equations of piezoelectric nano-beam are derived using principle of minimum potential energy and variational method. To evaluate the formulation derived, static deformation and free vibration of the hinged-hinged piezoelectric nano-beam is investigated in the special case. The results of the derived formulation are investigated under different parameters, and particularly, the ability and performance of the beam type piezoelectric low power energy harvesting was evaluated in nanoscale.

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NOMENCLATURE		Greek Symbols	
g_{ijklmn}	Strain gradient elasticity constants	ρ	Density
e_{kij}	Piezoelectric constants	ω	Nonlinear natural frequency
D_i	Electric displacement	σ_{ij}	Stress tensor
c_{ijkl}	Elastic constants	τ_{ijk}	Higher order stress tensor
a_{ij}	Dielectric constants	η_{jkl}	Strain gradient tensor
f_{jkli}	Electric field-strain gradient coupling coefficient tensor	ε_{ij}	Strain tensor
E_i	Electric field	φ	Electric potential

1. INTRODUCTION

There are a lot of energy in the environment that are dissipated most of time without any applications such as wind energy, solar energy, walking energy, sound and vibration energy, etc. The energy harvesting technology can use the wasted energies from the environment and convert them to electrical energy. Energy harvesting from the environment is the focus of researchers recently and many studies have been conducted to improve them [1-4].

Karami and Inman [5] approximated electromechanical coupling as equivalent changes in damping and frequency to simplify the analysis of energy harvesting systems. They used hybrid piezoelectric and electromagnetic energy harvester system in linear, softly nonlinear and stable cases. Their research showed during uses an optimal resistant load,

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the amplitude of mechanical vibrations is the smallest. Mohammadpour et al. [6] focused on nonlinear energy harvesting for electro-mechanical system. They used semi analytical method for analysis the behavior of a multi model nonlinear electromechanical system and used arc length method for achieve frequency response of the system.

Nano structures have higher specific surface than other materials therefore classic theory unable explain nanomaterials behavior. However, nonclassic theory based on continum mechanics utiliezed by researchers in the literature to model the large ratio of surface to bulk in the nanoscale materials. One of these non classic theory is modified couple stress theory (MCST). Many researchers used of this theory for modeling the nano and micro structures [7, 8].

Simsek [9] used the modified couple stress theory for Euler Bernouli beam to obtain dynamic response. results compared with previously studies and good agreement observed. Bakhshi Khaniki and Hosseini [10] by modified couple stress theory modeled the microbeams and solved for different types of boundary conditions analytically and the effects of nonuniformity and microscale effects were investigated.

Researchers have been studied vibration, deflection and buckling for different type of beam with nonclassical theories [11-15]. For example, Akgoz and Civalek [16] studied vibration response for Euler-Bernouli microbeam with modified couple stress theory. They used Rayleigh-Ritz solution method to solve free vibration. They results exibited that natural frequency observed from classic theory is smaller than MCST and concluded microbeams modeled on MCST become stiffer than classic theory.

Mehralian et al. [17] used higher-order theory to functionally graded piezoelectric cylindrical nanoshell and investigated buckling behavior.

Nonlinear static and dynamic response of laminated plates are also investigated by Baltacıoglu et al. [18].

Today's nanoscale piezoelectric materials used in nano electromechanical systems, thus understanding the behavior of these structures is important [19]. Tadi Beni [20] developed size-dependent model for a piezoelectric nano-beam. He used the piezoelectric couple stress theory proposed by Hadjesfandiari as well as Hamilton's principle to obtained the governing equations and boundary conditions. Also Tadi Beni et al. [21] investigate size effect in micro and nano structures with higher order continuum theory. On another article, Tadi Beni [22] used size dependent piezoelectricity theory and developed nonlinear equations of piezoelectric nano-beam.

Shah-Mohammadi-Azar et al. [23] modeled sandwiched piezoelectric nano-beam based on the nonlocal elasticity theory. Wang and Wang [24] purposed model for nano size energy harvester and investigated flexoelectric effect for voltage and power output.

