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Providing an Analytical Model in Determining Nanofluids

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ABSTRACT

The influence of temperature, mean nanoparticle size and the nanoparticle concentration on the dynamic viscosities of nanofluids are investigated in an analytical method followed by introduction of modified equations for calculating the nanofluids' viscosities. A new correlation is developed for effective viscosity based on the previous model where the Brownian movement of the nanoparticles is considered as the key mechanism. In previous studies, the proposed models were not appropriate for nanoparticles larger than 36 nm. They were also focused on low concentrations of nanoparticles up to 5%. The possibility of homogeneous dispersion of the nanoparticles and the Stokes law are observed here. This new model is explained in terms of temperature, mean nanoparticle diameter, nanoparticle volume concentration and both the nanoparticle and base fluid thermophysical characteristics for the effective viscosity of nanofluids. A combined correction factor is introduced to take into account the simplification for a free stream boundary condition outside the boundary layer. A good agreement is observed between the effective viscosity obtained in this new model and those of recorded experiments conducted for different nanofluids. The results show that the present model is valid for large volume concentration ($0\% < \varphi < 11\%$), mean nanoparticle size ($13\ nm < d_p < 95\ nm$), temperature variations ($290\ K < T < 350\ K$) and various types of nanoparticles.

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1. INTRODUCTION

Heat transfer is an essential phenomenon in mechanical engineering and industry in general due to its frequency of occurrence in various engineering applications like furnaces, double pane windows, heat exchangers, heating of buildings, solar technology, electronic equipment cooling, etc. In the last few decades, studies were carried out and are being conducted on simulating natural convection heat transfer in various geometries with different fluids in extensive experimental, numerical and analytical methods [1-3].

The low thermal conductivity of basic fluids like water, oil and ethylene glycol is a major restriction in enhancing the performance and the compactness of such systems. Typically, solids have a higher thermal conductivity compared to liquids. More than a century ago, numerous studies on suspensions containing solid particles were conducted by Maxwell's theoretical approaches [4]. He showed the possibility of increasing the thermal conductivity of a fluid–solid mixture by greater volume fraction of micro and mill sized solid particles. Particles with millimeter or even micrometer dimensions may cause sedimentation, clogging and erosion in the systems. Choi and Eastman [5] are among the pioneers who proposed an innovative and new technique for the improvement of heat transfer by applying nanoscale solid particles (usually less than 100 nm in size) dispersed in a base fluid known as nanofluid.

The mixture of the base fluid and nanoparticles have unique physical and chemical properties like high thermal conductivity, high stability with low sedimentation compared to millimeter or micrometer sized particles and not causing any clogging in micro-channels. Moreover, some engineering advantages like reduction in pumping power, long term stability, homogeneity, design of small heat exchanger systems and little or no pressure drop when flowing through the passages makes nanofluids an intriguing research area in heat transfer enhancement [6-8].

Various models are proposed for viscosity by

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researchers for calculating the nanofluid effective viscosity as a function of one or more variables. The variables consist of volume fraction, nanoparticle mean diameter, nanofluid temperature, thermophysical properties of the base fluid and the solid nanoparticles. Various models can be classified by number of variables used in the model in two general categories:

i) Most models in the literature exclusively depend on the volume concentration of nanoparticles regarding effective viscosity. In the case of very low concentrations of fine spherical particles, Einstein [9] was the first one who introduced a simple equation for estimating the viscosity of particle suspensions as following:

$$\mu_{eff} = \mu_f (1 + 2.5\varphi) \tag{1}$$

where, φ is the volume fraction (or volume concentration) and subscripts f and eff represent the property for base fluid and nanofluid, respectively. Equation (1) is applicable in very low volume concentration ($\varphi \le 0.02$) and when there is no interaction between the particles. Brinkman [10] presented a viscosity correlation that extended Einstein's formula to be applicable at higher concentrations as follows:

$$\mu_{eff} = \frac{\mu_f}{(1-\varphi)^{2.5}} \tag{2}$$

Likewise, Lundgren [11] proposed the following equation based on a Taylor series expansion for the higher concentration:

$$\mu_{eff} = \frac{\mu_f}{(1-2.5\varphi)} = \mu_f\ (1 + 2.5\varphi + 6.25\varphi + 0(\varphi^3)) \tag{3}$$

