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## A Possibilistic Bi-objective Mode I for A Competitive Supply Chain Network Design under Variable Coverage

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#### ABSTRACT

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Keywords: Facility location on the plane Chain-to-chain competition Radius of influence Cannibalization effect Capacity planning Multi-objective possibilistic programming demand of each demand zone is divided equally between the retailers which can cover that market. However, the demand of distant customers who are not in the retailers' radius of influence, will be lost. This competition is modelled for a real case application of a super-market chain. It is assumed that the chain's owner wants to expand retail outlets to improve its market share. Since this expansion could affect the current customers of existing retailers, the owner wants to avoid attacking the market share of its current retailers. A bi-objective fuzzy mixed integer nonlinear model is proposed. For solving the model, it is first reformulated to a mixed integer linear program and then an interactive approach is devised to handle the fuzzy bi-objective model. Four expansion strategies are analysed from which useful managerial insights are drawn.

In this paper, the strategic planning of a supply chain under a static chain-to-chain competition on the

plane is addressed. It is assumed that each retailer has a coverage area called the radius of influence. The

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NOMENCLATURE								
Ι	Set of current manufacturers $\;i\in\left\{ 1,\ldots,I^{L}\right\}$ .	${\boldsymbol{\tilde{b}}}_k$	Annual buying power of demand zone k					
J	Set of retailers in which $j \in \{1,, j_c\}$ are current retailers and $j \in \{j_c + 1,, J^L\}$ . are new candidate sites	$\widetilde{r}_k^d$	Fuzzy radius that defines areas near a demand zone					
К	Set of demand zones $k \in \{1,, K\}$ .	$\tilde{\mathbf{D}}_k$	Fuzzy demand of demand zone k					
$x_k^d, y_k^d$ .	Location coordinates of market points	c <sup>s</sup>	Fixed cost of establishing a retailer					
$\mathbf{x}_{i}^{m},\mathbf{y}_{i}^{m}$	Location coordinates of manufacturers	$\tilde{\boldsymbol{c}}_k^d$	The fuzzy extra fund (in percent) for locating the retailers in near the demand zones					
$\begin{bmatrix} x^{L}, x^{U} \end{bmatrix}, \\ \begin{bmatrix} y^{L}, y^{U} \end{bmatrix}$	Limits of the plane under consideration	$c_{ij}^t$	The transportation cost between an ufacturer $i$ and retailer $j$					
r <sup>max</sup>	Maximum radius of influence of retailers	c <sup>r</sup>	Unit cost of improving of radius of influence					
CAP <sub>r</sub> <sup>Max</sup>	Maximum capity of retailers	c <sup>capr</sup>	Unit cost of improving of storage capacity					
$T_k^1$	Fixed marketing cost	М	Big M					
$T_k^2$	Variable marketing cost	З	A very small value					
$z_j^0$	1 for $j \in \{1,, j_c\}$ ; otherwise 0	$x^r_j \ y^r_j$	Continuous variable; the location of the retailer $j \in \left\{ j_c,, J^L \right\}$					

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$r_j^0$	Radius of influence of existing retailers; $r_j^0 > 0$ for $j \in \{1,, j_c\}$ ; otherwise 0	r <sub>j</sub>	Ctinuous variable; radius of influence of the retailer j.
$CAP_{j}^{r0}$	Capacity of existing retailers; $cap_{j}^{r0}>0 \mbox{ for } j \in \left\{1,\ldots,j_{c}\right\};$ otherwise $0$	$cap_{j}^{r}$	Continuous variable; the storage capacity of the retailer in location j
$CAP_i^m$	Storage capacity of manufacturer i	s <sub>jk</sub>	Continuous variable; amount of market share allocated to retailer j from k th market
F <sub>k</sub>	Number of rival's retailers which the demand zone $k$ is in their catchment area	q <sub>ij</sub>	Continuous variable; amount of goods transported from the manufacturer i to the retailer j
$S_{jk}^0$	Market share of existing retailer j from the market point k before expansion of SC	$\mathbf{z}_{j}$	Binary variable; 1, if a retailer is established in location $j$ , and 0 otherwise
$O_{jk}^0$	1 if the retailer had been serving the demand zone; otherwise 0.	O <sub>jk</sub>	Binary variable; 1, if the market point $k$ is in the radius of influence of the retailer of location $j$ and 0 otherwise
В	Total available budget	g <sub>jk</sub>	Binary variable; 1, if the new retailer $j$ is located near the demand zone k and 0 otherwise

#### **1. INTRODUCTION**

A major body of the literature on supply chain network design (SCND) and facility location (FL) problems investigate those situations where no other SC exists [1]. Nevertheless, in a real world market, there are often several players. Moreover, the competition in traditional markets were among corporations. However, it has evolved to competition among supply chains (SCs). In this new chain-to-chain competition, firms compete as dependent entities of SCs [2]. A competitive SCND problem includes considering the effects of strategic design of a supply chain (SC) in a competitive situation on its market share alongside the conventional long-term decisions.

One classical extension of the FL problem is the coverage problem that aims to ensure each demand zone is served by a set of facilities within reasonable distance. This threshold in the literature of competitive FL is called radius of influence [3]. In the real world, the customers may refuse to buy from distant retailers. The term 'distant' depends on the radius of influence of retailers. The radius of influence includes the region where the retailer remains attractive for customers. It is endogenously related to the service level of retailers [4]. By this definition, the demand of those customer zones, which are outside the covering radius of retailers are lost. Generally, retailers with larger size can construct more attractive facilities like parking area and food court [5]. Consequently, they have larger radius of influence and attract more distant customers.

Consider a situation that a demand zone is in the coverage area of more than one retailer. There are various allocation rules to formulated the market share of the rival SCs like the price dependent rule, the gravity rule and covering approach. We have categorized the literature of the chain-to-chain competition in Table 1

based on these allocation rules. Unlike the price dependent rule that concentrates on a tactical decision in estaimating the market share of a rival, the second and third rules concentrate on the effect of location and service level which are strategic (i.e. long-term) decisions. In the gravity rule, the total demand of each market is split among the available facilities proportional to the utility they provide to that market. However, the covering approach equally splits it among the covering retailers.