Liang et al. [25] studied buckling and vibration of piezoelectric nanowires. Continuum model for Euler-Bernoulli beam considered and external voltage applied for buckling analysis. They concluded that resonant frequency dependent on external applied voltage on the beam thickness. Shams Nateri [26] analyzed mathematical modeling to consider the effect of material properties on the sensitivity of the micro electromechanical systems. Sensitivity of the sensor has been improved from -201.3 dB to -192.6 dB by choosing the proper material with higher piezoelectric coefficients.

According to previous work [27-29], one concluded that the nonlinear analyzed for nano-beam with nonclassic theory has not been done yet. Therefore, we choose one of the nonclassic theories, specially modified couple stress theory, and investigate size effect on nonlinear property of the nano-beam.

In this paper, Euler-Bernouli nano-beam is considered as isotropic and piezoelectric. Nonlinear formulation is developed based on the size-dependent piezoelectricity theory. For deriving the governing equation, principle of minimum potential energy and variational method is used. static deformation and vibration of the hinged-hinged piezoelectric nano-beam is investigated and numerical results for Polyvinylidene flouride(PVDF) nano-beam obtaind.

2. PRIELIMINARIES

In this section, nonlinear equations with using of size dependent piezoelectricity model and modified couple stress theory for isotropic piezoelectric nano-beam is derived. Also, Euler-Bernoulli beam model is used that is acceptable model of beam in the literature.

A nano-beam of length L, width b and height h is considered in Figure (1). The longitudinal axis is denoted by x_1 and transverse axis by x_3. Poling direction is along x_3-axis.

Displacement field for Euler-Bernoulli beam model is [22]:

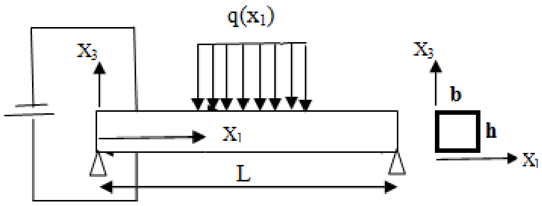

Figure 1. Schematic of piezoelectric nano-beam

$$u_1 = u_0(x_1) - x_3 \frac{\partial w(x_1)}{\partial x_1}, u_2 = 0, u_3 = w(x_1). \tag{1}$$

where u_1 and u_3 are the displacement in longitudinal and transverse directions and u_0 is the displacement of midplane of nano beam. The constitutive equations for piezoelectric material are proposed as follows [30, 31]:

$$\sigma_{ij} = c_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \tag{2}$$

$$\tau_{ijk} = -f_{lijk}E_l + g_{ijklmn}\eta_{lmn}, \tag{3}$$

$$D_i = a_{ij}E_j + e_{ijk}\varepsilon_{jk} + f_{jkli}\eta_{jkl}. \tag{4}$$

where $D_i, \sigma_{ij}, \tau_{ijk}, c_{ijkl}, e_{kij}$ and a_{ij} represent the components of electric displacement, stress tensor, higher order stress tensor, elastic coefficients, piezoelectric coefficients and dielectric coefficients ,respectively. g_{ijklmn} is strain gradient elasticity tensor and f_{jkli} is the electric field-strain gradient coupling tensor. Also, η_{jkl} is strain gradient tensor that defined as:

$$\eta_{ijk} = \varepsilon_{ij,k} = \frac{1}{2}(u_{i,jk} + u_{j,ik}). \tag{5}$$

Strain tensor ε_{ij} and electric field E_i are defined as follows:

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{6}$$

$$E_i = -\varphi_{,i}. \tag{7}$$

where, φ is electric potential. In modified couple stress theory that used in this study, by setting strain gradient elasticity tensor g_{ijklmn}, and the electric field-strain gradient coupling tensor f_{jkli} (the higher-order electro-mechanical coupling tensor) to zero, the formulation of classical continuum theory can be obtained.

