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Application of Multivariate Control Charts for Condition Based Maintenance

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ABSTRACT

Condition monitoring is the foundation of a condition based maintenance (CBM). To relate the information obtained from the condition monitoring to the actual state of the system, it is usually required a stochastic model. On the other hand, considering the interactions and similarities that exist between CBM and statistical process control (SPC), the integrated models for CBM and SPC have been developed. These models apply control charts as a condition monitoring technique, and the inference about the operational states of the system is based on the collected information about the quality of the produced items. Finally, it is decided whether to implement certain type of maintenance actions. This paper describes the application of multivariate control charts as a condition monitoring technique for CBM purposes. To this end, an integrated model is developed, while it is used a chi-square control chart. Also, to determine the inspection time points, a constant hazard policy is applied.

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NOMENCLATURE

R_i	Expected revenue for the system operation per time unit, when the system is in operational state i (i=0,1) ($R_0>R_1$)	t	Equipment age at the start of each production cycle
C_{QC}	Sampling inspection cost	k	dimension of the observations
C_{PM}	Preventive maintenance cost	t_i (i=1,..,m-1)	time points of the sampling inspection (they are the decision variables of the model)
C_{CM}	Corrective maintenance cost	α	Probability of type I error for the control chart
C_I	The cost of the maintenance inspection	β	Probability of type II error for the control chart
Z_{PM}	Expected time required for the preventive maintenance	N	The sample size in each sampling inspection (it is a decision variable of the model)
Z_{CM}	Expected time required for the Corrective maintenance	t_m	Maximum duration of each production cycle (decision variable)
Z_I	Expected time required for the maintenance inspection	m	Maximum number of the inspection periods (decision variable)
$f(t)$	Density function of time of quality shift	$E[T_0]$	The expected time that the system operates in state 0 during each production cycle
$F(t)$	Cumulative distribution function (c.d.f) of the time of quality shift ($\bar{F}(t)=1-F(t)$)	P_{PM}	Probability that a production cycle is terminated due to the preventive maintenance
$\varphi_i(t)$	Density function of time to failure state if the system is in state i (i=0,1) at t=0	P_{CM}	Probability that a production cycle is terminated due to the corrective maintenance
$\phi_i(t)$	Cumulative distribution function (c.d.f) of time to failure state if the system is in state i (i=0,1) at t=0		

1. INTRODUCTION

In the most models developed for condition based maintenance (CBM), it is usually assumed that the system has several operational states plus a failure state. In the failure state the system stops, thus, this state is directly and immediately observable. It can be easily distinguished from the operational states. On the other hand, the difference between the operational states is undetectable and inference about the actual operational state of the system is based on the condition monitoring. Thus, in CBM it is usually assumed that the actual

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operational state of the system is not observable and the obtained information in the condition monitoring only partially informs about the system state. This type of condition monitoring is called indirect condition monitoring. Hence, in the indirect condition monitoring, it is necessary to establish a stochastic model for relating the obtained information from the condition monitoring to the actual system state [1-5].

On the other hand, as stated by many authors, there are great interactions and interrelations between CBM and SPC [1, 6-8]. Considering the interactions and similarities that exist between CBM and SPC, the integrated models for CBM and quality control have been developed. These models usually apply the control chart as a condition monitoring technique. In these models, the inference about the operational states of the system is based on the collected information about the quality of the produced items. Finally, it is decided whether or not to implement certain types of maintenance actions. Indeed, the integrated models for CBM and SPC have been developed based on the fact that the product quality can partially indicates the actual operational state of the system.

Wang [2] used the multivariate Bayesian control chart for CBM. Panagiotidou and Tagaras [6], Panagiotidou and Tagaras [9] proposed an integrated model for CBM and SPC based on the $\bar{X}$ control chart. Yin et al. [10] developed an integrated model for CBM and SPC, while it was used a delayed monitoring for the CBM purposes. Panagiotidou and Nenes [11] used an adaptive Shewhart chart for the CBM purposes and proposed an integrated model for maintenance planning and economical design of an adaptive Shewhart chart. Ardakani et al. [12] developed an integrated maintenance planning and SPC model based on the multivariate exponentially weighted moving average (MEWMA) chart. Jamshidi and Madie [13] proposed an integrated model for maintenance and work-rest scheduling. Lie et al. [14], according to the geometric process, proposed a model for CBM and SPC. Wu and Makis [1] proposed a model for the economic design of a chi-square control chart for a CBM application. Wu and Makis assumed that the system has three states including: in-control state, out-of-control state and failure states. Also, transition time between these states is based on the exponential distribution, and it is used Markov chain for deriving the integrated model.