Batchelor [12] in his theoretical examination considered the effect of the Brownian motion and the interaction between rigid and spherical particles for isotropic structure of suspension, the formula which was developed by Lundgren [11] as follows:

$$\mu_{eff} = \mu_f (1 + 2.5\varphi + 6.2\varphi^2) \tag{4}$$

Chandrasekar et al. [13] considered the effects caused by both the electromagnetic and mechanical–geometrical aspects on Al_2O_3-water nanofluid with a nominal diameter of 43 nm and adopted the following model:

$$\mu_{eff} = \mu_f (1 + b(\frac{\varphi}{1-\varphi})^n \tag{5}$$

where, b and n were calculated as 1631 and 2.8, respectively.

ii) The second model, determines the effective viscosity of nanofluids as a function of all main parameters: temperature, concentration of nanoparticle, the mean nanoparticle diameter and both the nanoparticle and base fluid thermophysical properties. Masoumi et al. [14] proposed the following equation based on the Brownian motion of particles and the determination correction factor by validation for Al_2O_3-water nanofluids.

$$\mu_{eff} = \mu_f + \frac{\rho_P V_B d_P^2}{72C\delta} \tag{6}$$

where C is assumed as the correction factor, calculated by:

$$C = \mu_f{}^{-1}\left[(c_1 d_P + c_2)\varphi + (c_3 d_P + c_4)\right] \tag{7}$$

where c_1, c_2, c_3 and c_4 are determined as:

$$\begin{aligned} &c_1 = -0.000\,001133,\ c_2 = -0.000\,002\,771 \\ &c_3 = 0.000\,000\,090,\ c_4 = -0.000\,000\,393 \end{aligned} \tag{8}$$

Although the effect of all important parameters are considered in Masoumi et al. model [14], however it should be noted that this model is not accurate for particles larger than 100 nm, and the nanoparticle concentration should be less than 5%.

By using various experimental data available in the literature, a general correlation was developed by Khanafer et al. [15] as a function of volume fraction, nanoparticles diameter, and temperature as follows:

$$\begin{aligned} \mu_{eff} = &-0.4491 + \frac{28.837}{T} + 0.574\varphi - 0.1634\varphi^2 + \\ &23.053\frac{\varphi^2}{T^2} + 0.0132\varphi^3 - \frac{2354.735\,\varphi}{T^3} + \frac{23.498\,\varphi^2}{d_P{}^2} \\ &- \frac{3.0185\,\varphi^3}{d_P{}^2} \end{aligned} \tag{9}$$

$$1\% < \varphi < 9\%,\ 20 < T(^0C) < 70,\ 13\,nm < d_p < 131nm$$

The model proposed by Khanafer et al. [15] consisted of all major parameters' effects for calculating the viscosity with temperature variation to a certain degree. However, this model can be merely adopted for Al_2O_3-water nanofluid. Furthermore, analyzing the variations of the volume fraction in ambient temperature does not predict accurate behavior for different diameters. Therefore, an analytical model for calculation of nanofluid viscosity, based on the Masoumi et al. [14] model is presented in this article where the effective viscosity is explained in terms of temperature, mean nanoparticle diameter, the nanoparticle concentration and thermophysical characteristics of nanoparticle and the base fluid. There exists a good agreement between the results obtained from the equation' developed in this article for effective viscosity and those reported experimentally regarding different nanofluids.

2. DERIVING MODEL

Attempt is made to develop a general equation based on the Masoumi et al. [14] model, regarding the effective

viscosity of nanofluid where the effects of temperature, mean nanoparticles diameter and volume fraction of nanoparticles are considered. For this purpose, the experimental data available in the published articles are applied.