3. GOVERNING EQUATIONS AND RELATED BOUNDARY CONDITIONS

Here, principle of minimum potential energy is used to obtain the equilibriums equations and related boundary conditions that defined as follows:

$$\delta U - \delta W = 0. \tag{8}$$

where δW is the variation of external load works on nano-beam and δU is the variation of strain energy. The modified couple stress strain energy density for piezoelectric nano-beams defined as [32]:

$$U = \frac{1}{2}\int_V [\sigma_{ij}\varepsilon_{ij} + \tau_{ijk}\eta_{ijk} - D_k E_k]. \tag{9}$$

By substituting Equation (1) into Equations (5)-(7), the nonzero components of the strain and strain gradient tensors are as follow:

$$\varepsilon_{11} = \frac{\partial u_0}{\partial x_1} - x_3 \frac{\partial^2 w}{\partial x_1^2} + \frac{1}{2}(\frac{\partial w}{\partial x_1})^2, \tag{10}$$

$$\eta_{11} = \frac{\partial^2 u_0}{\partial x_1^2} - x_3 \frac{\partial^3 w}{\partial x_1^3}, \tag{11}$$

$$\eta_{13} = -\frac{\partial^2 w}{\partial x_1^2}. \tag{12}$$

Now, by using non-zero components calculated in Equations (10)-(12) and substituting them into Equations (2)-(4), the resulted equations are present as below:

$$\sigma_{11} = c_{1111}\varepsilon_{11} - e_{311}E_3, \tag{13}$$

$$\tau_{11} = g_{111111}\eta_{11}, \tau_{13} = g_{113113}\eta_{13} - f_{3113}E_3, \tag{14}$$

$$D_1 = a_{11}E_1, D_3 = a_{33}E_3 + e_{311}\varepsilon_{11} + f_{3113}\eta_{13}. \tag{15}$$

The work of external loads is expressed as:

$$\delta W = \int_0^L q(x_1)\delta w dx_1. \tag{16}$$

According to Equations (13)-(15) the variation of strain energy is as follows:

$$\begin{aligned} \delta U &= \int_V \{\sigma_{11}\delta\varepsilon_{11} + \tau_{11}\delta\eta_{11} + \tau_{13}\delta\eta_{13} \\ &- D_1\delta E_1 - D_3\delta E_3\} dV \\ &= \int_0^L \int_A \{\sigma_{11}\delta\varepsilon_{11} + \tau_{11}\delta\eta_{11} + \tau_{13}\delta\eta_{13} \\ &- D_1\delta E_1 - D_3\delta E_3\} dA dx_1. \end{aligned} \tag{17}$$

In above equation A is cross section of beam. By substituting Equations (13)-(15) into Equation (17) and used variation method, the longitudinal, transverse and elecrical governing equations are developed as follows:

$$\begin{aligned} &A g_{111111}\frac{\partial^4 u_0}{\partial x_1^4} - A c_{1111}(\frac{\partial^2 u_0}{\partial x_1^2} + \frac{1}{2}\frac{\partial}{\partial x_1}(\frac{\partial w}{\partial x_1})^2) \\ &- b e_{311}\frac{\partial}{\partial x_1}(\int_{-h/2}^{h/2}\frac{\partial \varphi}{\partial x_3}dx_3) = 0, \end{aligned} \tag{18}$$

$$
\begin{aligned}
&-Ig_{111111}\frac{\partial^6 w}{\partial x_1{}^6}+(Ic_{1111}+Ag_{113113})(\frac{\partial^4 w}{\partial x_1{}^4})-q(x_1)\\
&-(\frac{\partial^2 w}{\partial x_1{}^2})\left(Ac_{1111}(\frac{\partial u_0}{\partial x_1}+\frac{1}{2}(\frac{\partial w}{\partial x_1})^2)+be_{311}(\int_{-h/2}^{h/2}\frac{\partial \varphi}{\partial x_3}dx_3)\right)\\
&-(\frac{\partial w}{\partial x_1})\left(Ac_{1111}(\frac{\partial^2 u_0}{\partial x_1{}^2}+\frac{1}{2}\frac{\partial}{\partial x_1}(\frac{\partial w}{\partial x_1})^2)+be_{311}(\frac{\partial}{\partial x_1}(\int_{-h/2}^{h/2}\frac{\partial \varphi}{\partial x_3}dx_3))\right)\\
&-bf_{3113}\frac{\partial^2}{\partial x_1{}^2}(\int_{-h/2}^{h/2}\frac{\partial \varphi}{\partial x_3}dx_3)=0,
\end{aligned}
\tag{19}
$$

$$
a_{11}\frac{\partial^2\varphi}{\partial x_1{}^2}+a_{33}\frac{\partial^2\varphi}{\partial x_3{}^2}+e_{311}\frac{\partial^2 w}{\partial x_1{}^2}=0. \tag{20}
$$