In this paper, the proposed model by Wu and Makis is developed based on three main contributions:

1. while in the Wu and Makes' study, the deterioration mechanism is assumed to follow the exponential distribution; we place no restrictive assumption on the deterioration mechanism of the system. In other words, the time to quality shift as well as the time to the failure state from each operational states were assumed as a general continuous distribution function.

2. considering the memory less property of the exponential distribution, Wu and Makis applied Markov chain in deriving their model. While in this paper, developing the integrated CBM and SPC model is based on the recursive equations and renewal reward process.

3. the proposed model in this paper is applicable for different types of inspection policy, while the proposed model by Wu and Makis is based on the fixed time interval inspection policy. Thus, by releasing many assumptions of the Wu and Makis' model, our proposed model can be applied in more practical situations and has a wider application domain. Indeed, the proposed model by Wu and Makis can be considered as a special state of the proposed model in this paper.

The rest of the paper is organized as follows: section 2 describes the considered system. In section 3, the proposed model is derived. In section 4, the proposed model in section 3 is applied for a situation that a chi-square control chart is used as a condition monitoring technique. Also, constant hazard policy is introduced in this section. Section 5 presents two examples of the application of the model. Some sensitivity analyses are presented in section 6. Finally, section 7 concludes the paper.

2. DESCRIPTION

Consider a production process or a single production machine which may operate under different conditions. Specifically, the system has two operational states, as well as a failure state that is non-operational. The operational states include in-control state (denoted as state 0) and out-of-control state (denoted as state 1). Every production cycle starts from the in-control state and zero equipment age. The system may shift from state 0 to state 1 or directly shift from state 0 to the failure state due to the usage and age. Having shifted the system to state 1, if this state is not identified, the system eventually shifts from state 1 to the failure state.

The system operation in state 1 is undesirable in comparison with its operation in state 0, due to the lower level of the produced item quality, the lower revenue and the higher chance for the system complete failure. In the failure state, the system stops and cannot produce the item. No matter what is the operational states, the system may transit from an operational state to the failure state. It is assumed that the transition time from state 0 to 1, from state 0 to the failure state and from state 1 to the failure state are based on the probability functions that follow a general continuous distributions. Also, the failure rate of the system in state 1 is higher than the failure rate in state 0.

The failure state is directly and immediately observable because the system stops, while the inference about the operational states of the system (whether the system is in state 1 or 0) is based on the condition monitoring. More specifically, the operational condition of the system is inferred based on the quality of the produced item. Condition monitoring is implemented as follows: at the specific time points, such as $t_1, t_2, \ldots, t_{m-1}$, that are decision variables in the model, a sample with size n is taken from the produced item of the system.

Based on the information collected from the quality of this sample, an appropriate statistic is computed and plotted on a suitable multivariate control chart. If the value of this statistic exceeds the control limits of the chart, the chart alarms meaning that the system probably operates in the out-of-control state. To determine the actual state of the system, an error free inspection is conducted after releasing each alarm from the control chart. This inspection is called maintenance inspection to distinguish it from the sampling inspection. If the maintenance inspection indicates that the system is in state 1 then preventive maintenance (PM) is implemented on the system. But if the maintenance inspection concludes that the system state is 0 (in other words the chart alarm is incorrect) then the system will continue its operation without any further actions.

Once the system transits to the failure state, the corrective maintenance (CM) is conducted and the system renews. It is possible for the system to operate without transiting to the failure state until the end of the production cycle (at time point t_m). In this situation the system may be in state 0 or 1 however, regardless of the system state at t_m , the PM is implemented. Thus, PM is conducted in two general situations: 1- after releasing a true out-of-control signal from the control chart and 2- if the system reaches the time point t_m. Hence, two types of maintenance is implemented on the system: PM and CM.

It is assumed that both types of the maintenance are perfect such that they can renew the system to the as-good-as new state. Once each type of the maintenance is implemented on the system, the system renews and a new production cycles starts.