Here, the relative viscosity (μ_r) is defined as:

$$\mu_r = \frac{\mu_{eff}}{\mu_f} = 1 + \mu_{pr} \tag{10}$$

Although the produced viscosity (μ_{pr}) is established due to the presence of nanoparticles, it is assumed that the viscosity of the base fluid is effective in these term as well.

The Brownian motion is the random movement of microscopic particles suspended in a liquid or gas, due to collision of molecules with the surrounding medium. It is found that the Brownian motion of nanoparticles in molecular and nanoscale level constitute a key factor governing the nanofluid properties [16].

The Brownian velocity (V_B) is presented by [17]:

$$V_B = \sqrt{\frac{3k_bT}{m_P}} = \frac{1}{d_P}\sqrt{\frac{18k_bT}{\pi\rho_P d_P}} \tag{11}$$

where Equation (11) is calculated and for spherical particles and m_P is the particle mass.

In the Masoumi et al. model [14] for each particle a cubic space is assumed. However, it is reasonable to consider that all particles are distributed uniformly in the fluid and the space between particles is almost equal in all directions relative to each other (see Figure 1). Here, each particle is considered in a hexagonal space where the length of each side is equal to delta letter (δ). According to the definition given for the nanoparticles' concentration, the distance between the nanoparticles' centers is obtained as Equation (12). In this equation, instead of the spherical space the hexagonal space is approximated. Here, delta is the covered distance where the kinetic energy of the Brownian velocity is dissipated due to friction.

$$\delta = \sqrt[3]{\frac{4}{\varphi}}d_P \tag{12}$$

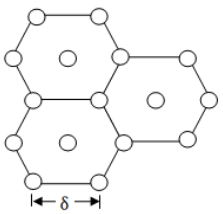


Figure 1. Distribution of particles in the fluid relative to each other

Thus according to the first law of thermodynamics the following equation can be expressed as:

$$V_{B2}^2 - V_{B1}^2 = 2\frac{F_D}{m_P}\delta \tag{13}$$

where, F_D is the drag force. The Brownian-Reynolds number (Re_B), based on the Brownian velocity, is defined as [18]:

$$\mathrm{Re}_B = \frac{1}{\vartheta_f}\sqrt{\frac{18k_bT}{\pi\rho_P d_P}} \tag{14}$$

where, (ϑ_f) is the kinematic viscosity of the liquid. Thus, for 10 nm Al_2O_3 nanoparticle for example, Re_B = 0.029, which is much less than one and the Stokes flow conditions is sensible. The drag force for the stokes regime can be expressed as follows:

$$F_D = 3\pi d_P \mu_f V_B = 3\pi d_P \mu_{\mathrm{Pr}} C V_B \tag{15}$$

where, C is the correction factor applied in simplifying the flow boundary conditions. The following equation is derived from combining and simplifying Equations (5) and (6) for viscous product.

$$\mu_{\mathrm{Pr}} = \frac{V_B d_P^2 \rho_P}{36C\delta} \tag{16}$$

By inserting Equations (16) and (12) into Equation (10), the following equations are obtained:

$$\mu_r = 1 + \frac{\rho_P}{36C}\sqrt{\frac{18k_bT}{\pi\rho_P d_P}}\sqrt[3]{\frac{4}{\varphi}} \tag{17}$$

After rearranging Equation (17), this equation will be reduced to:

$$\mu_r = 1 + \frac{A}{B-\varphi}\left(\frac{V_B d_P^2 \rho_P}{36\delta\sqrt{T}}\right) \tag{18}$$

where, *A* and *B* are defined as Equations (19) and (20).

$$A = \frac{36\delta\sqrt{T}\varphi^{\frac{1}{3}}}{V_B d_P^2 \rho_P}\left(9.193*10^{28}\ \mathrm{d_P^4} - 2.404*10^{22}\ \mathrm{d_P^3} + 2.0250*10^{15}\ \mathrm{d_P^2} - 5.8860*10^{7}\ \mathrm{d_P} + \frac{1}{488774}\rho_{\mathrm{P}} + \frac{2507}{4446}\right) \tag{19}$$

$$B = -8.344*10^{36}\ \mathrm{d_P^5} + 2.212*10^{30}\ \mathrm{d_P^4} - 2.16*10^{23}\ \mathrm{d_P^3} + 9.492*10^{15}\ \mathrm{d_P^2} - 1.804*10^{8}\ \mathrm{d_P} - \frac{2}{327327}\rho_{\mathrm{P}} + \frac{3575}{291360} \tag{20}$$