The mechanical and electrical boundary conditions that used for hinged-hinged nano-beam are as follows:

$$
u_0\big|_{x_1=0,\mathrm{L}}=0, \tag{21}
$$

$$
(\frac{\partial^2 u_0}{\partial x^2{}_1})\bigg|_{x_1=0,L}=0, \tag{22}
$$

$$
w\big|_{x_1=0,L}=0, \tag{23}
$$

$$
\left(-Ig_{111111}\frac{\partial^4 w}{\partial x_1{}^4}-bf_{3113}(\int_{-h/2}^{h/2}\frac{\partial \varphi}{\partial x_3}dx_3)+(Ic_{1111}+Ag_{113113})(\frac{\partial^2 w}{\partial x_1{}^2})\right)\bigg|_{x_1=0,L}=0, \tag{24}
$$

$$
(\frac{\partial^2 w}{\partial x_1{}^2})\bigg|_{x_1=0,L}=0, \tag{25}
$$

$$
\int_0^L (a_{33}E_3+e_{311}\varepsilon_{11}+f_{3113}\eta_{13}))dx_1\bigg|_{x_3=h/2,-h/2}=0, \tag{26}
$$

$$
\varphi\big|_{x_1=0,L}=0. \tag{27}
$$

In above equations, I express area moment of inertia about y-axis. Equations (18)-(20) together with boundary conditions in Equations (21)-(27) are nonlinear equations for hinged-hinged nano-beams with modified couple stress theory. It should be noted that, on the assumption of isotropic nanobeam e_{311} is zero.

4. STATIC BENDING OF PIEZOELECTRIC NANO-BEAM

The equilibrium equations can be transformed for the dimensionless case by assuming dimensionless parameters as follows:

$$
\begin{aligned}
&\bar{w}=\frac{w}{h},\bar{x}_1=\frac{x_1}{L},\bar{x}_3=\frac{x_3}{h},\bar{u}_0=\frac{u_0}{h},\\
&\xi=\frac{Al^2}{I},\beta=\frac{Ah^2}{I},\mathrm{S}=\frac{L}{h},\\
&\lambda_1=\frac{a_{33}}{a_{11}}\mathrm{S}^2,\lambda_2=(\frac{l}{L})^2,\lambda_3=\frac{c_{1313}}{c_{1111}},\lambda_4=\frac{f_{3113}}{c_{1111}}(\frac{L^2}{h^4})
\end{aligned}
$$

Equlibirum equations in the dimensionless case defined as following:

$$
\xi\frac{\partial^4\bar{u}_0}{\partial\bar{x}_1^4}-\beta S(S\frac{\partial^2\bar{u}_0}{\partial\bar{x}_1^2}+\frac{1}{2}\frac{\partial}{\partial\bar{x}_1}(\frac{\partial\bar{w}}{\partial\bar{x}_1})^2)=0, \tag{28}
$$

$$
\begin{aligned}
&-\lambda_2\frac{\partial^6\bar{w}}{\partial\bar{x}_1^6}+(1+\xi\lambda_3)(\frac{\partial^4\bar{w}}{\partial\bar{x}_1^4})-\bar{q}(\bar{x}_1)\\
&-\lambda_4\frac{\partial^2}{\partial\bar{x}_1^2}(\int_{-1/2}^{1/2}\frac{\partial\varphi}{\partial\bar{x}_3}d\bar{x}_3)\\
&-(\frac{\partial^2\bar{w}}{\partial\bar{x}_1^2})\left[\beta(S\frac{\partial\bar{u}_0}{\partial\bar{x}_1}+\frac{1}{2}(\frac{\partial\bar{w}}{\partial\bar{x}_1})^2)\right]\\
&-(\frac{\partial\bar{w}}{\partial\bar{x}_1})\left[\beta(S\frac{\partial^2\bar{u}_0}{\partial\bar{x}_1^2}+\frac{1}{2}\frac{\partial}{\partial\bar{x}_1}(\frac{\partial\bar{w}}{\partial\bar{x}_1})^2)\right]=0,
\end{aligned}
\tag{29}
$$

$$
\lambda_1\frac{\partial^2\varphi}{\partial\bar{x}_3^2}+\frac{\partial^2\varphi}{\partial\bar{x}_1^2}=0. \tag{30}
$$