3. MODEL DEVELOPMENT

In this section, based on the renewal reward process and recursive equations, an integrated stochastic model is developed for CBM and SPC

3. 1. System Evolution During an Inspection Interval

In each inspection interval, six different scenarios are possible for the system operation. These scenarios are illustrated in Table 1. Also, in this table, the pertinent

3. 2. System State at the Start of Each Inspection Interval

Let denote $P_{t_i}^0$, $P_{t_i}^1$ as the probabilities that, immediately after inspection at t_i, the system operates in the in-control state or out-of-control state, respectively. In this subsection, the computation of these probabilities is described. $P_{t_i}^0$ is given by:

$$P_{t_i}^0 = \bar{F}(t_i)\bar{\Phi}_0(t_i); \ ; \ 1 \le i \le m \quad (1)$$

This equation is obtained based on the fact that the system operates in state 0 at t_i, if and only if the time to quality shift as well as the transition time from state 0 to the failure state be greater that t_i. $P_{t_i}^1$ is calculated based on this recursive formula:

$$P_{t_i}^1 = \beta P_{t_{i-1}}^0 . P\left(b_{t_{i-1}}\right) + \beta P_{t_{i-1}}^1 . P\left(e_{t_{i-1}}\right); \ 1 \le i \le m-1 \quad (2)$$

This equation is obtained as follows: with respect to Table 1, it is clear that the system will operate in state 1 at t_i in two cases: (1) if the system is in state 0 at t_{i-1} , scenario b occurs and the control chart cannot identify this state; or (2) the system is in state 1 at t_{i-1}, scenario e occurs and the control chart cannot identify this state. It is assumed that there is no sampling inspection at t_m. Hence, for the last inspection time we have:

$$P_{t_m}^1 = P_{t_{m-1}}^0 . P\left(b_{t_{m-1}}\right) + P_{t_{m-1}}^1 . P\left(e_{t_{m-1}}\right) \quad (3)$$

Both maintenance types are assumed to be perfect. Hence, at the start of each production cycle the following equation is held:

$$P_{t_0}^0 = 1; \ P_{t_0}^1 = 0 \quad (4)$$

Equation (4) indicates that, as it was assumed, each production cycle starts with a new zero-age system. In other words, at the start of each production cycle the system is in state 0.

3. 3. Expected In-control and out-of-control Time

Expected time during each production cycle that the system operates in the in-control state can be obtained using the following equation:

$$E[T_0] = \int_0^{t_m} \bar{F}(t)\bar{\phi}_0(t)dt \quad (5)$$

Expected time during the inspection interval (t_{i-1}, t_i) that the system operates in the out-of-control state is denoted by T_1^i . Using the following equation, T_1^i can be computed:

$$T_1^i = P(t_{i-1}^0)\left[\int_{t_{i-1}}^{t_i}(t_i - t)\frac{f(t)}{\bar{F}(t_{i-1})}\frac{\bar{\phi}_0(t)}{\bar{\phi}_0(t_{i-1})}\frac{\bar{\phi}_1(t_i)}{\bar{\phi}_1(t)}dt\right] +$$

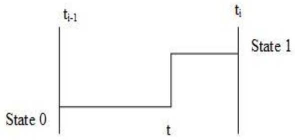
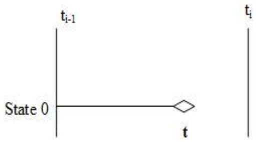
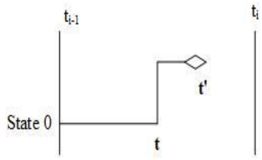
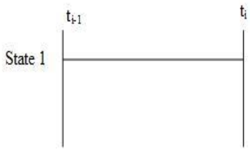
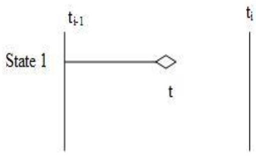
$$P(t_{i-1}^0)\int_{t_{i-1}}^{t_i}(t' - t)\frac{f(t)}{\bar{F}(t_{i-1})}\frac{\bar{\phi}_0(t)}{\bar{\phi}_0(t_{i-1})}\int_t^{t_i}\frac{\varphi_1(t')}{\bar{\phi}_1(t)}dt'dt \quad (6)$$

$$+ P(t_{i-1}^1)[\frac{\bar{\phi}_1(t_i)}{\bar{\phi}_1(t_{i-1})}(t_i - t_{i-1}) + \int_{t_{i-1}}^{t_i}(t - t_{i-1})\frac{\varphi_1(t)}{\bar{\Phi}_1(t_{i-1})}dt];$$

$$1 \le i \le m$$

This equation is obtained considering the depicted scenarios in Table 1 and their corresponding in-control and out-of-control duration.