One critical question is: which allocation rule can better estimate the market shares of rival SCs? Drezner et al. [3] analysed the sensitivity of the market shares to different allocation rules. They showed that the covering rule has significant effect on the market shares. However, there is no evidence to support that more sophisticated rules like the gravity rule necessarily yield more accurate results. Consequently, the simple equal division rule provides a reasonable estimation of market shares.

Suppose that there is a retail SC whose owner wants to expand its chain. Following the expansion, the customers can buy required goods from either the new retailers of this SC or the pre-existing retailers of each SC rival. Resulting from the complex competition, establishing new facilities may lead to the market reduction effect on pre-existing retailers [5]. Managers have to control the number of customers of current facilities who may switch to the new facilities after the expansion plan [16].

This research aims to develop a model for the competitive strategic planning of retail outlets for an expanding SC considering the variable coverage of retailers and the market reduction effect while retail outlets are scattered in the plane.

As you can see from Table 1, SCND problem with chain-to-chain competition is a new stream of the literature.

Competitive Characteristic				(	D		
Location (distance)	Price	Service level	Radius of influence	Price dependent	Gravity rule	Covering Approach	Reference
	*			*			[6]
	*			*			[7]
	*	*		*			[8]
*	*			*	*		[9]
	*			*			[10]
	*			*			[11]
	*			*			[12]
	*	*		*			[13]
		*			*		[2]
*					*		[5]
			*			*	[14]
	*			*			[15]
*			*			*	This paper

TABLE 1. The literature of SCND problems under chain-to-chain competition based on customer choice rules used

Moreover, incorporating the radius of influence under covering approach is also a new development for this field. From the academic point of view, the motivations and contributions for this paper are as follows: 1) considering the effect of strategic decisions of SCND phase in the market share of a SC in the form of a chainto-chain competition, 2) incorporating the variable coverage of facilities and the concept of lost demand which is a new approach for market share estimation, 3) considering market reduction effect as an objective function and 4) accounting for epistemic uncertainty in critical input data and providing a suite for trade-off analysis of two important objectives via formulating a biobjective possibilistic model.

The rest of this paper is organized as follows. In the next section, the literature of chain-to-chain competition, market reduction effect and the covering approach in SCs are reviewed. In section 3, the problem addressed in this research is defined in detail and formulated. In section 4, the solution approach is provided. In section 5, some numerical experiments are concluded. Finally, section 6 concludes this paper and offers some directions for future research.

## 2. LITERATURE REVIEW

The literature of chain-to-chain competition in SCND problems can be categorized based on customer choice rules. A part of the literature used price dependent rule to formulate competitive SCND problem [6-13]. Also a part

of the literature used gravity rule [2, 5]. The covering approach that considers the concept of lost demand in competition is a new rule in the literature. Drezner [17] proposed a method for estimating the radius of influence of shopping malls. Drezner et al. [3, 18] considered competitive FL problem with the covering approach. Drezner et al. [19] proposed a leader–follower FL based on this rule. As it can be seen, there is a gap in the literature of SCND problem under chain-to-chain competition for applying the covering rule to estimate the market shares of rivals.

Drezner [20] and Plastria [16] did the basic works in introducing the market reduction effect. Also, others expand it to the field of FL problems by analysing the interactions between the market expansion and market reduction effect [21, 22]. The literature shows a gap in addressing this effect in SCND problems under chain-tochain competition.

#### **3. PROBLEM DEFINITION AND FORMULATION**

We consider a two-echelon SC whose owner decides to expand its SC in a competitive market. The expanding SC has a basic structure including a number of manufacturers and retailers. Each facility has a basic capacity and radius of influence. On the other hand, there are some rival retailers competing directly with the expanding SC. Since there is no reliable data about the maximum radius of influence of each retail outlet, we use appropriate possibilistic distributions in the form of fuzzy numbers to model these imprecise parameters. The proposed model is as follows in which imprecise (fuzzy) parameters are associated with a tilde sign:

$$Z_{1} = \max \sum_{j \in J} \sum_{k \in K} b_{k} S_{jk} - \sum_{i \in I} \sum_{j \in J} c_{ij}^{t} q_{ij}$$
  
$$-\sum_{j \in J} \sum_{k \in K} (T_{k}^{1} + F_{k} T_{k}^{2}) O_{jk}$$
(1)

$$Z_2 = \min \sum_{j \in \{1, \dots, j_c\}} \sum_{k \in K} b_k \left( S^0_{jk} - O^0_{jk} s_{jk} \right)$$

$$\tag{2}$$

$$\sum_{k \in K} s_{jk} \leq \sum_{i \in I} q_{ij}; \forall j$$
(3)

$$\sum_{i \in I} q_{ij} \le cap_j^r \; ; \forall j \tag{4}$$

$$\sum_{j \in J} q_{ij} \leq CAP_i^m \; ; \forall i \tag{5}$$

$$\left|\mathbf{x}_{j}^{r}-\mathbf{x}_{k}^{d}\right|+\left|\mathbf{y}_{j}^{r}-\mathbf{y}_{k}^{d}\right|\leq r_{j}+\mathbf{M}\left(1-\mathbf{O}_{jk}\right);\forall j,k$$
(6)

$$r_{j} \leq \left| x_{j}^{r} - x_{k}^{d} \right| + \left| y_{j}^{r} - y_{k}^{d} \right| + MO_{jk} ; \forall j,k$$

$$\tag{7}$$

$$s_{jk} \le \frac{O_{jk}}{\epsilon + F_k + \sum_j O_{jk}} \tilde{D}_k \quad ; \forall j,k$$
(8)

$$\left|\mathbf{x}_{j}^{r}-\mathbf{x}_{k}^{d}\right|+\left|\mathbf{y}_{j}^{r}-\mathbf{y}_{k}^{d}\right| \leq \tilde{\mathbf{r}}_{k}^{d}+\mathbf{M}\left(\mathbf{1}-\mathbf{g}_{jk}\right); \forall j,k$$

