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Detailed Scheduling of Tree-like Pipeline Networks with Multiple Refineries

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A B S T R A C T

In the oil supply chain, the refined petroleum products are transported by various transportation modes, such as rail, road, vessel and pipeline. The latter provides one of the safest and cheapest ways to connect production areas to local markets. This paper addresses the operational scheduling of a multi-product tree-like pipeline connecting several refineries to multiple distribution centers under demand uncertainty. A new deterministic mixed-integer linear programming (MILP) model is first presented, and then a two-stage stochastic model is proposed. The aim of this model is to meet depot requirements at the minimum total cost including pumping and stoppages costs. The efficiency and utility of the proposed model is shown by two numerical examples, which one of them uses the industrial and real data.

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1. INTRODUCTION

Nowadays, operations of a supply chain and logistics are among the most important activities in companies [1]. Transportation is an important supply chain driver because products are rarely produced and consumed in the same location [2]. Multi-product pipeline conveying a variety of petroleum products are the most common means of transportation in the oil industry to transfer the products from refineries to depots. Transportation planning of petroleum products via pipelines is one of the most challenging management problems in the oil industry and can increase the annual profit in several millions of dollars. Multi-product pipelines stay full with product at any time. Consequently when some products enter inside a pipeline, the same volume should leave that line. Some product sequences are forbidden inside the pipelines; for instance, gas and oil should never be adjacent to gasoline. Pipeline structures vary from straight to treelike and mesh structures. This paper is concerned with a tree-like pipeline system that must distribute a number of petroleum products from multiple refineries to several depots. Finding the best sequence of product injections at refineries, which product removals at depots and the length of pumping operations for multi-product ducts to satisfy product demands requested by distribution centers on time at the minimum total cost, is the most important element of pipeline scheduling problem. Several operational constraints should also be considered. These include keeping the inventory level of depots at the acceptable range and maintaining the flow rate in different segments at the feasible range.

In succeeding years, two types of popular approaches based on time representation of the planning horizon have been studied in the operational planning of pipeline networks, namely discrete and continuous time mixed-integer linear programming (MILP) approaches. In discrete approaches, both pipeline segments and time horizon are apportioned into slots of equal size, while continuous approaches relax such assumptions. Using these tools, oil products distribution scheduling problems have been addressed from rather simpler

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structure including a single refinery and single distribution center Relvas et al. [3] and Cafaro and Cerda [4], or single refinery with multiple destinations Hane and Ratliff [5] and Rejowski and pinto [6] and Cafaro and Cerda [7] Mostafaei and Ghaffari Hadigheh [8], or multiple refineries and distribution centers, Cafaro and Cerda [9] Mostafaei et al. [10, 11] to more complex cases with branching configuration Castro and Grossman [12] and Cafaro and Cerda [13] and Herran et al. [14] Desouza Filho et al. [15] and Boschetto et al. [16]and Mostafaei et al. [17] and bidirectional pipeline systems connecting a refinery to a harbor [18, 19].

All paper mentioned above have focused on the aggregate planning providing the optimal sequence of batch injections during a known planning horizon. None of them focuses on determining the optimal sequences of stripping operations at receiving terminals to generate a detailed delivery schedule and to get savings in pump operating and maintenance costs. Cafaro et al. [20, 21] used hierarchical decomposition approaches for detailed scheduling of straight of pipeline networks. Mostafaei et al. [22-25]developed monolithic MILP frameworks for detailed scheduling of straights pipeline that achieve better solution with respect to decomposition approaches.

Mostafaei et al. [26] presented an MILP model for detailed scheduling of tree-like pipeline connecting a single refinery to multiple depots. The model is based on a continuous time representation in both volume and time scales. Given product demands at depots, the proposed model determines both input and output operations of the pipeline in a single step. However, the model can only be applied to tree-like pipeline with a single refinery and with deterministic product demands at depots. The contribution of this paper is as follows. Firstly, a generalization of the model presented by Mostafaei et al. [26] from a unique refinery at the origin of the pipeline to multiple refineries with dual purpose stations is developed. Secondly, contrarily to the previous studies considering product demands at pipeline depot as deterministic data, the proposed approach uses stochastic datum for a product requirement.

The rest of this paper is organized as follows. Section 2 gives a concise description of the problem under study. Section 3 presents the model assumptions. Section 4 presents the deterministic MILP model for a tree-like scheduling problem. Section 5 proposes a stochastic counterpart of the MILP model. Section 6 gives two case studies to show the validation of the proposed model. The paper ends with the conclusions.