TABLE 1. Different scenarios that may occur for the evolution of the considered system during each inspection interval

Scenario	Figure	Probability	In-control time	Out-of-control time
a		$P(a_{t_{i-1}}) = \frac{\bar{F}(t_i)\bar{\Phi}_0(t_i)}{\bar{F}(t_{i-1})\bar{\Phi}_0(t_{i-1})}$	$t_i - t_{i-1}$	0
b		$P(b_{t_{i-1}}) = \int_{t_{i-1}}^{t_i} \frac{f(t)}{\bar{F}(t_{i-1})} \frac{\bar{\Phi}_0(t)}{\bar{\Phi}_0(t_{i-1})} \frac{\bar{\Phi}_1(t_i)}{\bar{\Phi}_1(t)} dt$	$t - t_{i-1}$	$t_i - t$
c		$P(c_{t_{i-1}}) \int_{t_{i-1}}^{t_i} \frac{\varphi_0(t)}{\bar{\Phi}_0(t_{i-1})} \frac{\bar{F}(t)}{\bar{F}(t_{i-1})} dt$	$t - t_{i-1}$	0
d		$P(d_{t_{i-1}}) = \int_{t_{i-1}}^{t_i} \frac{f(t)}{\bar{F}(t_{i-1})} \frac{\bar{\Phi}_0(t)}{\bar{\Phi}_0(t_{i-1})} \int_t^{t_i} \frac{\varphi_1(t')}{\bar{\Phi}_1(t)} dt' dt$	$t - t_{i-1}$	$t' - t$
e		$P(e_{t_{i-1}}) = \frac{\bar{\Phi}_1(t_i)}{\bar{\Phi}_1(t_{i-1})}$	0	$t_i - t_{i-1}$
f		$P(f_{t_{i-1}}) = \int_{t_{i-1}}^{t_i} \frac{\varphi_1(t)}{\bar{\Phi}_1(t_{i-1})} dt$	0	$t - t_{i-1}$

3. 4. Performance Probability of Each Type of the Maintenance Probability of performance of the PM action at the end of the inspection interval (t_{i-1}, t_i) is denoted by P_{PM}^i. Using the following equation P_{PM}^i can be calculated:

$$P_{PM}^i = P_{t_{i-1}}^0 (1-\beta) P(b_{t_{i-1}}) + P_{t_{i-1}}^1 (1-\beta) P(e_{t_{i-1}}); \quad 1 \le i \le m-1 \tag{7}$$

This equation is obtained as follows: with respect to Table 1, it is clear that at t_i, PM is implemented on the system in two cases: (1) if the system is in state 0 at t_{i-1}, scenario b occurs and the control chart identify this state; or (2) the system is in state 1 at t_{i-1}, scenario e occurs and the control chart identify this state. At time point t_m, neither maintenance inspection nor sampling inspection is implemented on the system. Thus, the probability for conducting the PM action at t_m is computed as follows:

$$P_{PM}^m = P_{t_{m-1}}^0 [P(a_{m-1}) + P(b_{m-1})] + P_{t_{m-1}}^1 P(e_{m-1}); \tag{8}$$

Based on the description presented in Section 2, a production cycle may be terminated by conducting PM or CM action. Thus, the probability for terminating a production cycle by conducting CM on the system is as follows:

$$P_{CM} = 1 - \sum_{i=1}^m P_{PM}^i \tag{9}$$

3. 5. Probability of Conducting the Sampling Inspection The probability of conducting the sampling inspection at the end of the inspection interval (t_{i-1}, t_i) is denoted by P_{QC}^i. Using the following equation P_{QC}^i can be obtained:

$$P_{QC}^i = P_{t_{i-1}}^0 . [P(a_{t_{i-1}}) + P(b_{t_{i-1}})] + P_{t_{i-1}}^1 P(e_{t_{i-1}}); \quad 1 \le i \le m-1 \tag{10}$$

Also, in the special case that m=1, $P_{QC}=0$. Equation (10) is true because the sampling inspection is performed at t_i, if and only if, the system is in state 0 or 1 at t_i. Considering Table 1, it is clear that the system is in state 0 or 1 if one of the scenarios a, b or e occurs.