The static deflection of nano beam in the above equations can be obtained by using the Galerkin method. It should be noted that for the Galerkin method, the assumed displacements in the longitudinal and transverse directions must be satisfied the boundary condituions in Equations (21)-(25). First, Laplaces Equation (30) solved by separation method and using boundary condition (26), the electric potential can be expressed as follows:

$$
\varphi(\bar{x}_1,\bar{x}_3)=\sum_{n=1}^{\infty}\left(A_n\sinh(\frac{n\pi}{\sqrt{\lambda_1}}\bar{x}_3)+B_n\cosh(\frac{n\pi}{\sqrt{\lambda_1}}\bar{x}_3)\right)\sin(n\pi\bar{x}_1), \tag{31}
$$

Also, beam deflection is assumed in the way that satisfied boundary equations in Equations (23), (25) as follows:

$$
\bar{w}(\bar{x}_1)=A\sin(\pi\bar{x}_1). \tag{32}
$$

By utilizing the Equations (31)-(32) and using remainder boundary conditions in Equations (24) and (26), the relationship between parametere A_n, B_n and A is obtained as follows:

$$
A_n=\frac{\frac{f_{3113}\pi}{S}}{\frac{na_{33}S}{\sqrt{\lambda_1}}\cosh(\frac{n\pi}{2\sqrt{\lambda_1}})}A, \tag{33}
$$

$$B_n = 0. \tag{34}$$

By substituting parameter A_n based on constant A from Equation (33) into Equations (31) and (32), The longitudinal and transverse displacement of the beam as well as electric potential is expressed as:

$$\bar{u}_0(\bar{x}_1) = C\sin(\pi\bar{x}_1), \tag{35}$$

$$\bar{w}(\bar{x}_1) = A\sin(\pi\bar{x}_1), \tag{36}$$

$$\varphi(\bar{x}_1,\bar{x}_3) = A\sum_{n=1}^{\infty}\frac{f_{3113}\pi\sqrt{\lambda_1}}{na_{33}S^2\cosh(\frac{n\pi}{2\sqrt{\lambda_1}})}\sinh(\frac{n\pi}{\sqrt{\lambda_1}}\bar{x}_3)\sin(n\pi\bar{x}_1). \tag{37}$$

Now, by substituting $\bar{u}_0(\bar{x}_1), \bar{w}(\bar{x}_1)$ and $\varphi(\bar{x}_1,\bar{x}_3)$ into Equations (28) and (29) and using Galerkin method, the values of parameter A and C can be determined.

5. NON-LINEAR VIBRATION OF THE PIEZOELECTRIC NANO-BEAM

By rendering the nanobeam's equations of motion dimensionless and using the dimensionless parameter $\tau = t\sqrt{\frac{c_{1111}I}{\rho AL^4}}$ and dismissing the effect of external forces, governing equations for the nanobeam free vibrations are expressed as:

$$\begin{aligned}&\frac{\partial^2\bar{w}}{\partial\tau^2} - (\frac{\partial^2\bar{w}}{\partial\bar{x}_1^2})\left[\beta(S\frac{\partial\bar{u}_0}{\partial\bar{x}_1} + \frac{1}{2}(\frac{\partial\bar{w}}{\partial\bar{x}_1})^2)\right] \\ &-(\frac{\partial\bar{w}}{\partial\bar{x}_1})\left[\beta(S\frac{\partial^2\bar{u}_0}{\partial\bar{x}_1^2} + \frac{1}{2}\frac{\partial}{\partial\bar{x}_1}(\frac{\partial\bar{w}}{\partial\bar{x}_1})^2)\right] \\ &+(1+\xi\lambda_3)(\frac{\partial^4\bar{w}}{\partial\bar{x}_1^4}) - \lambda_2\frac{\partial^6\bar{w}}{\partial\bar{x}_1^6} \\ &-\lambda_4\frac{\partial^2}{\partial\bar{x}_1^2}(\int_{-1/2}^{1/2}\frac{\partial\varphi}{\partial\bar{x}_3}d\bar{x}_3) = 0,\end{aligned} \tag{38}$$