2. PROBLEM STATEMENT

This paper addresses the scheduling of a tree-like pipeline network with a mainline (pipeline n_0), several secondary lines, multiple refineries, depots and dual purpose stations. Figure 1 depicts a tree-like pipeline network with two refineries, two secondary lines (i.e., pipeline n_1 and n_2). A dual purpose node composes of refinery R_2 and depot N_2. The refineries inject refined petroleum products into the pipeline while depots receive them. The dual purpose station performs both injecting and receiving operations. We use terms “secondary line” or “branching line” for pipelines that are emerged from the mainline. In Figure 1, the first secondary line leaves the mainline at point 3000 m^3 while the second one branches at point 15000 m^3. The volume of a product is regarded as a batch, in which each batch only conveys one product and can be at most adjacent to two products. For example, in Figure 1, batch I_3 in the mainline (i.e., yellow rectangle) conveys product P_3 and touches two batch I_2 and I_4 conveying products P_2 and P_1, respectively. Note that each pipeline is divided into segments, in which each segment ends with a depot or branch point. The mainline composes of five segments, in which the segments of the pipeline network and their volume are specified by left and right arrows.
The existence batches inside the pipeline n at the start time of the time horizon are given by old batches (I_n^{old}).

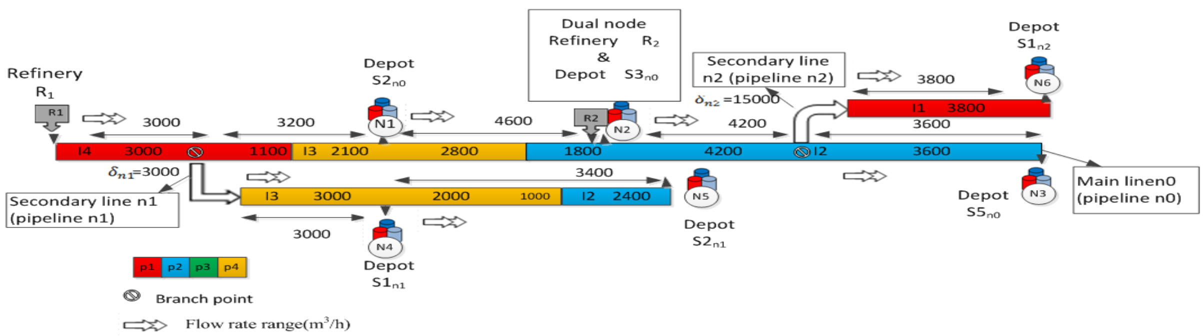

Figure 1. Tree-like pipeline

The set of old batches of pipeline n_0, n_1 and n_2 is $\mathrm{I}_{n0}^{old} = \{I_2, I_3, I_4\}$, $\mathrm{I}_{n1}^{old} = \{I_2, I_3\}$ and $\mathrm{I}_{n2}^{old} = \{I_1\}$, respectively. The set of new batches, which enter in to pipeline n during the scheduling horizon, is new batches (I_n^{new}). In this paper, the number of new batches to be injected into the mainline is known beforehand. For instance, if we assume that two new batches I_5 and I_6 are injected into the mainline of Figure 1 during the planning horizon, we have $\mathrm{I}_{n0}^{new}\{I_5, I_6\}$, $\mathrm{I}_{n1}^{new}\{I_4, I_5, I_6\}$ and $\mathrm{I}_{n2}^{new}\{I_2, I_3, I_4, I_5, I_6\}$. Set $\mathrm{I}_n = \mathrm{I}_n^{new} \cup \mathrm{I}_n^{old}$ is the set of batches to move in pipeline n during planning horizon. The set of batches that can be injected from refinery r into mainline during planning horizon is given by I_r.

In Figure 1, refinery R_1 can only inject batches I_4, I_5 and I_6 and so $I_{r1} = \{I_4, I_5, I_6\}$ whereas refinery R_2 can increase the size of batch I_2, I_3,…, and so $I_{r2} = \{I_2, I_3, I_4, I_5, I_6\}$. The set I_n^S is set of batches that can be received by depot s_n (depot s on pipeline n). For example, in Figure 1, depot $s1_{n0} = \mathrm{N1}$ can only receive product from batch I_3 and batches I_4, I_5 and I_6 that will pass from the output facility of this depot in the future; however, batch I_2 can never be transferred to depot N_1, and so we have $I_{n0}^{s1} = \{I_3, I_4, I_5, I_6\}$.

3. MATHEMATICAL MODEL

To develop a mathematical tool based on an MILP framework, the following assumptions are to be considered.