3. 6. Probability of Releasing a False Alarm The probability of releasing a false alarm from the control chart at inspection time point t_i is denoted by P_α^i. Using the following equation P_α^i can be obtained:

$$P_\alpha^i=\alpha\bar{F}(t_i)\bar{\Phi}_0(t_i);\ ;\ 1\leq\ i\leq m-1 \tag{11}$$

Equation (11) is obtained based on the fact that the control chart releases a false alarm at t_i ,if and only if, the time to quality shift as well as the time to failure state from state 0, be greater that t_i. Also α is the probability of releasing a false alarm from the control chart. For the special state that *m=1*, $P_\alpha=0$.

3. 7. Expected Profit Per Time Unit The integrated model consists of independent and stochastic identical cycles. Thus the expected profit per time unit (*EPT*) for the system operation can be computed based on the renewal reward process. Let define *E[T]* and *E[P]* as the expected time for the system operation in each production cycle and the expected profit for the system operation in each production cycle, respectively. Then *EPT* is computed as follows:

$$EPT=\frac{E[P]}{E[T]} \tag{12}$$

Considering the descriptions presented so far *E[T]* and *E[P]* are computed as follows:

$$E[P]=R_0E[T_0]+R_1\sum_{i=1}^{m}T_1^i-C_{QC}\sum_{i=1}^{m-1}P_{QC}^i-C_I\sum_{i=1}^{m-1}P_\alpha^i-(C_I+C_{PM})\sum_{i=1}^{m}P_{PM}^i-C_{CM}P_{CM}+C_I.P_{PM}^m \tag{13}$$

$$E[T]=E[T_0]+\sum_{i=1}^{m}T_1^i-Z_I\sum_{i=1}^{m-1}P_\alpha^i-(Z_I+Z_{PM})\sum_{i=1}^{m}P_{PM}^i-Z_{CM}P_{CM}+Z_I.P_{PM}^m \tag{14}$$

Finally, optimization of Equation (12) determines the decision variables of the integrated model.

4. CBM USING A MULTIVARIATE CONTROL CHART AND DETERMINING THE INSPECTION POLICY

In this section, it is assumed that a chi-square control chart is applied as a condition monitoring technique. Also, the application of the constant hazard policy as an inspection policy is elaborated.

4. 1. Application of a Chi-square Control Chart for CBM Consider a chi-square control chart is employed for the CBM purposes. In the following the details are for the CBM purposes. In the following the details are described. It is assumed that when the system is in state 0, $\mathbf{X_i}$, the quality characteristic of the item at time t_i, is a k-dimensional vector has a multivariate normal distribution with parameters $(\boldsymbol{\mu_0},\boldsymbol{\Sigma_0})$. In the out-of-control state $\mathbf{X_i}$ has a multivariate normal distribution with parameters $(\boldsymbol{\mu_1},\boldsymbol{\Sigma_0})$. Thus, occurring of the assignable cause only affects the mean of the system while has no influence on the covariance matrix. In the sampling time points $t_1,t_2,\dots,t_{m-1}$, a sample with size n is taken from the system and the value of the statistic $\chi_i^2=n(\bar{\boldsymbol{X}}_i-\boldsymbol{\mu_0})'\boldsymbol{\Sigma_0^{-1}}(\bar{\boldsymbol{X}}_i-\boldsymbol{\mu_0})$ is computed and plotted on the chi – square control chart. If the system is in state 0 then χ_i^2 has a central chi-square distribution with k degrees of freedom. In the out-of-control state, χ_i^2 has a non - central chi-square distribution with k degrees of freedom and non-centrality parameter $\delta=n(\boldsymbol{\mu_1}-\boldsymbol{\mu_0})'\boldsymbol{\Sigma_0^{-1}}(\boldsymbol{\mu_1}-\boldsymbol{\mu_0})$.

As any control chart, the chi-square control chart also has three parameters that includes the sample size, sampling interval and control limit. The chi-square control chart only has an upper control limit denoted by *UCL*. The probability of type I error is computed using the following equation:

$$\alpha=\int_{UCL}^{\infty}f(x)dx \tag{15}$$

where, f(x) is a chi-square distribution with k degrees of freedom. When the system operates in state 1 the probability of type II error is computed as follows:

$$\beta=\int_{0}^{UCL}f_\delta(x)dx \tag{16}$$

while, $f_\delta(x)$ is a non-central chi-square distribution with δ as a non-centrality parameter and k degrees of freedom.