$$\tag{9}$$

$$\tilde{r}_{k}^{d} \leq \left| x_{j}^{r} - x_{k}^{d} \right| + \left| y_{j}^{r} - y_{k}^{d} \right| + Mg_{jk} ; \forall j,k$$

$$(10)$$

$$\sum_{j \in J} c^{r} (1 + \sum_{k \in K} \tilde{c}_{k}^{d} g_{jk}) (cap_{j}^{r} - cap_{j}^{r0}) + \sum_{i \in J} c^{s} (1 + \sum_{k \in K} \tilde{c}_{k}^{d} g_{jk}) (z_{j} - z_{0}) < B$$
(11)

$$\operatorname{cap}_{j}^{r} \leq \operatorname{CAP}_{r}^{Max} z_{j} \; ; \forall j$$
 (12)

$$\mathbf{r}_{j} \le \mathbf{r}^{\max} \mathbf{z}_{j} \quad (13)$$

$$z_j^0 \le z_j \; ; \forall j \tag{14}$$

$$\mathbf{r}_{j}^{0} \le \mathbf{r}_{j} \quad ;\forall j \tag{15}$$

$$\operatorname{cap}_{j}^{r0} \le \operatorname{cap}_{j}^{r} ; \forall j$$
(16)

The first objective function is the maximization of revenue minus transportation and marketing costs. Also, marketing cost of a demand zone depends on the number of rival retailers there (i.e.  $F_{k}$ ). The second objective

function minimizes the market reduction effect. For those markets that the expanding SC was not present before the expansion plan,  $O_{jk}^0$  and  $s_{jk}^0$  are zero and consequently,  $\left(s_{jk}^0 - O_{jk}^0 s_{jk}\right)$  is zero. Nonetheless, in those markets that the expanding SC was attracting customers before the expansion plan,  $O_{jk}^0 = 1$  and  $s_{jk}^0 > 0$ . In this case,  $\left(s_{jk}^0 - O_{jk}^0 s_{jk}\right)$  equals to the market reduction effect. Contraint (3) is the flow conservation constraint. Constraints (4) and (5) are capacity limitations. Constraints (6) and (7) ensure that if the facility *j* covers the demand zone *k*,  $O_{jk}$  equals to 1; otherwise, it is 0. Constraint (8) ensures that the demand of a demand zone (i.e.  $D_k$ ) is divided equally between the facilities that can cover that demand zone.

Typically, owners of SCs try to establish new facilities as close as possible to the center of demand zones that require higher investment. Constraints (9) and (10) determine that whether the retailer is located near the demand zone k or not. There is a normal cost  $c^s$  for establishing a facility. If the facility is located near the demand zone k, an extra cost  $\tilde{c}_k^d$  % is charged to the SC. Constraint (11) is the budget constraint. Improving either the radius of influence or the capacity of current facilities requires a budget proportional to the improvement level. Establishing a new facility requires a fixed charge (i.e.  $c^{s}$  ) plus a variable cost, which is related to its radius of influence (i.e.  $c^r$ ) and capacity of the retailer (i.e.  $c^{capr}$ ). Also, it can establish or improve a retailer near demand zones with  $\tilde{c}_k^d$  % more than the normal cost. Constraints (12) and (13) ensure that the radius of influence and capacity of a retailer are greater than zero, if and only if it is established. Constraints (14)-(16) ensure that downgrading or closing a facility is not allowed.

## 4. SOLUTION STRATEGY

The presented model in this paper consists of nonlinear terms. That is, constraints (6) -(7) and (9) -(10) involve absolute terms and constraint (8) includes fractional term with binary variables. Also, constraint (14) consists of nonlinear cost terms. Here, the nonlinear terms are linearized to reformulate the model to a mixed integer linear one. Then, the procedures used for deriving the crisp counterpart of the linear model are elaborated.

**4. 1. Linearizing of Fractional Term** Consider the auxiliary variable  $w_k$  as follows:

$$w_{k} = \frac{1}{\varepsilon + F_{k} + \sum_{j} O_{jk}} ; \forall k$$
(17)

By incorporating  $w_k$  in constraint (8), it is reformulated as follows:

$$s_{jk} \le w_k O_{jk} \tilde{D}_k \; ; \forall j,k \tag{18}$$

By defining the continuous auxiliary variable  $w'_{jk} = w_k O_{jk}$ , constraints (8), (17) and (18), are rewritten as follows:

$$s_{jk} \le w'_{jk} \tilde{D}_k$$
;  $\forall j \in J, \forall k \in K$  (19)

$$\mathbf{M}\left(\mathbf{O}_{jk}-\mathbf{1}\right)+\mathbf{w}_{k}\leq\mathbf{w}_{jk}'\leq\mathbf{M}\left(\mathbf{1}-\mathbf{O}_{jk}\right)+\mathbf{w}_{k}\;\;;\;\forall j,k \tag{20}$$

$$w_{k}\left(F_{k}+\epsilon\right)+\sum_{j}w'_{jk}=1 ; \forall k$$
(21)

$$w'_{jk} \le MO_{jk}$$
;  $\forall j,k$  (22)

$$w'_{jk} \ge 0 \tag{23}$$

The resulted linear constraints (19) -(23) are replaced by nonlinear fragmental constraint (8) in the model.

