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Solving a New Multi-objective Inventory-Routing Problem by a Non-dominated Sorting Genetic Algorithm

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ABSTRACT

This paper considers a multi-period, multi-product inventory-routing problem in a two-level supply chain consisting of a distributor and a set of customers. This problem is modeled with the aim of minimizing bi-objectives, namely the total system cost (including startup, distribution and maintenance costs) and risk-based transportation. Products are delivered to customers by some heterogeneous vehicles with specific capacities through a direct delivery strategy. Additionally, storage capacities are considered limited and the shortage is assumed to be impermissible. To validate this new bi-objective model, the ε-constraint method is used for solving problems. The ε-constraint method is an exact method for solving multi-objective problems, which offers Pareto's solutions, such as meta-heuristic algorithms. Since problems without distribution planning are very complex to solve optimally, the problem considered in this paper also belongs to a class of NP-hard ones. Therefore, a non-dominated sorting genetic algorithm (NSGA-II) as a well-known multi-objective evolutionary algorithm is used and developed to solve a number of test problems. In this paper, 20 sample problems with the ε-constraint method and NSGA-II are solved and compared in different dimensions based on Pareto's solutions and the time of resolution. Furthermore, the computational results showed the better performance of the NSGA-II.

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1. INTRODUCTION

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An inventory-routing problem (IRP) is derived from a vehicle routing problem (VRP), in which inventory control and routing decisions are merged. Numerous studies and analyses have been carried out on IRPs previously; for example, a comprehensive review has been presented by Anderson [1]. Having analyzed the industrial aspects of the problem, an inclusive classification and review of previous studies, direct shipment is discussed. That is one of many distribution strategies used in IRP; in which each vehicle only delivers products to retailers once during each cycle. Due to the simplicity of implementation associated with this distribution strategy, it is normal for such a strategy to be considered first in IRP [2].

Li et al. [3] considered IRP with a condition of a supplier only holding one means of transport and only being able to deliver products to one customer in each period. They developed an innovative algorithm to obtain the feasible sequence. Campbell and Hardin [4] examined the minimum number of required vehicles in IRP with direct shipment and proposed a greedy algorithm as a solution. Cheng and Duran [5] proposed a model for the IRP in global crude oil supply chain, in which customer demand and shipment duration are indecisive, and a customer demand is dynamic. In this paper for planning and controlling the inventory and transportation system, a decision support system method was used.

Niyakan and Rahimi [6] studied the multi-objective IRP with a fuzzy approach in a health environment. They proposed a combined fuzzy approach to solve the problem in a state of uncertainty and proved the performance and efficiency of this proposed algorithm. Cheng et al. [7] also studied the multi-period IRP based on environmental issues. They used a hybrid genetic algorithm (GA) and obtained the significance of the

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environmental issue (i.e., the amount of carbon dioxide emissions).

Cardenas-Barron et al. [8] proposed a new approach to solve the multi-product multi-period inventory lot sizing with a supplier selection problem. They applied a new algorithm based on a reduced and optimized approach. They proposed a new valid inequality to solve the model.

Moubed and Mehrjerdi [9] suggested a hybrid dynamic programming method for an inventory-routing problem in collaborative reverse supply chains. They used a hybrid heuristic algorithm combining dynamic programming, ant colony optimization and tabu search in order to solve the problem. Esmaili and Sahraeian [10] proposed a new bi-objective model for a two-echelon capacitated IRP for perishable products with the environmental factor. They solved the model with the simple additive weighting (SAW) and compared with GAMS software.

Fattahi et al. [11] proposed a bi-objective model for a multi-period two-echelon perishable product IRP with production and lateral transshipment. They used the NSGA-II to solve the model.

One of the important issues in IRPs at tour selection is the manner of risk issue. In the IRP, we should consider the risk of transporting hazardous material and environmental and human, which can cause damage. Given the importance of this issue, in our model, also the risk was considered.

It is to be noted that except for the investigations by Niakan and Rahimi [6] and Nolz *et al.*[12], who introduced a risk factor in IRP by considering it in their problems. There was no studies that have considered such issues in IRPs. Therefore, our contributions in this paper are as follows:

- Considering the transportation risk in the model.
- Considering the backhaul transportation option on IRP model.
- Considering the problem in a bi-objective multi-product, multi-period and heterogeneous fleet form.