4. 2. Constant Hazard Policy In this subsection, it is discussed how the inspection time points, $(t_1,t_2,\dots,t_{m-1})$, can be determined. As states by [15] different types of inspection policy may be applied for process monitoring and determining the inspection time points. Constant hazard policy is an inspection policy that appropriate for a system that its deterioration does not follow the exponential distribution [6]. Based on this policy, the inspection time points are determined such that the probability for quality shift remains constant in each inspection interval given that the system operates in the in-control state at the start of that interval. The inspection times are determined using the following equation:

$$\int_{t_{i-1}}^{t_i}h(t)dt=\int_{t_i}^{t_{i+1}}h(t)dt;\ \ i=1,2,\dots,m \tag{17}$$

In this formula, h(t) is the hazard rate function that is obtained as follows:

$$h(t)=\frac{f(t)}{\bar{F}(t)} \tag{18}$$

By assigning an arbitrary value to t_1 the other inspection time points can be determined by Equation (17).

5. NUMERICAL EXAMPLE

In the numerical examples presented in this section, it is assumed that time to quality shift, time to transit from state 0 to the failure state as well as time to transit from state 1 to the failure state are based on a Weibull distribution as the following cumulative distribution function:

$$F(x) = 1 - \exp[-(\lambda t)^{v}]; \quad v, \lambda, t \geq 0 \tag{19}$$

where, λ and v are the scale and shape parameter of the Weibull distribution, respectively.

Example 1. In this example, the observation vector has two dimensions as follows: $\boldsymbol{\mu_0} = (0,0)$, $\boldsymbol{\mu_1'} = (2,5.25)$ and $\boldsymbol{\Sigma_0} = \begin{bmatrix} 2 & 1 \\ 1 & 2.5 \end{bmatrix}$,hence δ is obtained 11.

Also, the sample size is assumed 1 as the We and Makis' study [1]. The other parameters of the numerical example are shown in Table 2. In this table C_f and C_v are the fixed and variable sampling cost, respectively Thus, C_{QC} for n units is $C_f+n\times C_v$.

It is used a grid search algorithm coded in MATLAB program for optimizing the proposed integrated model. The result of the integrated model optimization is as follows:

$EPT=351;\ t_1=3.3;\ UCL= 9.2;\ m=19;\ t_m=14.38$

This result indicates that monitoring of the process is started after passing 3.3 time unit from the start of each production cycle. Other inspection time points are obtained based on Equation (17). In each inspection time, a sample is taken from the process and for each item a vector of observation is obtained and the value of the statistic χ_i^2 is computed. The upper limit of the chi – square control chart is 9.2. After passing 19 inspection periods and at time 14.38 the production cycle is terminated. Applying these approach leads to maximize the expected profit per time that is equal to 351. The control chart used in this example is illustrated in Figure 1.

Example 2. Suppose that the observation vector has three dimensions. The means of the process in the in-control state and out-of-control state are: $\boldsymbol{\mu_0} = [1.5, 2.3, 2.5]$; $\boldsymbol{\mu_1'} = [4.7, 4.3, 6.6]$ respectively. The covariance matrix is: $\Sigma = \begin{bmatrix} 3 & 2.5 & -1 \\ 2.8 & 6.1 & 4 \\ 4.2 & 3.8 & 5.1 \end{bmatrix}$.

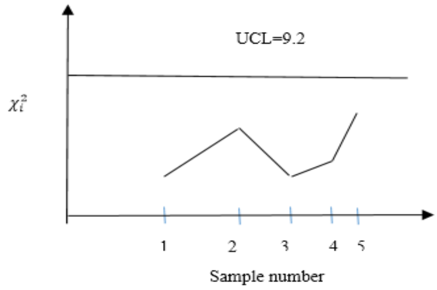


Figure 1. A Chi-square control chart to monitor the process

Based on equation $\delta = n(\boldsymbol{\mu_1} - \boldsymbol{\mu_0})'\boldsymbol{\Sigma_0^{-1}}(\boldsymbol{\mu_1} - \boldsymbol{\mu_0})$, $\boldsymbol{\delta = 3.29}$. The other parameters are similar to Example 1. The results are as follows:

$EPT = 97.79;\ t_1 = 6.9;\ UCL = 4.6;\ m = 3;\ t_m = 11.95.$

6. SENSETIVITY ANALYSIS

In this section, some sensitivity analysis is conducted on the key parameters of the process. Throughout this section, with respect to Example 1 in Section 5, some changes are made on the values of the process parameters and the result of the integrated model optimization is analyzed.