**4. 2. Linearizing of Absolute Terms** For linearizing, constraints (6) -(7) and (9) -(10), new auxiliary continuous variables  $p_{jk}^{1x}$  and  $p_{jk}^{2x}$  are defined. The following constraints are defined:

$$\mathbf{x}_{j}^{r} - \mathbf{x}_{k}^{d} \le \mathbf{p}_{jk}^{1x} \quad (\forall j, k$$

$$x_k^d - x_j^r \le p_{jk}^{lx} \ ; \ \forall j,k$$
(25)

$$\mathbf{y}_{j}^{r} - \mathbf{y}_{k}^{d} \le \mathbf{p}_{jk}^{ly} \quad ; \forall \mathbf{j}, \mathbf{k}$$

$$(26)$$

 $y_k^d - y_j^r \le p_{jk}^{ly} \quad ; \forall j,k \tag{27}$ 

$$p_{jk}^{1x}, p_{jk}^{1y}, p_{ij}^{2x}, p_{ij}^{2y} \ge 0$$
(28)

To force the auxiliary variables to take the right values,  $p_{jk}^{lx}$  and  $p_{jk}^{ly}$  should be appeared in the objective function to take values as minimal as possible. To do so, an artificial cost is incorporated in the objective functions as follows:

$$Z_{1} = \max \sum_{j \in J} \sum_{k \in K} (b_{k} S_{jk} - (T_{k}^{1} + F_{k}T_{k}^{2})O_{jk}) - \sum_{j \in J} \sum_{k \in K} \varepsilon(p_{jk}^{1x} + p_{jk}^{1y})$$
(29)

$$Z_{2} = \min \sum_{j \in \{1, \dots, j_{e}\}} \sum_{k \in K} \mathbf{b}_{k} \left( \mathbf{S}_{jk}^{0} - \mathbf{O}_{jk}^{0} \mathbf{s}_{jk} \right)$$
  
+ 
$$\sum_{j \in J} \sum_{k \in K} \varepsilon(\mathbf{p}_{jk}^{1x} + \mathbf{p}_{jk}^{1y})$$
(30)

By defining these auxiliary variables and incorporating constraints (24)-(28) in the model, constraints (6) -(7) and (9) -(10), should be replaced by the following constraints:

$$p_{jk}^{lx} + p_{jk}^{ly} \le r_j + M(1 - O_{jk})$$
;  $\forall j, k$  (31)

$$r_{j} \leq p_{jk}^{1x} + p_{jk}^{1y} + MO_{jk} \quad ;\forall j,k$$

$$(32)$$

$$p_{jk}^{lx} + p_{jk}^{ly} \le \tilde{r}_k^d + M\left(1 - g_{jk}\right); \forall j, k$$
(33)

$$\tilde{r}_k^d \le p_{jk}^{1x} + p_{jk}^{1y} + Mg_{jk} \quad ; \forall j,k$$
(34)

**4. 3. Linearizing the Budget Constraint** For linearizing the budget constraint, continuous variables  $g_{jk}^{r}$  and  $g_{jk}^{cap}$  and binary variables  $g_{jk}^{s}$  are defined and incorporated in the model as follows:

$$g_{jk}^{r} \le Mg_{jk} \quad ;\forall j,k \tag{35}$$

$$r_{j} \leq g_{jk}^{r} + M\left(1 - g_{jk}\right) \; ; \forall j,k$$
(36)

$$g_{jk}^{cap} \le Mg_{jk} \quad ;\forall j,k \tag{37}$$

$$\mathbf{r}_{j} \le \mathbf{g}_{jk}^{cap} + \mathbf{M} \left( 1 - \mathbf{g}_{jk} \right) \; ; \forall j,k \tag{38}$$

$$2g_{jk}^{s} \le g_{jk} + z_{j} \quad ;\forall j,k \tag{39}$$

$$g_{jk} + z_j - 1 \le g_{jk}^s \quad ;\forall j,k \tag{40}$$

The budget constraint is reformulated as follows:

$$\sum_{j \in J} c^{r} (\mathbf{r}_{j} - \mathbf{r}_{j}^{0}) + \sum_{j \in J} c^{r} (\sum_{k \in \mathbf{K}} \tilde{c}_{k}^{d} (\mathbf{g}_{jk}^{r} - \mathbf{g}_{jk} \mathbf{r}_{j}^{0})) + \sum_{j \in J} c^{capr} (\operatorname{cap}_{j}^{r} - \operatorname{cap}_{j}^{r0}) + \sum_{j \in J} c^{capr} (\sum_{k \in \mathbf{K}} \tilde{c}_{k}^{d} (\mathbf{g}_{jk}^{cap} + g_{jk} cap_{jk}^{r0})) + \sum_{j \in J} c^{s} (z_{j} - z_{j}^{0}) + \sum_{j \in J} c^{s} (\sum_{k \in \mathbf{K}} \tilde{c}_{k}^{d} (\mathbf{g}_{jk}^{s} - \mathbf{g}_{jk} z_{j}^{0})) < \mathbf{B}$$

$$(41)$$

**4. 4. Crisp Counterpart Formulation** To deal with the possibilistic constraints, several methods have been proposed. Here, the method developed by Jiménez et al. [23] is applied. It has a simple while reliable structure and and preserves the linearity of the model. It uses the fuzzy expected value (EV) and the fuzzy expected interval (EI) of each fuzzy number when defuzzying the possibilistic model. Due to space limitations, interested readers can refer to literature [23]. In this way, the possibilistic

objective function (29) and constraints (19), (33), (34)and (41) are defuzzified as follows:

$$Z_{1} = \max \sum_{j \in J} \sum_{k \in \mathbf{K}} \left[ (1 - \alpha) E_{2}^{b_{k}} + (\alpha) E_{2}^{b_{k}} \right] \mathbf{S}_{jk}$$

$$- \sum_{j \in J} \sum_{k \in \mathbf{K}} (T_{k}^{1} + F_{k} T_{k}^{2}) \mathcal{O}_{jk} ) - \sum_{j \in J} \sum_{k \in \mathbf{K}} \varepsilon(\mathbf{p}_{jk}^{1x} + \mathbf{p}_{jk}^{1y})$$

$$(42)$$

$$s_{jk} \le w'_{jk} \left[ \left( 1 - \alpha \right) E_2^{\tilde{D}_k} + \left( \alpha \right) E_1^{\tilde{D}_k} \right] \; ; \forall j,k \tag{43}$$

$$p_{jk}^{lx} + p_{jk}^{ly} \le \left[ \left( 1 - \alpha \right) E_2^{\tilde{t}_k^d} + \left( \alpha \right) E_1^{\tilde{t}_k^d} \right] + M\left( 1 - g_{jk} \right); \forall j, k$$
(44)