The problem discussed in this paper is considered NP-hard based on previous research. Thus, solving the problem in reasonable computational time using exact methods is not possible, especially for large-scale problems. However, it is possible to use meta-heuristic algorithms in order to obtain near-optimal (or Pareto-optimal) solutions in a reasonable amount of computational time. To the best of our knowledge, heuristic/meta-heuristic algorithms have been considered a few studies in solving IRP under some mentioned constraints. Therefore, NSGA-II algorithm is employed to solve the given problem, and its efficiency is compared to the epsilon-constraint method on several random sample problems.

2. MATHEMATICAL MODEL

2. 1. Modeling Framework

This paper proposes a multi-objective multi-period and multi-product IRP with backhaul transportation, which can be formulated in the form of a mixed-integer programming (MIP) model. The model consists of two objective functions. The first objective function minimizes the total system cost (i.e., startup, distribution and maintenance costs) and the second objective function reduces transportation risk.

In connection with this issue, for each route that transit happens, risk variables are used. These variables are different for specific periods and products. In this model, each route is associated with a specific transportation risk r_{ijvt}. This risk is usually considered for the transportation of hazardous materials (e.g., gas pipes and oil). Risk calculations have been carried out using the findings of Marhavilas [13].

However, for simplicity, we use random numbers with a uniform distribution in the calculation of the transportation risk.

Finally, the proposed model is based on the following assumptions:

- Model is a single depot that services to all of the customers.
- Distribution fleet is heterogeneous.
- Distances between points are known.
- Demands of customers are predetermined.
- Shortages are not allowed.

Before describing the model, the notations used to describe the model are defined below.

2. 2. Sets and Indices

i, j, μ, λ:	Demand nodes index
A:	Total number of customers (A=1,…,U, $U+1$,…, $U+W$)
u:	Index of customers on the inhaul trip
w:	Index of customers on the backhaul trip
$\{0,U+W+1\}$:	Depot index
v:	Vehicle index
P:	Distributable product index
t:	Index of time periods

2. 3. Parameters

cf_v^t: Fixed cost of using vehicle v in period t

cv_v^t: Variable costs of using vehicle v in period t

du_{it}^p: Demand of the i-th customer on the initial trip for product p in period t

dw_{it}^p: Demand of the i-th customer on return trip for product p period t.

Q_v: Capacity of vehicle v per unit weight

r_{ijvt}: Risk between customers i and j by vehicle v in period t
C_i: Inventory capacity of the i-th customer
S_{ij}: Edge length between customers i and j
C_{ijvt}: Costs of traverse between customers i and j by vehicle v in period t.
y_i^p: Storage and maintenance costs undergone by the i-th customer product p
q^p: Weight of product p.

2. 4. Variables

M_{ivt}^p: Amount of product p delivered to the i-th customer by vehicle v in period t (inhaul trip).
N_{ivt}^p: Amount of product p received from the i-th customer by vehicle v in period t (backhaul trip).
I_{it}^p: Amount of inventory of product p held by the i-th customer at the end of period t.
B_{ijvt}^p: The amount of product p transported from customer i to j by vehicle v during period t.

2. 5. Mathematical Model

$$\text{Min } Z_1 = \sum_{t=1}^{T}\sum_{v=1}^{V}\sum_{j=1}^{U+W} cf_v^t . x_{0jvt} + \sum_{t=1}^{T}\sum_{i=0}^{U+W}\sum_{j=0}^{U+W}\sum_{v=1}^{V} c_{ijvt}. x_{ijvt} + \sum_{t=1}^{T}\sum_{i=1}^{U+W}\sum_{p=1}^{P} y_i^p . I_{it}^p \quad (1)$$

$$\text{Min } Z_2 = \sum_{t=1}^{T}\sum_{i=0}^{U+W}\sum_{j=0}^{U+W}\sum_{v=1}^{V} r_{ijvt}. x_{ijvt} \quad ; \quad \forall\, v,t \quad (2)$$

$$I_{i,t-1}^p - I_{i,t}^p + \sum_{v=1}^{V} M_{ivt}^p = du_{it}^p \; ; \; \forall\, i \in \{1,\dots,U\}, v, t \quad (3)$$

$$I_{i,t-1}^p - I_{i,t}^p - \sum_{v=1}^{V} N_{ivt}^p = -dw_{it}^p \; ; \; \forall\, i \in \{U+1,\dots,U+W\}, v, t \quad (4)$$