$$\lambda_1\frac{\partial^2\varphi}{\partial\bar{x}_3^2} + \frac{\partial^2\varphi}{\partial\bar{x}_1^2} = 0. \tag{39}$$

For the case of nonlinear vibration, first the beam's vibration response and electric potential are considered as follows:

$$\bar{w}(\bar{x}_1,\tau) = \bar{w}(\bar{x}_1)r(\tau) \tag{40}$$

$$\phi(\bar{x}_1,\bar{x}_3,\tau) = \varphi(\bar{x}_1,\bar{x}_3)r(\tau) \tag{41}$$

The boundary conditions are similar to the previous section and all of them satisfied with assuming above equations. Then by substituting Equations (40), (41) into Equation (38) and using Galerkin method, the following nonlinear ordinary differential equation is obtained:

$$\ddot{r}(\tau) + \alpha_1(r(\tau))^3 + \alpha_2 r(\tau) = 0 \tag{42}$$

There are different methods for solving nonlinear vibrations. In this paper the He's variational method used that done as follow:

$$F(r(\tau)) = \int_0^{T/4}\{-\frac{1}{2}\dot{r}^2(\tau) + \frac{\alpha_1}{4}r^4(\tau) + \frac{\alpha_2}{2}r^2(\tau)\}d\tau \tag{43}$$

In the above equation, T is nonlinear vibration period in the He's variational method and $r(\tau)$ assumed as follows:

$$r(\tau) = a\cos(\omega\tau) \tag{44}$$

In the above equation, a and ω represent nanobeam initial deflection and nonlinear natural frequency. By substituting Equation (44) into Equation (43), yeilds:

$$\begin{aligned}F(r(\tau)) = \int_0^{T/4}\{&-\frac{1}{2}a^2\omega^2\sin^2(\omega\tau) \\ &+\frac{\alpha_1}{4}a^4\cos^4(\omega\tau) + \frac{\alpha_2}{2}a^2\cos^2(\omega\tau)\}d\tau\end{aligned} \tag{45}$$

According to He's variational method, for finding nonlinear natural frequency we must have $\frac{\partial F(r(\tau))}{\partial a} = 0$, Therefor:

$$\begin{aligned}\frac{\partial F(r(\tau))}{\partial a} = \frac{1}{\omega}\int_0^{T/4}\{&-a^2\omega^2\sin^2(\omega\tau) \\ &+\alpha_1 a^3\cos^4(\omega\tau) + \alpha_2 a\cos^2(\omega\tau)\}d\tau = 0\end{aligned} \tag{46}$$

Now, the nonlinear natural frequency find as follows:

$$\omega_{NL} = \sqrt{\alpha_2 + \frac{3}{4}a^2\alpha_1} \tag{47}$$

Here a is taken as $\sqrt{3}/3$.

6. RESULTS AND DISCUSSION

In this study, nanobeam made of PVDF film that dimension is $(100nm \times 5nm \times 5nm)$. The properties of PVDF film used are E=238.24 GPa, ρ=1.74 g/cm^3 [33]. According to previous studies, the size effect has considerable affect on the nanoscale materials. Maximum deflection of PVDF nano-beam at different height-to-size effect parameter ratio (h/l) obtained for linear couple stress theory (LCST) and nonlinear couple stress theory (NLCST) and drawn in Figure 2. It should be noted that to derive the nano-beam responses, assumed here h=5nm, L=100nm, $f_{3113} = 5pC/m$ and $\bar{x}_1 = 0.5$.