6. 1. The Effect of the Parameters of the Weibull Distribution

In this subsection, the effect of the parameters of the Weibull distribution is studied. First, the effect of the shape parameter, v, is studied. Figure 2 illustrates the effect of the shape parameter. As can be seen, an increase in the value of the shape parameter has an increasing effect on the values of EPT and t_1. On the other hand, the increase in the value of the shape parameter leads to a decrease in the values of m and t_m. Increase in the value of t_1 can be justified based on the fact that in a Weibull distribution (for a fixed value of the scale parameter), increase of the shape parameter leads to the reduction of the variance of the distribution. Hence, for the larger values of the shape parameter, it is easier to predict the failure time. Also, change on the value of the shape parameter has no significant effect on the value of UCL. Now, we proceed to study the effect of the scale parameter of the Weibull distribution. The result is indicated in Figure 3.

TABLE 2. The parameters of the numerical example

parameter	δ	C_f	C_v	R_I	R_0	C_1	C_{CM}	C_{PM}	Z_I	Z_{RM}	Z_{PM}	$v = v_0 = v_1$	$\lambda = \lambda_0$	λ_1
value	11	5	1	50	500	50	1000	500	0.5	2	1	2	0.059	0.089

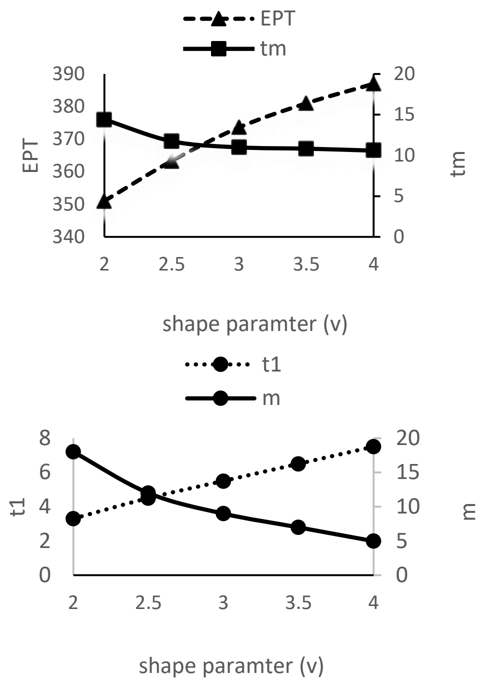


Figure 2. the effect of the shape parameter of the Weibull distribution

Decreasing the values of the scale parameter, as expected, leads to an increase in the value of *EPT*, because in a Weibull distribution, the smaller values of the scale parameter yield to a larger values of the mean value.

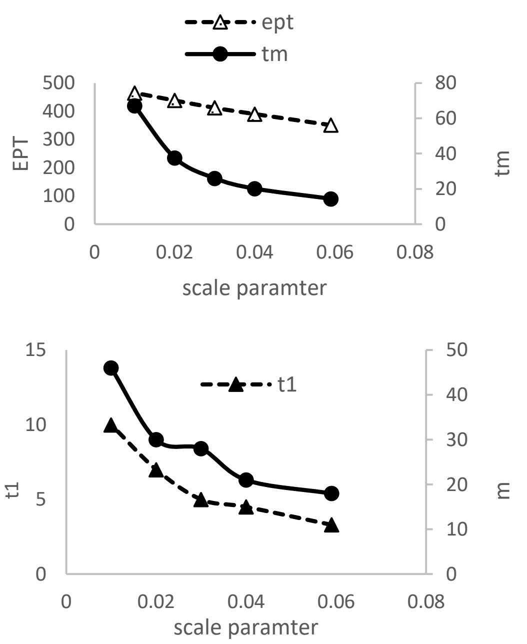


Figure 3. The effect of the scale paramter of the Weibull distribution

6. 2. The Effect of the Revenue Parameters In this subsection, the effect of change on the values of R_0 and R_1 is analyzed. First, assume that the values of R_1 increases from 50 to 100. The results of optimization are as follows: EPT=352; t_1=3.5; UCL=9.2; m=17; t_m=14.3.