$$\sum_{v=1}^{V}\sum_{\mu=0}^{U} B_{\mu ivt}^p - \sum_{v=1}^{V}\sum_{\lambda=1}^{U+W+1} B_{i\lambda vt}^p = \sum_{v=1}^{V} M_{ivt}^p \; ; \; \forall\, i \in \{1,2,\dots,U\}, \forall\, t,p \;,\; i \neq \mu \neq \lambda \quad (5)$$

$$\sum_{v=1}^{V}\sum_{\lambda=U+1}^{U+W+1} B_{i\lambda vt}^p - \sum_{v=1}^{V}\sum_{\mu=1}^{U+W} B_{\mu ivt}^p = \sum_{v=1}^{V} N_{ivt}^p \; ; \; \forall\, i \in \{U+1,\dots,U+W\}, \forall\, t,p, i \neq \mu \neq \lambda \quad (6)$$

$$\sum_{v=1}^{V}\sum_{i=0}^{U+W} x_{ijvt} \leq 1 \; ; \; \forall\, j \in \{1,\dots,u+w+1\}, \forall\, t \quad (7)$$

$$\sum_{j=1}^{U+W} x_{0jvt} \leq 1 \; ; \; \forall\, v,t \quad (8)$$

$$\sum_{i=1}^{U+W} x_{i,(U+W+1)v,t} = \sum_{j=1}^{U+W} x_{0jvt} \; ; \; \forall\, v,t \quad (9)$$

$$\sum_{i=1}^{U+W} x_{i\mu vt} - \sum_{j=1}^{U+W} x_{\mu jvt} = 0 \; ; \; \forall\, \mu \in \{1\dots,u+w\}, t, v \quad (10)$$

$$\sum_{i=1}^{U}\sum_{j=U+1}^{U+W}\sum_{v=1}^{V} x_{ijvt} \geq 1 \; ; \; \forall\, t \quad (11)$$

$$\sum_{p=1}^{P} I_{it}^p \leq C_i \; ; \; \forall\, i \in A \quad (12)$$

$$\sum_{\mu=0}^{U+W} q^p . B_{\mu ivt}^p - \sum_{\lambda=0}^{U+W} q^p . B_{i\lambda vt}^p \geq 0 \quad (13)$$

$$0 \leq \sum_{p=1}^{P} q^p . B_{ijvt}^p \leq Q_v . x_{ijvt} \quad (14)$$

$$M_{ivt}^p \geq 0 \,,\; N_{ivt}^p \geq 0, X_{ijvt} \in \{0,1\}, I_{it}^p \geq 0, B_{ijvt}^p \geq 0 \quad (15)$$

The first objective function includes fixed routing costs, shipment and delivery, and maintenance costs undergone by customers. The second objective function minimizes transportation risks on routes taken by vehicles. Equations (3) and (4) express the inventory balance for customers on a round trip with respect to their demand, respectively. Equations (5) and (6) represent the difference between the input and output of each node for customers on a round trip. Equation (7) shows that each customer is visited by a vehicle utmost once during each period. Equations (8) and (9) indicates that each vehicle starts at the central depot and returns to that after each trip. Constraint (10) shows the continuity of the travel path. Equation (11) indicates that customers on the inhaul trip are to be visited and provided with service before customers on the backhaul trip. Equation (12) indicates compliance with the allowed storage capacity limit for each customer. Equation (13) is used to sub-tour elimination from vehicle routing problems. Constraint (14) shows the maximum and minimum load of variable products for each vehicle during inhaul and backhaul trips. Also, other constraints show the type of variables.

2. NSGA-II ALGORITHM

Highlights and significant facts regarding this NSGA optimization method are given below:

- The solutions to which no superior is found holds the most points. Solutions are ranked and sorted based on the number of answers superior to them.
- Suitability of a solution is determined based on its rank and lack of predominance and superiority of other solutions.
- Shared suitability method is used for solutions with close results in order to adjust distribution; allowing answers and solutions to be distributed uniformly in the searching space. Also, the NSGA-II flowchart is presented in Figure 1.