$$\left[ \left( 1 - \alpha \right) E_2^{\tilde{r}_k^d} + \left( \alpha \right) E_1^{\tilde{r}_k^d} \right] \le p_{jk}^{1x} + p_{jk}^{1y} + Mg_{jk} \quad ; \forall j,k$$

$$\tag{45}$$

$$\begin{split} &\sum_{j \in J} c^{r} (\mathbf{r}_{j} - \mathbf{r}_{j}^{0}) \\ &+ \sum_{j \in J} c^{r} (\sum_{k \in K} \left[ (1 - \alpha) E_{2}^{\tilde{c}_{k}^{d}} + (\alpha) E_{1}^{\tilde{c}_{k}^{d}} \right] (\mathbf{g}_{jk}^{r} - \mathbf{g}_{jk} \mathbf{r}_{j}^{0})) \\ &+ \sum_{j \in J} c^{capr} (\operatorname{cap}_{j}^{r} - \operatorname{cap}_{j}^{r0}) \\ &+ \sum_{j \in J} c^{capr} (\sum_{k \in K} \left[ (1 - \alpha) E_{2}^{\tilde{c}_{k}^{d}} + (\alpha) E_{1}^{\tilde{c}_{k}^{d}} \right] (\mathbf{g}_{jk}^{cap} + g_{jk} cap_{jk}^{r0})) \\ &+ \sum_{j \in J} c^{s} (z_{j} - z_{j}^{0}) \\ &+ \sum_{j \in J} c^{s} (\sum_{k \in K} \left[ (1 - \alpha) E_{2}^{\tilde{c}_{k}^{d}} + (\alpha) E_{1}^{\tilde{c}_{k}^{d}} \right] (\mathbf{g}_{jk}^{s} - \mathbf{g}_{jk} z_{j}^{0})) < \mathbf{B} \end{split}$$

$$(46)$$

here,  $\alpha$  denotes the minimum acceptable feasibility degree ensuring that each possibilistic constraint will be satisfied at least at level  $\alpha$ .

4. 5. Solution Approach for Multi-Objective Model

To solve the resulted bi-objective crisp counterpart, the solution procedure introduced by Pishvaee and Torabi [24] is adopted here. In this procedure, crisp counterpart is solved independently for each objective function to find the ideal solutions for each objective at  $\alpha$ -level (called  $\alpha$ -positive solution). The  $\alpha$ -negative solution for the other objective function is then calculated according to the constructed pay-off table. The linear membership function is then defined for each minimization objective function as follows:

$$\mu_{h}\left(x\right) = \begin{cases} 1 & W_{h} < W_{h}^{\alpha-PIS} \\ \frac{W_{h}^{\alpha-NIS} - W_{h}}{W_{h}^{\alpha-NIS} - W_{h}^{\alpha-PIS}} & W_{h}^{\alpha-PIS} < W_{h} < W_{h}^{\alpha-NIS} \\ 0 & W_{h}^{\alpha-NIS} < W_{h} \end{cases}$$
(47)

where  $\mu_h(x)$  is the satisfaction degree of h -th objective function,  $W_h^{\alpha-NIS}$  and  $W_h^{\alpha-PIS}$  are  $\alpha$ -negative and  $\alpha$ - positive solutions. For the maximization problems, they will be modified accordingly [25]. In order to convert the bi-objective crisp model into the single objective formulation, the TH aggregation function is applied [26]:

$$\max \lambda(x) = \gamma \lambda_0 + (1 - \gamma) \sum_{h} \theta_h \mu_h \tag{48}$$

$$\lambda_0 \le \mu_h \tag{49}$$

$$\mathbf{x} \in \mathbf{F}(\mathbf{x}) \tag{50}$$

$$\lambda_0 \ , \ \lambda \in \left[0,1\right] \tag{51}$$

where  $\gamma$  and  $\theta_h$  are the compensation coefficient and the importance weigh of h-th objective function, respectively. Also,  $\lambda_0$  denotes the minimum satisfaction degree of objective functions, and F(x) is the feasible region of the original model. By manipulating the above mentioned coefficients, a set of compromising solutions can be found through solving the above model for each set of parameters.

## **5. CASE ILLUSTRATION**

Due to high level of competition in retail industry, locating retail outlets depends on the location of rival retailers as well as the distribution of population (demand) in that area [27]. A case inspired by a real world application in a retail-chain is investigated. This sector in Iran is growing dramatically such that it could attract huge amount of investments in recent years. Five SCs supplying several products like foods, detergents, dairy products, etc. can be identified in Tehran. They are growing rapidly through their expansion plans. Due to severe competition, rivals have to adjust their price level in response to the price list of other rivals. As a consequence, accessibility of customers to retailers has a high impact on the market share of rival retailers. Here, the rivals are named in the abstract form as "H", "SH", "R", "G" and "OF". They can be categorized as follows:

- Type 1: Three of the SCs including "H", "SH" and "R" have focused on constructing large retail outlets to attract high distant customers. These retail outlets are capital intensive because of higher establishing costs (e.g. the costs for purchasing the land for providing parking areas).
- Type 2: Two SCs including "OF" and "G" construct the retailers with enough shop space, but without parking space. They focus on those customers near to the location of retailers.

The situation in which the SC "G" is going to expand its retail outlets in the regions 1 and 4 of Tehran city is considered as the case study. There are 41 retailers in the selected regions including 12 retailers owned by "G". There are four possible expansion strategies:

- Strategy A: The owner of the SC focuses on the improvement of existing facilities via upgrading their radius of influence and/or capacities. So, the owner will plan to construct parking area for its existing retailers by providing required spaces.
- **Strategy B**: The owner will focus on geographical expansion of the SC by establishing new retailers with current design policy of the "G" without parking space.
- Strategy C: The owner will construct new facilities with higher radius of influence. The plan in strategy C will require more space in comparison to the basic plan such that the extra space will be allocated to the parking area.
- Strategy D: The owner considers improvement of current facilities and construction of new facilities with higher radius of influence.