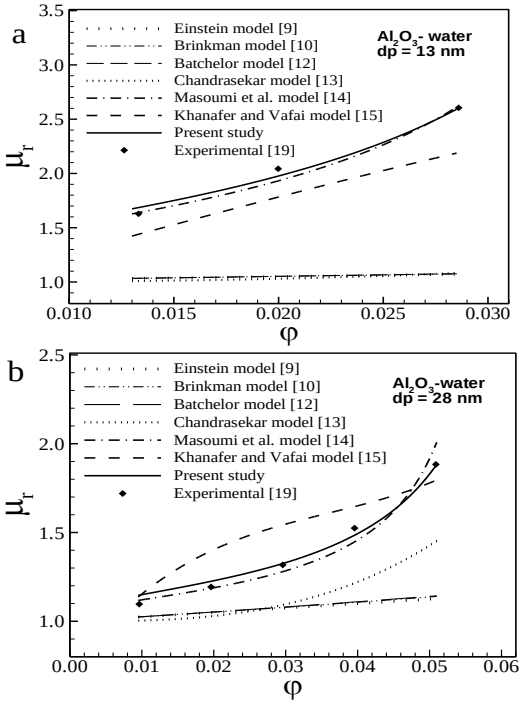
3. RESULTS AND DISCUSSION

The analytical equation obtained in this study is compared with the experimental results. A comparison

between this newly introduced model with the experimental results and other theoretical models regarding the mean nanoparticle diameter indicated that this developed model could well predict the viscosity of effective nanofluid at different volume fractions. Moreover, this model provides a more realistic and accurate behavior than that of the Masoumi et al. model [14]. A comparison between the effective viscosity obtained in the new model and other analytical and available experimental results based on low to medium mean nanoparticle diameter, i.e. d_p =13, 28 and 47 nm for Al_2O_3-water nanofluids, is shown in Figure 2.

As observed in Figure 2a, with respect to Al_2O_3-water nanofluid with low volume fraction of (φ<0.05) and d_p=13 nm, some of the presented models [9, 10, 12, 13] fail to show the correct behavior of the viscosity changes; this problem is for d_p=28 nm. The behavior that has been presented in the work of Khanafer et al. [15] is more accurate than previous models, while the behavior introduced by Masoumi et al. model [14] and this model have better functionality. Among the previous models, Masoumi et al. model [14] is the only appropriate model, while the model introduced here regarding the mean nanoparticle diameter of 28 nm is slightly better (Figure 2b). A good agreement is found in the literature [14] for d_p = 13 and 28 nm, however it is clearly observed that this model is not valid when the diameter or volume fraction of nanoparticles is increased (Figure. 2c).

A comparison of this new theoretical model and the available models are illustrated in Figure 3.



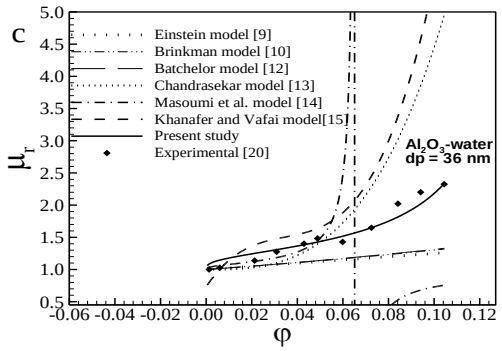


Figure 2. Comparison between the results based on new correlation and experimental data and other theoretical models for the Al_2O_3–water: (a) d_p = 13 nm, (b) d_p = 28 nm and (c) d_p = 36 nm