3. 1. Presenting the Particles One of the most important decisions taken during the design of a meta-heuristic algorithm is how to present the solutions and provide an effective, unique and identifiable relation between these solutions and search space of the analysis. In this paper, a common string method is used to display the chromosomes. For example, in a transport network consisting of m origins and n destinations, a feasible chromosome is considered as a permutation of m+n bits (i.e., gens),

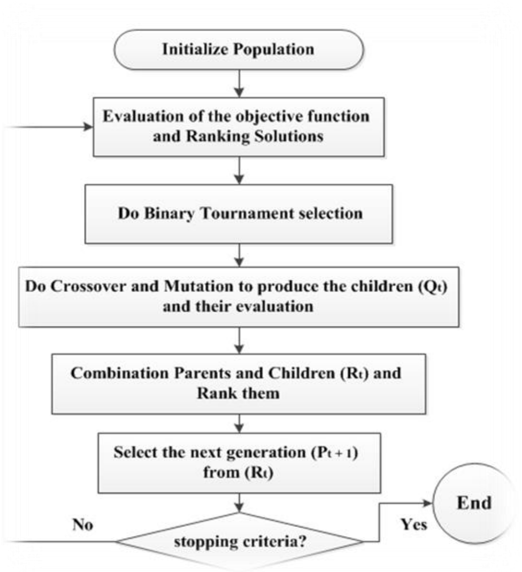

Figure 1. Flowchart of the NSGA-II

which in fact represents the order of the nodes for participation in the transport network tree. Thus, in our problem, a feasible chromosome assuming u=4 and w=4 is as shown in Figure 2.

3. 2. Creating the initial population In general, the quality of early solutions may vastly affect the performance of a meta-heuristic algorithm. Therefore, designing an effective method to produce the initial solutions is considered of great significance. Hence, a quasi-random method is designed to produce initial chromosomes in the proposed algorithm. This method consists of the following steps:

- A random sequence of vehicles, along with another random sequence of customers, is created in order to form the initial population.
- Starting with the first vehicle, different customers are assigned to each vehicle. It is noted that customers are assigned to vehicles depending on the timing of delivery for each customer, and the capacity of each vehicle. The same procedure is repeated for each customer, and so, all the remaining customers are assigned to a vehicle with regard to the capacity and timing schedule of the vehicle.

Finally, the amount of inventory and delivery or receiving the shipment for all customers are determined according to the matrices described in the next section.

3. 3. Position Matrix This matrix shows the timing of vehicles and pinpoints, which vehicle is to be used in each period.

2	5	7	3	1	6	8	4

Figure 2. Presentation of a feasible chromosome

The position matrix contains U+W+V-1 columns and T rows consisting of V-1 zeros, and ones for all remaining values. Hence, with the use of this matrix, vehicle timing schedules during each period of time may be observed from each row in the position matrix. For example, if the following numbers are found in one such matrix, it may be concluded that during this time period the first vehicle will cover the 2nd and 3rd customers, while the 1st and 4th customers are assigned to the second vehicle, and the third vehicle is to provide service to the 5th customer only as shown in Figure 3.

3. 4. Distribution Matrix The distribution matrix is a matrix with the size of $P\times(U+W)\times T$, which is related to the distribution of products. In other words, for each period, the distribution matrix is defined as a two-dimensional matrix with the size of $P\times(U+W)$. As an example, the distribution matrix for the first period of time is shown in Figure 4. This figure indicates that during the first period of time, the vehicle has delivered 17 units of product Type I, 23 units of product Type II, etc. to the 1st customer.

After calculating the initial population, the best available solution needs to be found and the acts of crossover and mutation need to be carried out in order to complete the algorithm and achieve the optimal solution.

3. 4. 1. Crossover Operator in position matrix A **c**lever single-point method is used for the crossover operator in this model. The clever single point method is explained in the following example (suppose U=W=3).

$Parent_1$	1	4	3	2	6	5
$Parent_2$	6	2	4	3	1	5

The hatched cells result in the following offspring:

$Child_1$	1	4	3	3	1	5
$Child_2$	6	2	4	2	6	5

It is easily noticed that for child number one, the third customer is repeated twice, which is incorrect. The intellectual property of the operator used in this model ensures that the 6th customer comes after the third (according to the sequence of child No. 2), the 5th customer after that, and finally the 2nd customer at the end. Consequently, the resulting chromosome is as follows

T=1	3	2	0	1	4	0	5

Figure 3. Example of allocating vehicles to customers

	C_1	-----	
	17	-----	P_1
	23	-----	P_2
T=1:	15	-----	P_3
	12	-----	P_4
	14	-----	P_5

Figure 4. Example of a distribution matrix by a vehicle

$Child_1$	1	4	3	6	2	5

$Child_2$	6	2	4	1	3	5

Finally, given that customers on the inhaul trip must be provided with service to customers on the backhaul, the adjusted chromosome will be as shown in Figure 5.