In none of the above strategies, downgrading is allowed. The important question is which strategy is the best for the owner regarding the revenue and market reduction effect? To answer these questions, the required information is first gathered. The neighborhoods of these regions according to the official classification are assumed as the demand zones whose annual buying powers are categorized in 4 classes starting from 2 to 5 based on their perceived social class. There are 12 existing retailers with basic design policy of "G". Eight new retailers can be established anywhere in the region. There is a standard plan for the shop floor of the retailers. Also, there is one depot center (i.e. the manufacturer) for delivering the products to the retailers. One percent of the total demand multiplied by the annual buying power of each demand zone is set as the marketing cost of that zone. It is assumed that for supplying a zone with 20000 population requires a storage area of 300 m<sup>2</sup>. The details of the information are shown in Table 2.

The linear model is solved via GAMS 24.7.3 using CPLEX solver. The weight for the objective function  $Z_1$ , (i.e.  $\theta_1$ ) is set as 0.8 with  $\lambda_0 = 0.1$  and  $\gamma = 0.2$ . To better analyze the case, MR is defined as  $\sum_{j \in Jk \in K} b_k s_{jk}$ . It shows the total market share of the SC multiplied by the annual buying power.  $MR^0 = \sum_{j \in \{1, \dots, j_k\} k \in K} \sum_{jk} b_k s_{jk}^0$  shows the initial MR before the expansion plan. After solving the problem for obtaining the  $W_h^{\alpha - PIS}$ , this indicator is recorded as  $MR^1$ . This shows that the maximum obtainable MR via solving a single objective problem regarding  $Z_1$ . Then, by maximizing the TH aggregation

function, this indicator is recorded as  $MR^2$ . Table 3 shows the results found with different strategies and values of  $\alpha = 0.2$  and  $\theta_1 = 0.8$ .  $MR^0$  is the same for all the strategies.

<b>TABLE 2.</b> Parameters of the real case problem	em
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Sets and Parameters	Description
$I^{L}.j_{c}$ , $J^{L}$ , $K$	1,12,15,46
$\tilde{\boldsymbol{b}}_k$	$b_k^m$ equal to 2-5 based on income class of the neighborhood and $(b_k^l, b_k^u) =$ $b_k^m (1 - uniform (0, 0.1), 1 + uniform (0, 0.1))$
$\tilde{D}_k$	Population of each demand zone
$T_k^1$ , $T_k^2$	$1\%\!\times\!b_k\!\times\!D_k$ , $0.1\%\!\times\!b_k\!\times\!D_k$
number of rivals	41 rival retailers (including 12 retailer for expanding SC of "G")
r <sup>m,max</sup>	r <sup>m,max</sup> equal to 10 in scale of the space
$\tilde{r}_k^d$	$r_k^{dm}$ equal to 2 in scale of the space and $\left(\tilde{r}_k^d\right) = r^{m,max} \begin{pmatrix} 1 - uniform(0,0.1), \\ 1 + uniform(0,0.1) \end{pmatrix}$
c <sup>s</sup> , c <sup>r</sup> , c <sup>capr</sup>	Land price, 15 land price for each unit, 5% of land price for each unit and maximum storage area for serving zone with 2000 population
$\tilde{c}^d_k$	$c_k^{md}$ equal to 2 in scale of the space and $\left(\tilde{c}_k^d\right) = c_k^{md} \begin{pmatrix} 1 - \text{uniform}(0, 0, 1), \\ 1 + \text{uniform}(0, 0, 1) \end{pmatrix}$

Since in strategies B and C no modification for the current facilities is allowed,  $W_2^{\alpha-PIS}$  is assumed equal to zero for both. Also,  $W_2^{\alpha-PIS}$  for strategies A and D is similar.  $W_1^{\alpha-NIS}$  is the best solution obtained while maximizing the first objective function.  $W_2^{\alpha-NIS}$  shows the market reduction effect of this solution. As it is clear in Table 3, strategy D that includes all possible modifications is superior to the other strategies in  $Z_1$  and  $\ensuremath{\mathsf{MR}}^1.$  On the other hand, strategy A is superior to the strategies B and C in both objective function  $Z_1$  and  $MR^1$ . This means that concentrating on improving the current facilities is more effective in comparison to geographical expansion. However, the market reduction effect of strategies A and D are more than other strategies. This means that although the geographical strategy will not yield higher revenue, but will avoid attacking the market share of the current facilities. The same analysis is done with  $\alpha = 0.8$  in Table 4.

To analyze the effect of weights in the TH aggregation function, all the strategies are solved in different levels of  $\theta_1 = 0.2$  to  $\theta_1 = 1$ . Also, the  $\lambda_0$  and

 $\gamma$  is set to zero with the  $\alpha$ -level of 0.5. The results are shown in Table 5. It shows the relation between two objective functions. It is interesting that the second objective function  $\,Z_2\,$  is very sensitive to the value of  $\,\theta_1\,$ when decreasing from  $\theta_1 = 1$  to  $\theta_1 = 0.8$ .

Consider a situation that there are more rival facilities owned by the competitors. Which of the strategies are more sensitive to the number of rival facilities? In other

words, which one is more resistant to the rival's power in the market. For analyzing this question, sensitivity of Z<sub>1</sub> is analyzed in comparison to increasing the number of rival retailers.  $F_k$  is increased by one unit and the value of  $Z_1$  is recorded. The results are shown in Figure 1. As it can be seen, strategy B is very sensitive to the attack of rivals.