Figure 2 exhibites that with deceasing the size effect parameter, deformation increased. In reverse, with increasing the size effect parameter, nano-beam becomes stiffer therefore deformation decreased.
As is clear from Figure 3 the closer the size of length scale parameter to the constractive dimensions of the nanobeam such as its height, the effect of length scale parameter is considerable. Another point worth noting is that in greater dimensions of the nanobeam or greater values of h/l, the results are closer to each other. However, the purpose of Figure 3 is to show the importance of size effect and nonlinearity in nano-scale for nano-beams. The result also has displayed in Table 1 in another view.

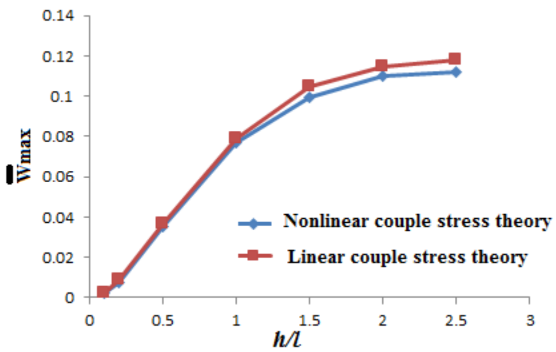

Figure 2. Effect of material size scale parameter (l) on the maximum deflection

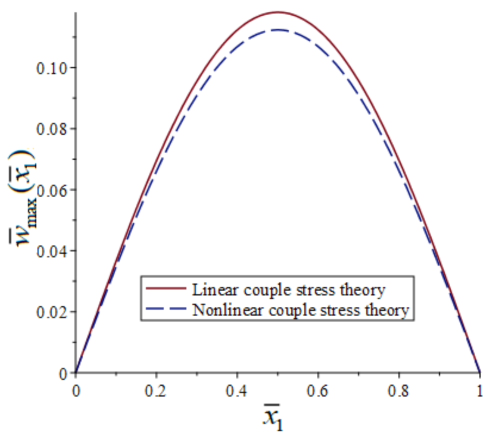

Figure 3. Static deflection of piezoelectric nanobeam

TABLE 1. Comparison of size effect parametere on deflection nano-beam.

$\bar{w}_{max}(\bar{x}_1)$	$l=10h$	$l=2h$	$l=h$	$l=0.4h$
LCST	0.001937	0.03567	0.07727	0.1123
NLCST	0.001937	0.03572	0.07852	0.1181

Table 1 also has good consent about the considerable effect of length scale parameter on the deflection of nanobeam.

By applying the force $q(x_1)$ on the nano-beam deformation occurred. The shape of deformation of the nano-beam drawn on Figure 3.

Deflection shown in two modes of linear couple stress theory and non linear couple stress theory. As is clear, the response of the nano-beam deflection equation in the nonlinear case is generally lower than that in the linear case which is due to increased bending stiffness of the beam in the nonlinear couple stress case compared to the linear one.

Figure 4 shows the changes of voltage in thickness direction of the nanobeam.

It is clear that on both sides of nanobeam the maximum voltage occurred due to the maximum stress occurred in the top and bottom of nanobeam. It should be noted that, potential difference on two sides of the nanobeam is zero (closed circuit condition).

The changes of voltage in length of the beam for different x_3 shown in Figure 5.
The results show that by farfroming the neutral axis or nearing the top or bottom of the nano-beam the stress incresed and caused increasing the output voltage.

Table 2 illustrate increases size effect parameter leads to decreses natural frequency in both cases (linear and nonlinear couple stress theory).

In Figure 6, the nano-beam natural frequency variation has also been illustrated similar to Table 2 in another view.

This can be inferred from Figure 6. First, as clear, the natural frequency in the nonlinear case is higher than that in the linear case. Second, the change of natural frequency for couple stress theory for closer the size of length scale parameter to the constractive dimensions of the nanobeam such as its height is considerable.