3. 4. 2. Mutation Operator in the Position Matrix In this model, a swap method is used for the mutation operation (e.g., swapping the chromosomes from the previous example will result in a new child) as shown in Figure 6. However, the priority of inhaul customers over backhaul customers is to be maintained.

3. 4. 3. Crossover Operator in Distribution Matrix An arithmetic mean method is used in this model for the distribution matrix crossover operator. For example, assuming that the following chromosomes in Figure 7 indicate a part of a distribution matrix, resulting children can be obtained from the following Equation (16).

$$Child_i=Parent_i\ (\alpha)+Parent_j\ (1-\alpha);\ 0\leq\alpha\leq1 \qquad (16)$$

$Parent_1$	20	10	15	16	5	11
$Parent_2$	14	12	6	3	11	9

3. 4. 4. Mutation Operator in the Distribution Matrix In this model, the insertion pattern is used as shown in Figure 8.

3. 5. Reasonability In order to ensure the feasibility of the model, a repair process is used.

$Child_1$	1	3	2	4	6	5
$Child_2$	2	1	3	6	4	5

Figure 5. Crossover operator for the position matrix

$Parent_1$	2	1	3	4	6	5
$Child_1$	2	1	3	5	6	4

Figure 6. Mutation operator for the position matrix

$Child_1$	18	11	13	13	7	11
$Child_2$	16	12	9	7	10	10

Figure 7. Crossover operator for the distribution matrix (*a*= 0.7)

$Parent_1$	20	10	15	16	5	11
$Child_1$	20	10	16	15	5	11

Figure 8. Mutation operator for the distribution matrix

In other words, the vehicle capacity, storage capacity and the like should be checked and inspected for each case, separately. In addition, deliveries to each customer are not to be less than demanded by the customer, as lacking and insufficiency are not considered allowed in the problem. If this occurs; however, the customer must be removed.

3. 6. Algorithm Iteration After the initial population of parents is randomly created and evaluated, a population of children that is equal to the population of parents will generate in accordance with the selection method and genetic operators described in previous sections. The combination of these two sets of populations according to the previously presented structure will result in the next generation. This procedure will be repeated until the termination criterion of the algorithm is met. Finally, the first fronts of the last generation, which in fact represent the non-dominated solutions of the problem, are obtained as the output of the algorithm.

3. 7. Termination Criteria of the Algorithm This condition can be defined as criteria determining how far the algorithm iteration loop will continue. Depending on the designer, these criteria may be different for each algorithm. The most common criterion is the number of iterations (e.g., the algorithm may be designed to terminate after K iterations). This termination criterion (i.e., number of iterations) is also used in the model presented in this paper.

2. EPSILLON-CONSTRAINT METHOD

The ε-constraint method is one of the best-known methods for solve the MOP. In this method, one of the objective functions must be considered as the main objective function (randomly) and other objective functions must be converted to model constraints. Marhavilas et al. [13] proposed the ε-constraint method with Relations (17).

$$\begin{aligned} &Max\ f_1(x)\\ &\text{st.}\\ &f_2(x)\geq e_2\\ &f_m(x)\geq e_m\\ &x\in S \end{aligned} \qquad (17)$$

The following steps are necessary to apply the proposed ε-constraint method:

- Create the payoff Table. To do this, optimize each objective function individually, and calculate the value of other objective functions at this optimal point. For each objective function, call the interval between the ideal value and the worst value (nadir value). In cases where finding nadir value is not straightforward, generate the payoff table with a lexicographic method.

- Choose one of the objective functions (f_j(x)) as the main objective function to be optimized, and transform other functions into constraints.
- Solve mathematical model, as provided in Equation (17). In this model, e_i is the nadir value of the objective function.

5. COMPUTATIONAL RESULTS

In this section, the computational results of the aforementioned model are analyzed. Therefore, the problem is solved with ε-constraint method in small and medium sizes. In order to validate the proposed meta-heuristics, the results obtained from ε-constraint method are compared to those obtained from the NSGA-II. The problem is solved on a personal computer with an Intel core2duo 2.67 GHz processor and 3GB internal storage, using GAMS software and MATLAB software for small and large-size problems, respectively.