<b>TABLE 3.</b> Numerical results for $\alpha = 0.2$ , $\theta_1 = 0.8$ , $\lambda_0 = 0.1$ and $\gamma = 0.2$							
$\mathbf{MR}^{0}$			$W_h^{\alpha-PIS}$	$\mathbf{MR}^1$	$W_h^{\alpha-NIS}$	$\mathbf{MR}^2$	
		Zl	695768	770989	370855	734559	
	A	$Z_2$	8919	416828	137404		
	р	Z <sub>1</sub>	623432	660958	339085	656735	
415401	D	$Z_2$	0	372540	55761		
	C	Z <sub>1</sub>	639486	677699	339085	675395	
	t	$Z_2$	0	372540	55420		
	D	Zl	801753	853165	370855	858047	
	D	$Z_2$	8919	416828	93282		

**TABLE 4.** Numerical results for  $\alpha = 0.8$ ,  $\theta_1 = 0.8$ ,  $\lambda_0 = 0.1$  and  $\gamma = 0.2$ 

$\mathbf{MR}^{0}$			$W_h^{\alpha-PIS}$	$\mathbf{MR}^1$	$W_h^{\alpha-NIS}$	$\mathbf{MR}^2$
	٨	$Z_1$	682457	764537	366183	724549
	A	Z <sub>2</sub>	13591	403156	145029	
407545	P	$Z_1$	616990	659817	340126	677776
	В	Z2	0	382580	54293	
	C	$Z_1$	624590	662116	340126	657776
	C	$Z_2$	0	382580	54750	
	D	$Z_1$	799351	845714	366178	854755
	D	Z <sub>2</sub>	13597	403156	95230	

TABLE 5. Sensitivity of the	Z	and Z <sub>2</sub>	to $\theta_1$	$\alpha = 0.5, \lambda$	$\gamma = 0.1, \gamma = 0$
<b>TIDLE C.</b> Schold (http://tile	~	und D)		( 0.0, 74	1 0.1,1 0)

		1	0.8	0.6	0.4
А	$Z_1$	684148	641620	629426	617535
	Z2	144638	36535	31924	27429
В	$Z_1$	621938	536997	534835	534484
	$Z_2$	55747	32938	31456	30660
С	$Z_1$	640305	635671	635671	635671
	$Z_2$	54451	29172	28724	27817
D	$Z_l$ .	801386	781758	782826	770943
D	$Z_2$ .	95739	27848	27801	27847



Figure 1. Sensitivity of the strategies to the number of rivals in demand zones

## **6. CONCLUSION**

In this paper, strategic planning of retail outlets of a SC under a chain-to-chain competition is addressed. It consists of locating and planning of retailers with variable coverage on the plane. The customers may patronize to a retailer if and only if they are in the catchment area of retailers called the radius of influence. It is assumed that the demand of each market is divided equally between those retailers who can cover that market. The demands of those customer zones which are not in the catchment area of any retailer, is lost. Here, it is considered an existing retail SC wants to expand its network. Resulting from the complex competitive environment, customers can buy required goods from either the new retailers of this SC or the pre-existing retailers of any rival. This may result to market reduction of pre-existing retailers. The first objective is to increase the total market share of the SC. The second objective is to minimize the market reduction effect. A bi-objective fuzzy MINLP is proposed to formulate this problem. To solve this problem, it is first reformulated as an MILP. An interactive method is devised to solve the resulted biobjective fuzzy problem. For numerical analysis, a real case inspired by a supermarket chain is analysed in different scenarios. Four expansion strategies are analysed. The results showed that improvement of current facilities beside the hybrid strategy can lead to higher market share. However they have higher levels of market reduction which means that they will capture part of current customers of the pre-existing retailers. The results showed that geographical expansion is more sensitive to the presence of rivals in the demand zones for this case. For the future research, the following directions can be proposed:

 Foresight competition: In this research, it was considered that the rivals may not react in the near future. However, there may be a situation that the rivals can react to the expanding SC.  Downgrading strategy: The point for downgrading option is to consider an asymmetric cost function which can differentiate between the costs of upgrading and downgrading.

## 7. REFERENCES

- Fernández, J., Pelegrín, B., Plastria, F., and Tóth, B., "Solving a Huff-like competitive location and design model for profit maximization in the plane", *European Journal of Operational Research*, Vol. 179, No. 3, (2007), 1274–1287.
- Rezapour, S., Zanjirani Farahani, R., and Drezner, T., "Strategic design of competing supply chain networks for inelastic demand", *Journal of the Operational Research Society*, Vol. 62, No. 10, (2011), 1784–1795.
- Drezner, T., Drezner, Z., and Kalczynski, P., "A cover-based competitive location model", *Journal of the Operational Research Society*, Vol. 62, No. 1, (2011), 100–113.
- Berman, O., Drezner, Z., and Krass, D., "Generalized coverage: New developments in covering location models", *Computers & Operations Research*, Vol. 37, No. 10, (2010), 1675–1687.
- Farahani, R.Z., Rezapour, S., Drezner, T., Esfahani, A.M., and Amiri-Aref, M., "Locating and capacity planning for retailers of a new supply chain to compete on the plane", *Journal of the Operational Research Society*, Vol. 66, No. 7, (2015), 1182– 1205.
- Fallah, H., Eskandari, H., and Pishvaee, M.S., "Competitive closedloop supply chain network design under uncertainty", *Journal of Manufacturing Systems*, Vol. 37, No. 37, (2015), 649–661.
- Rezapour, S., Farahani, R.Z., Fahimnia, B., Govindan, K., and Mansouri, Y., "Competitive closed-loop supply chain network design with price-dependent demands", *Journal of Cleaner Production*, Vol. 93, No. 93, (2015), 251–272.
- Rezapour, S., and Zanjirani Farahani, R., "Supply chain network design under oligopolistic price and service level competition with foresight", *Computers & Industrial Engineering*, Vol. 72, No. 72, (2014), 129–142.
- Rezapour, S., Farahani, R.Z., Dullaert, W., and De Borger, B., "Designing a new supply chain for competition against an existing supply chain", *Transportation Research Part E: Logistics and Transportation Review*, Vol. 67, No. 67, (2014), 124–140.
- Rezapour, S., Farahani, R.Z., Ghodsipour, S.H., and Abdollahzadeh, S., "Strategic design of competing supply chain networks with foresight", *Advances in Engineering Software*, Vol. 42, No. 4, (2011), 130–141.
- Rezapour, S., Farahani, R.Z., and Pourakbar, M., "Resilient supply chain network design under competition: A case study", *European Journal of Operational Research*, Vol. 259, No. 3, (2017), 1017–1035.
- Rezapour, S., and Farahani, R.Z., "Strategic design of competing centralized supply chain networks for markets with deterministic demands", *Advances in Engineering Software*, Vol. 41, No. 5, (2010), 810–822.
- Hafezalkotob, A., Makui, A., and Sadjadi, S.J., "Strategic and Tactical Design of Competing Decentralized Supply Chain Networks with Risk-Averse Participants for Markets with Uncertain Demand", *Mathematical Problems in Engineering*, Vol. 2011, (2011), 1–27.
- 14. Shamekhi Amiri, A., Torabi, S.A., and Ghodsi, R., "An iterative approach for a bi-level competitive supply chain network design problem under foresight competition and variable coverage",