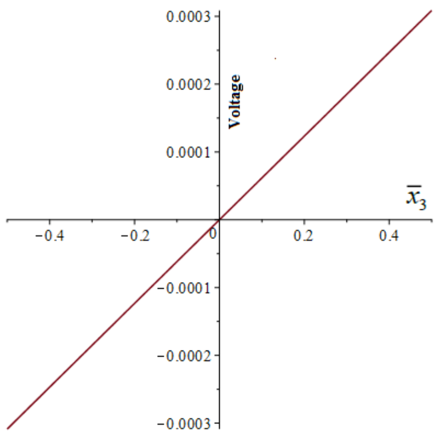

Figure 4. Voltage changes in the thickness of the beam

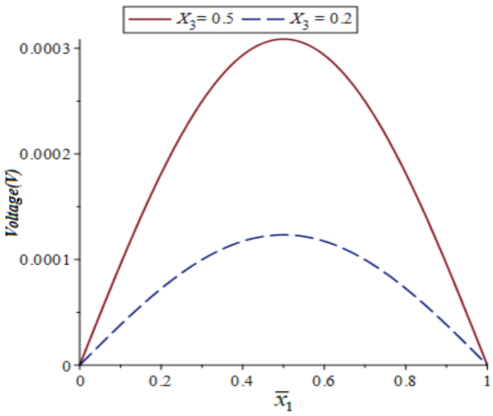

Figure 5. Voltage changes in the length of nano- beam

TABLE 2. Comparison of size effect parametere on natural frequency of the nano-beam

ω	$l=10h$	$l=2h$	$l=h$	$l=0.4h$
LCST	81.1422	18.8793	12.7229	10.4107
NLCST	81.8142	21.5873	16.4759	14.7631

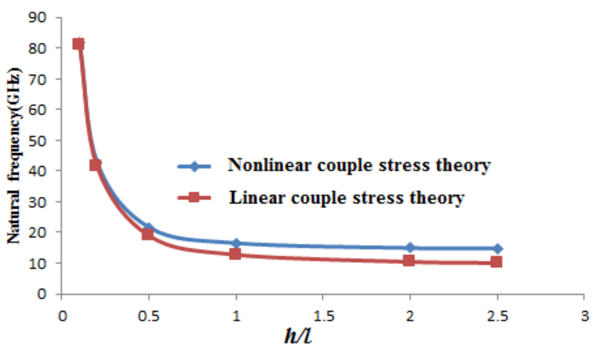

Figure 6. The changes of frequency variation with dimensionless length scale parameter

7. CONCLUSION

In this study, nonlinear formulation of isotropic piezoelectric Euler-Bernoulli nano-beam is developed based on the size-dependent piezoelectricity theory. Basic nonlinear equations of piezoelectric nano-beam are derived using principle of minimum potential energy and variation method. The effect of length scale parameter on the static deformation and vibration of the hinged-hinged piezoelectric nano-beam was investigated .

The most important observations are:

- The deformation of the nano-beam by the linear couple stress theory are larger than those by the nonlinear couple stress theory.
- With increasing the size effect parameter, nano-beam becomes stiffer therefore deformation decreased.
- Natural frequency decreased with increas of dimensionless length scale parameter in linear and nonlinear nano-beam.
- The natural frequency of the linear case is lower than the nonlinear case.
- The maximum output voltage occurred due to the maximum stress occurred in the top and bottom of nanobeam.

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Investigation of the Size Effect on the Nano-beam Type Piezoelectric Low Power Energy Harvesting

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چکیده

در این مقاله نانوتیر وابسته به اندازه پیزوالکتریک به عنوان برداشت‌کننده انرژی مورد بررسی قرار گرفته است. برای این هدف ابتدا فرمول غیرخطی از نانوتیر اویلر برنولی ایزوتروپ بر پایه تئوری وابسته به اندازه پیزوالکتریک به دست آمده و سپس برای نوع خاصی از این نانوتیر پارامترهای مختلف مورد بررسی قرار گرفته است. معادلات غیرخطی پایه با روش کمینه انرژی پتانسیل و وردش‌گیری به دست آمده است. برای ارزیابی معادلات به دست آمده خمش استاتیکی و ارتعاش آزاد نانوتیر پیزوالکتریک دو سر مفصل برای حالت خاصی بررسی گردیده و نتایج برای معادلات مذکور نشان می‌دهد که این تیر قابلیت برداشت کم انرژی را در مقیاس نانو دارد.

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