5. 1. Creating Sample Problems Sample problems for this section are designed as to provide a two-dimensional 25×25 unit square inside, which customers are randomly scattered. The distance between all points is calculated based on the Euclidean distance. Customer demand is generated within the range of 20 to 60 using a uniform distribution. The amount of risk associated with each travel path is also generated using a normal distribution, and range from 10 to 100. The number of customers for small and medium-scale problems is set between 5 and 25, and the number of planning periods is selected between 2 to 12. The vehicle capacity is assumed within the range of 50 and 200 units.

5. 2. Measuring Metrics In this section, we introduce the main measuring metrics used in the proposed meta-heuristic algorithms.

- *Number of Pareto solutions (NP)*: This criterion indicates the number of Pareto solutions obtained by each algorithm. According to this index, a higher number of Pareto solutions are associated with a higher algorithm quality.
- *Spacing metric (SM)*: This criterion measures the uniformity of a non-dominated solutions distribution within the solution spacing, and can be defined by:

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} \left(\bar{d} - d_i\right)^2} \quad \& \qquad (18)$$

$$d_i = min_{j \neq i}\left(\sum_{m=1}^{M} \left|f_i^m - f_j^m\right|\right)$$

- *Diversity metric (DM)*: This criterion evaluates the diversity and distribution of Pareto solutions, and is defined by:

$$D = \left[\sum_{i=1}^{n} \max(\|\widehat{x_i} - \widehat{y_i}\|)\right]^2 \qquad (19)$$

- *Mean Ideal Distance (MID)*: This criterion measures the average distance of Pareto solutions from the origin and is preferred to be as little as possible.

$$MID = \sum_{sol=1}^{n} \sqrt{\sum_{g=1}^{number\ OF} {F_{sol.g}}^2} \qquad (20)$$

5. 3. Setting the Parameters In this section, parameters required for the adjustment of the NSGA-II, with the aim of ensuring achievement of the best solution, are presented using the Taguchi method. Four parameters (i.e., the population size, mutation and crossover rate, and iteration number) are used in the NSGA-II, and three levels are defined for each of these parameters. Minitab software is used for experimental design and analysis of the results. Given the number of factors and levels selected for the analysis, standard interactive Table L9, provided by the Taguchi method, is chosen for this study. As may be observed from Figure 9, the minimum point of each parameter in the software output is usually considered the best level for that parameter (based on minimum signal noise); thus, the most appropriate levels and values for each parameter will be as presented in Table 1.

5. 4. Results 12 small and medium-sized problems are solved in this section by using the NSGA-II and ε-constraint method with due attention to Pareto Fronts. The results are presented in this Table for small-size problems, a meta-heuristic algorithm can easily achieve the optimal solution with the minimum error percentage. The efficiency of these methods in substantially reducing solution time is shown as well.

It is also to be concluded from Table 2 that the proposed algorithm provides a high yield and performance, as well as close-to-optimum Pareto solutions, and a very low average calculation error. However, as the calculation time in GAMS software increases exponentially with the enlargement of problem sizes, up to a maximum number of 24 customer knots are considered for solving the problem using the ε-constraint method. Solving the problem in larger sizes by using GAMS software is considered close to impossible, and requires an enhanced use of meta-heuristic algorithms, as further explained in the following section.

Figure 10 shows the computational time for two algorithms, which is increasing exponentially in GAMS software due to the enlargement of problem scales. For this reason, problems that take more than 30 minutes of time to be solved in GAMS are considered large-scale problems and are only solved using the NSGA-II.

The amounts of error for the algorithm, regarding the opt solution, can also be found with respect to the objective functions. The error values are obtained by:

$$Error\ (GAP) = \frac{OF_{NSGA} - OF_{GAMS}}{OF_{GAMS}} \qquad (21)$$

TABLE 1. Results of parameters for the NSGA-II

	Parameters			
	Population Size	Crossover Rate	Mutation Rate	Iteration Number
Level	2	3	1	3
Amount	100	0.8	0.2	200

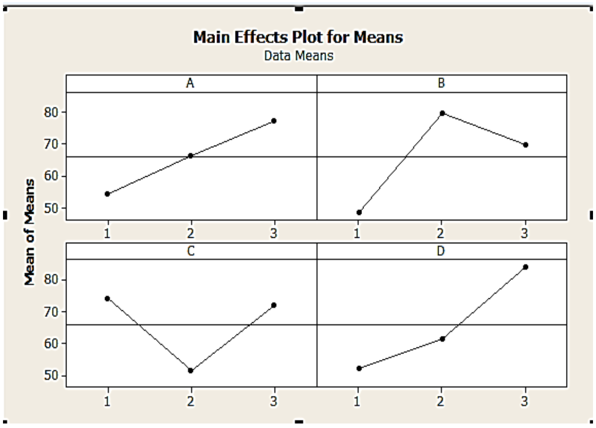

Figure 9. Minitab output for parameters setting

Table 3 presents the results of solving large-scale problems. These include eight sample problems, which take more than 30 minutes of time to be solved with ε-constraint method.