Transportation Research Part E: Logistics and Transportation Review, Vol. 109, No. November 2017, (2018), 99–114.

- Nagurney, A., "Supply chain network design under profit maximization and oligopolistic competition", *Transportation Research Part E: Logistics and Transportation Review*, Vol. 46, No. 3, (2010), 281–294.
- Plastria, F., "Avoiding cannibalisation and/or competitor reaction in planar single facility location", *Journal of the Operations Research Society of Japan*, Vol. 48, No. 2, (2005), 148–157.
- Drezner, T., "Derived attractiveness of shopping malls", *IMA Journal of Management Mathematics*, Vol. 17, No. 4, (2006), 349–358.
- Drezner, T., Drezner, Z., and Kalczynski, P., "Strategic competitive location: improving existing and establishing new facilities", *Journal of the Operational Research Society*, Vol. 63, No. 12, (2012), 1720–1730.
- Drezner, T., Drezner, Z., and Kalczynski, P., "A leader-follower model for discrete competitive facility location", *Computers & Operations Research*, Vol. 64, No. 64, (2015), 51–59.
- Drezner, T., "Cannibalization in a Competitive Environment", *International Regional Science Review*, Vol. 34, No. 3, (2011), 306–322.
- Aboolian, R., Berman, O., and Krass, D., "Competitive facility location and design problem", *European Journal of Operational Research*, Vol. 182, No. 1, (2007), 40–62.

- Pelegrín, B., Fernández, P., and Pérez, M.D.G., "Profit maximization and reduction of the cannibalization effect in chain expansion", *Annals of Operations Research*, (2014), 1–19.
- 23. Jiménez, M., Arenas, M., Bilbao, A., and Rodri 'guez, M.V., "Linear programming with fuzzy parameters: An interactive method resolution", *European Journal of Operational Research*, Vol. 177, No. 3, (2007), 1599–1609.
- Pishvaee, M.S., and Torabi, S.A., "A possibilistic programming approach for closed-loop supply chain network design under uncertainty", *Fuzzy Sets and Systems*, Vol. 161, No. 20, (2010), 2668–2683.
- Torabi, S.A., and Madadi, M., "A Fuzzy Multi Objective Programming Model for Power Generation and Transmission Expansion Planning Problem", *International Journal of Engineering*, Vol. 23, No. 1, (2010), 29–39.
- Torabi, S.A., and Hassini, E., "An interactive possibilistic programming approach for multiple objective supply chain master planning", *Fuzzy Sets and Systems*, Vol. 159, No. 2, (2008), 193–214.
- Hafezalkotob, A., Babaei, M.S., Rasulibaghban, A., and Noori-Daryan, M., "Distribution design of two rival decenteralized supply chains: A two-person nonzero sum game theory approach", *International Journal of Engineering, Transactions B: Applications*, Vol. 27, No. 8, (2014), 1233–1242.

## A Possibilistic Bi-objective Mode l for A Competitive Supply Chain Network Design under Variable Coverage

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Keywords: Facility location on the plane Chain-to-chain competition Radius of influence Cannibalization effect Capacity planning Multi-objective possibilistic programming در این مقاله، برنامه ریزی راهبردی یک زنجیره تامین با در نظر گرفتن رقابت از نوع استاتیک زنجیره – در مقابل – زنجیره در یک فضای پیوسته مورد بررسی قرار گرفته است. فرض می شود که هر خرده فروش در این زنجیره دارای یک فضای پوشش است که به آن شعاع اثر گفته می شود. تقاضای مشتریان هر منطقه بین خرده فروشانی که آن منطقه را پوشش می دهند، به طور مساوی تقسیم می شود. همچنین، تقاضای مشتریانی که دورتر از شعاع اثر خرده فروشانی که آن منطقه را پوشش می دهند، به طور رقابت برای یک کاربرد عملی در صنعت سوپر مارکتها مدل شده است. فرض شده که مالک یک زنجیره تأمین موجود در نظر دارد تا ساختار زنجیره را برای افزایش سهم بازار توسعه دهد. از آنجا که برنامه توسعه ممکن است بر روی مشتریان نظر دارد تا ساختار زنجیره را برای افزایش سهم بازار توسعه دهد. از آنجا که برنامه توسعه ممکن است بر روی مشتریان زنجیره اجتناب کند. یک مدل دو هدفه فازی غیر خطی عدد صحیح برای این مسئله پیشنهاد شده است. برای حل مدل، در ابتدا آن را تبدیل به یک مدل خطی نموده و سپس از یک متد تعاملی برای حل مدل خطی فازی دو هدفه استفاده شده است. چهار برنامه توسعه برای این زنجیره مورد تحلیل قرار گرفته شده که از آنها نکات می موجود شده است. پردا آن را تبدیل به یک مدل خطی نموده و سپس از یک متد تعاملی برای حل مدل خطی فازی دو هدفه استفاده شده است. په را برنامه توسعه برای این زنجیره مورد تحلیل قرار گرفته شده که از آنها نکات مدیریتی استخراج شده است. **doi:** 10.5829/ije.2018.31.09c. 14

چکيده