As observed, increase in the value of R1 from 50 to 100 does not have a significant effect on the decision variables of the model. In the next step, the values of R_0 changes from 500 to 200. The results are as follows: EPT=101; t_1=4.9; UCL=7.8; m=10; t_m=14.7

As can be seen, unlike the effect of R_1, a decrease in the value of R0 has a drastic decreasing effect on EPT , so that the value of EPT decreases from 351 to 101.

6. 3. The Effect of the Parameters of the Maintenance Costs Figure 4 illustrates the effect of the maintenance cost. To this end, the values of the corrective maintenance cost,C_{CM} is increased, while the preventive maintenance cost is unchanged. As expected, the increase in the value of C_{CM} leads to a decrease on the value of EPT. Also, this change has a decreasing effect on the values of t_m and m, while it has no significant effect on the values of UCL and t_1.

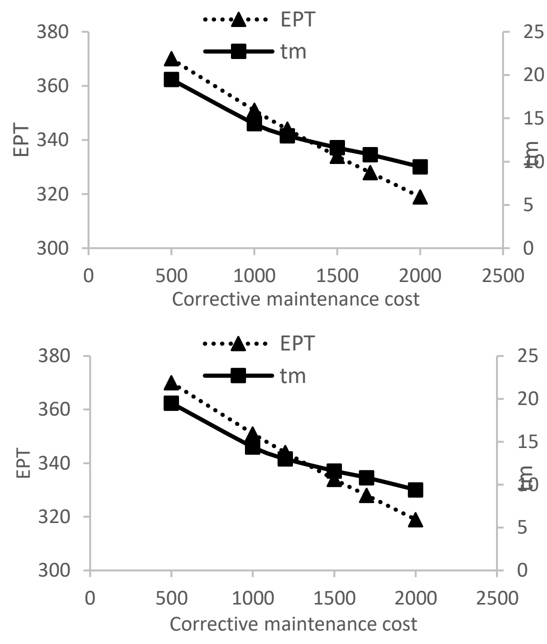


Figure 4. the effect of the values of the corrective maintenance cost

4. CONCLOSION

In this paper, an integrated model for CBM and SPC is developed. With respect to the current CBM models that have used the multivariate control charts as a condition monitoring technique, the proposed model has a more general structure and a wider application domain because this model has three main novelties:

1- we place no restrictive assumption on the deterioration mechanism of the system. In other words, the time to quality shift as well as the time to the failure state from each operational states are assumed as a general continuous distribution function;

2- developing the integrated CBM and SPC model in this paper is based on the recursive equations and renewal reward process. Thus, the model can be easily applied for the other control charts; 3- the model can be applied for different types of inspection policy, because the inspection time points are considered as the decision variables of the model.

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Application of Multivariate Control Charts for Condition Based Maintenance

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چکیده

اساس سیاست نگهداری و تعمیرات مبتنی بر وضعیت، پایش وضعیت است. معمولا برای ارتباط دادن اطلاعات بدست آمده از پایش وضعیت به حالت واقعی سیستم، نیاز به مدل های احتمالی است. از طرف دیگر، با درنظر گفتن ارتباط نزدیکی که بین نگهداری و تعمیرات مبتنی بر وضعیت و کنترل فرایند آماری وجود دارد، مدل های یکپارچه توسعه داده شده است. این مدل ها از نمودارهای کنترل به عنوان یک ابزار پایش وضعیت استفاده می کنند و استنباط در مورد وضعیت عملیاتی سیستم برمبنای اطلاعات بدست آمده از پایش وضعیت صورت می گیرد. درنهایت تصمیم گرفته می شود که چه نوع تدابیر نگهداری و تعمیراتی به کارگفته شود. این مقاله کاربرد نمودارهای کنترل چند متغیره را به عنوان یک تکنیک پایش وضعیت، در نگهداری و تعمیرات مبتنی بر وضعیت نشان می دهد. در این راستا، یک مدل یکپارچه توسعه داده شده در حالیکه از نمودار کنترل کای اسکوار استفاده می شود. همچنین برای تعیین دوره های بازرسی، یک سیاست تحت عنوان سیاست نرخ خرابی ثابت به کار گرفته می‌شود.

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