5. 5. Discussion

In small-sized problems, the Epsilon method was better than the NSGA-II algorithm, in terms of the value of the objective function, although the NSGA-II is faster than the Epsilon to get Pareto's front.

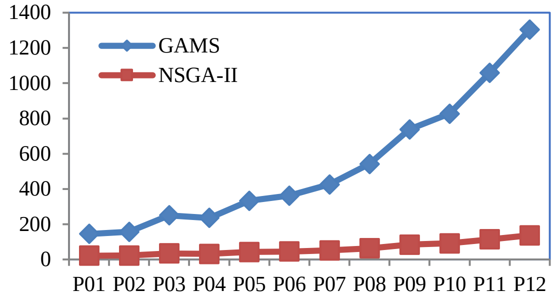

Figure 10. Comparison of computational time for two algorithms

But in large-sized problems, the computational time by ε-constraint method is exponentially increasing, and so we compare them to the following way. For example, we consider sample problem P_{15}. The sorted Pareto solutions (seven Pareto based on Table 3) that is related to the NSGA-II includes:

(626441, 190.5), (632386, 187.1), (634458, 184.1), (649473, 179.6), (682323, 179.4), (707899, 179.3) and (723777, 178.5).

Also, the Pareto solutions of this problem based on ε-constraint method are as follows:

(624707, 187.8), (638762, 187.2), (647890, 184.7), (665054, 182.1) and (689158, 179.4).

It should be noted that in Table 2, we consider the mean value of Pareto solutions as the objective function values. So, Z_{Trans} for ε-constraint method for P_{15} problem is 653116 and for the NSGA-II is 665251. Although the average Pareto cost of ε-constraint method is less than that of the NSGA-II; however, with respect to Figure 11, the NSGA-II finds relatively better responses in larger dimensions. In small-sized problems, the performance of NSGA-II is not superior to ε-constraint method.

TABLE 2. Sample problem results for small-sized problems

Prob.	Prob.info (U/W/P/V/T)	ε-constraint			NSGA-II				
								Relative GAP	
		Z_{Trans}	Z_{Risk}	*Time*	Z_{Trans}	Z_{Risk}	*Time*	Z_{Trans}	Z_{Risk}
P01	3×3×2×3×2	5808	29.5	145.5	6058	31.8	21.6	0.043	0.078
P02	4×3×3×2×3	8712	47.4	157.2	9144	50.3	22.5	0.049	0.061
P03	5×5×3×4×4	25435	98.3	250.8	27324	105.7	34.6	0.074	0.075
P04	6×4×4×3×3	19148	74.5	236.2	21429	79.9	31.5	0.119	0.072
P05	7×5×4×3×4	39984	129.8	333.2	43428	137.7	43	0.086	0.06
P06	7×7×3×4×4	47471	151.6	362.4	50024	159.4	45.3	0.053	0.051
P07	8×6×5×4×4	54018	158	425.7	57704	171.3	51.6	0.068	0.084
P08	9×7×5×5×6	107505	263.9	541.5	111359	278	63.7	0.035	0.053
P09	9×9×6×5×6	134289	311.6	737.6	138840	324.1	84.3	0.033	0.04
P10	11×9×6×7×7	198736	447.1	827	204148	463.1	91.9	0.027	0.035
P11	11×11×8×6×7	220732	508.7	1059.1	225486	524	114.5	0.021	0.03
P12	13×11×9×9×8	374528	714.8	1304.3	380410	728.7	137.3	0.015	0.019