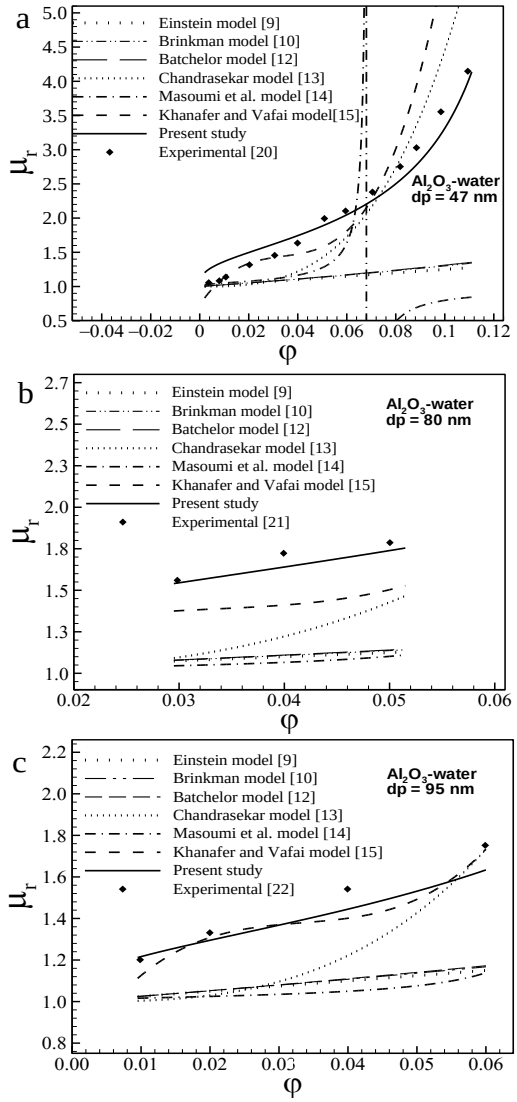


Figure 3. Comparison between the results based on new correlation and experimental data and other theoretical models for the Al_2O_3–water: (a) d_p = 47 nm, (b) d_p = 80 nm and (c) d_p = 95 nm

This comparison is made on a wide range of volume fractions and nanoparticles with larger diameters. Other models cannot behave accurate for large diameter or high volume fraction, but this model has a good potential for such conditions.

While most models are limited to one type of nanofluids, a comparison is made between this model and the experimental results for the viscosity of CuO-water. According to Figurre 4 a good agreement is observed between this presented model and the experimental data.

It is observed that, although the model is reviewed at room temperature, according to the experimental data, the new coefficients are obtained and by inserting these coefficients in Equation (18), the current model will be adopted for temperature variation. The validity of the new correlation (Equation (18)) for temperature variation is illustrated in Figure 5. As can be seen in Figure 5, the model could well predict the influence of the temperature variations on nanofluid viscosity at different mean diameters and volume fractions.

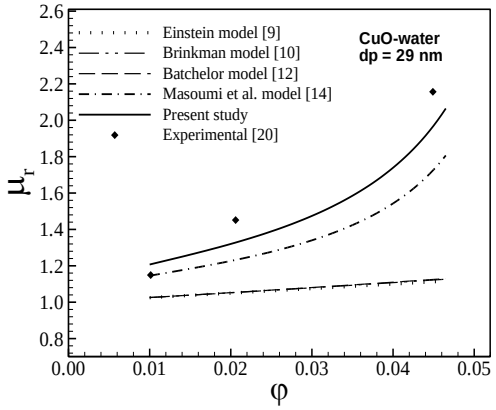
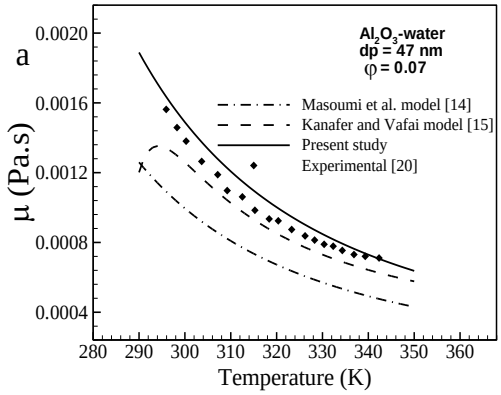