TABLE 3. Computational results of the NSGA-II in terms of four criteria

Prob.	Prob.info (*U*/*W*/*P*/*V*/*T*)	Time [min] (NSGA-II)	Number of Pareto	Diversity metric	Spacing metric	MID metric
P13	15×15×9×8×6	3.15	8	262.5	0.74	13.34
P14	18×17×10×8×7	5.21	6	227.9	0.77	21.96
P15	20×20×12×9×8	8.84	7	317.5	0.64	23.07
P16	25×25×13×10×9	11.45	9	293.9	1.11	20.87
P17	30×30×15×11×10	15.36	5	365.1	0.81	13.28
P18	35×35×18×11×11	19.77	8	474.7	0.73	56.13
P19	40×40×20×11×12	25.63	5	339	1.4	30.3
P20	50×50×22×12×12	32.5	7	352.3	0.91	26.5
Average	–	15.23	7	329.1	0.88	25.67

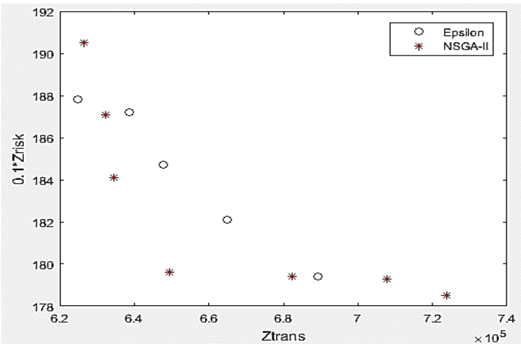

Figure 11. Pareto solutions of two proposed algorithms

6. CONCLUSION

This paper has addressed the multi-period, multi-product inventory-routing problem with the aim of minimizing the total system costs and transportation risks. For this problem, it is assumed that numerous products are transported to a set of retailers, through direct distribution using a fleet of heterogeneous vehicles with limited capacities. Due to the high computational complexity of the problem in this paper, a non-dominated sorting genetic algorithm (NSGA-II) has been used along with ε-constraint method for sample problems. The efficiency of the proposed NSGA-II has been compared to the ε-constraint method using several randomly generated sample problems. In small-sized problems, the multi-objective meta-heuristic algorithm, namely NSGA-II, has roughly found good Pareto-optimal solutions than the ε-constraint method; however, it has been able to answer in less computational time. By increasing the problem size, the ε-constraint method could not be reached within a reasonable time. Because of this, the efficiency of the NSGA-II has been evident in this case, which could reach Pareto solutions in much less time. Additionally, in large-sized problems, the Pareto front of the NSGA-II has had a higher quality according to the criteria mentioned in Section 5.2.

For future research, the given problem can be developed for conditions, in which lacking of products or sending multiple vehicles to one retailer in each period of time are allowed. Also developing any exact solution method (e.g., branch-and-price), and solving the model in an uncertain condition by a fuzzy (or robust optimization) method can be taken into account for future studies.

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Solving a New Multi-objective Inventory-Routing Problem by a Non-dominated Sorting Genetic Algorithm

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چکیده

در این مقاله، یک مسئله مسیریابی- موجودی چند هدفه، چند محصولی و چند دوره‌ای در یک زنجیره تامین دو سطحی شامل یک توزیع کننده و جندین مشتری ارائه می‌شود. مسئله با هدف حداقل سازی کل هزینه‌های سیستم (شامل هزینه راه‌اندازی، توزیع و نگهداری) و حداقل کردن ریسک حمل و نقل، مدل می‌باشد. محصولات توسط یک ناوگان حمل ناهمگن با ظرفیت حمل مشخص توسط استراتژی حمل مستقیم توزیع می‌شوند. ضمنا ظرفیت حمل نیز معین و محدود است و کمبود موجودی نیز غیر مجاز فرض می‌شود. برای اعتبارسنجی مدل دو هدفه جدید ارائه شده، از روش محدودیت اپسیلون که روشی دقیق در حل مسائل چندهدفه به شمار می‌آید، استفاده می‌شود. همچنین به دلیل سخت بودن مسئله، از الگوریتم ژنتیک چندهدفه مبتنی بر مرتب‌سازی نامغلوب (NSGA-II) که مانند روش اپسیلون بر مبنای جبهه‌های پارتو عمل می‌کند، استفاده می‌شود. در این مساله، تعداد 20 نمونه تولید شده و سپس با روش‌های مذکور حل و مورد تجزیه و تحلیل قرار می‌گیرد. در نهایت، نتایج محاسباتی نشان می‌دهد که عملکرد NSGA-II در حل مسئله مورد نظر، کارایی بالایی دارد.

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