Figure 4. Comparison between the results based on new correlation and experimental data and other theoretical models for the Cu-water (29 nm)



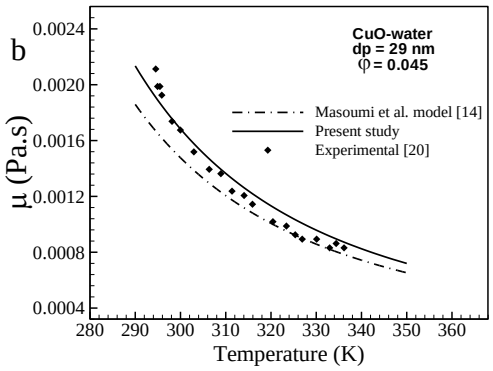


Figure 5. Comparison between the results based on new correlation and experimental data and other theoretical models for the Cu-water and Al_2O_3-water.

4. CONCLUSION

An analytical study of the variants within the effective viscosity of nanofluids is conducted in this study. A new correlation for nanofluids viscosity is developed based on the Masoumi et al. equation [14]. The equation for effective viscosity is developed in terms of temperature, mean nanoparticle diameter, the nanoparticle concentration and both the nanoparticle and base fluid thermophysical characteristics.

A comparison between this model and other available models indicate that the available models were subjected to at least one of the following restrictions:

- Valid only for a very small volume concentration, as Brinkman models and the Einstein's formula. Moreover it is found that these models are not appropriate for nanofluid viscosity, particularly for intermediate to high particle volume fractions and mean nanoparticle diameter
- Reliable only for a small volume fraction (less than 5%) as Masoumi et al. model [14]
- A valid model for low- and medium –diameter nanoparticle, for example, Masoumi et al. model [14]
- The correlations are true only for one type of nanoparticle, as Chandrasekar model [13] and Khanafer et al. model [15]

According to the results obtained here, it can be claimed that all the above-mentioned problems can be solved through this proposed model.

This study clearly demonstrates that this model is applicable for effective viscosity of nanofluid with higher accuracy and precision.

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Providing an Analytical Model in Determining Nanofluids

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چکیده

در کار حاضر تأثیر متغیرهای درجه حرارت، اندازه متوسط نانوذره و غلظت نانوذره روی ویسکوزیته دینامیکی نانوسیال بررسی شده است که بر یک روش تحلیلی معادلات اصلاح‌شده ویسکوزیته نانوسالات مبتنی است. مدل جدید بر اساس مدل‌های قبلی که اثر حرکت براونی را به‌عنوان یک پارامتر کلیدی مدنظر قرار می‌دادند، توسعه یافته است. نتایج نشان داد که استفاده از مدل‌های قبلی برای نانوذرات بزرگ‌تر از 36 نانومتر مناسب نیست. هم‌چنین مطالعات قبلی تنها در حجم کسری کم نانوذرات تا 5% متمرکز شده است. در کار حاضر امکان پراکندگی همگن نانوذرات و قانون استوکس برای نانوذرات لحاظ شده است. در مدل جدید اثر دما، قطر متوسط نانوذرات، غلظت نانوذره و نیز ویژگی‌های ترموفیزیکی نانوذره و سیال پایه برای ویسکوزیته مؤثر نانوسیال در نظر گرفته شده است. یک ضریب تصحیح ترکیبی برای لحاظ کردن ساده‌سازی شرط مرزی جریان آزاد در خارج از لایه مرزی معرفی شده است. توافق خوب نتایج بین ویسکوزیته مؤثر به‌دست‌آمده از مدل جدید و نتایج آزمایشگاهی برای نانوسیالات مختلف مشاهده شده است. نتایج نشان می‌دهد که مدل حاضر برای محدوده وسیعی از کسر حجمی ($0\% < \varphi < 11\%$)، اندازه متوسط نانوذرات ($13\ nm < d_p < 95\ nm$)، تغییرات دمایی نانوسیال ($290\ K < T < 350\ K$) و انواع مختلفی از نانوذرات معتبر است.

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