- A tree-like pipeline with a single mainline and several branch lines only emerged from the mainline is considered. Flow in all pipelines (mainline ant its branches) is unidirectional.
- Several refineries can be considered on the mainline; however, there is no refinery on branch point and branch line.
- All pipeline segment stay full of product at any time.
- Sequence, content and coordinate of old batches in each segment are known.
- Maximum and minimum flow rates in pipeline segments are given.
- Because of vast product contamination, some products can never touch each other inside the pipeline.
- At each pumping operation, an active depot can receive material from a single batch.
- Demand for each product is random variable ($\xi_{p,s,n}$) with a continuous uniform distribution ($\xi_{p,s,n} \cap [a_{p,s,n}, b_{p,s,n}]$).

This section presents the proposed MILP formulation for the short term scheduling of a tree-like pipeline system with several refineries and depots. The base model is the mathematical formulation of Mostafaei et al. [26] with the following differences: (1) our approach considers a tree-like pipeline network with several refineries and even dual purpose nodes whereas their approach deals with a single refinery at the origin of the mainline and (2) our approach considers product demands at depots as stochastic variables whereas their approach treats product demand as a deterministic datum. The sets, parameters and continuous and binary variables are all defined from Tables 1 to 3.

3. 1. Sequencing Pumping Runs The start time of pumping run k will be equal to the start time of pumping run $k-1$ plus the length of run $k-1$.

$$ST_k = ST_{k-1} + L_{k-1}, \qquad \forall k \geq 2 \tag{1}$$

3. 2. Location of Batch $i \in I$ in Mainline and Branch Lines Continuous variable $LPV_{i,k,n}$ is the upper coordination of batch i, which is equal to the total volume between the origin of pipeline n to the end of batch i. In Figure 2, the upper coordinate of batch I_2 in the mainline is 5000+2000=7000 m^3. Note that $LPV_{i,k,n} - SPV_{i,k,n}$ is the lower coordinate of batch i.

$$LPV_{i,k,n} = \sum_{i' \geq i} SPV_{i',k,n}, \qquad \forall i \in I_n, k \in K, n \in N \tag{2}$$

3. 3. Main line feeding operation By Equation (3), only a single batch $i \in I_r$ can be injected from refinery r during each pumping operation and the coordinates of this batch satisfy Equations (4) and (5). The volume injected from refinery r will always be between $[IPV_{r,n0}^{\min}, IPV_{r,n0}^{\max}]$.

TABLE 1. List of sets and indices

Sets	Description
K	Set of pumping operations indexed by $k, k = 0, 1, \dots, \lvert K \rvert$
N	Set of pipelines indexed by $n, n', n'' = 0, 1, \dots, \lvert N \rvert$
R	Set of refineries indexed by $r, r' = 1, \dots, \lvert R \rvert$
I	Set of all product batches indexed by $i, i', j = 1, \dots, \lvert I \rvert$
S_n	Set of depots on pipeline n indexed by s, s', s''
I_r	Set of batches that can receive material from refinery r ($I_n \subset I$)
I_n^{old}	Set of old batches inside pipeline n ($I_n^{old} \subset I$)
I_n^{new}	Set of new batches of pipeline n ($I_n^{new} \subset I$)
I_n	Set of batches to be moved in pipeline n ($I_n = I_n^{old} \cup I_n^{old}$)
I_n^s	Set of batches that can be diverted into depot s_n ($I_n^s \subset I$)
P	Set of oil products indexed by p, p'

TABLE 2. List of parameters

Parameters	Description
$h_{\max}$	The length of schedule horizon (h)
$CP_{p,r}$	Pumping cost of product p at refinery r (\$/m^3)
$CB_{p,s,n}$	Backorder cost of product p in the depot s_n t (\$/m^3)
$vr_r^{\min}$ / $vr_r^{\max}$	Min / max injection rate at refinery r (m^3/h)
$IPV_r^{\max}/IPV_r^{\max}$	Min / max batch injected size in mainline in each run (m^3)
$IRV_n^{\min}/IRV_n^{\max}$	Min / max batch size transferred to branch line n in each run (m^3)
$DPV_{s,n}^{\max}/DPV_{s,n}^{\min}$	Min / max batch size transferred to depot s_n in each run (m^3)
$vs_{s,n}^{\max}/vs_{s,n}^{\min}$	Min/max flow rate in segment s_n
θ_r	Volumetric coordination of refinery r from the origin of mainline (m^3)
σ_n	Volumetric coordination of secondary line n from the origin of mainline (m^3)
$\tau_{s,n}$	Volumetric coordinate of depot s_n from the origin of pipeline n (m^3)
PV_n	Volume of pipeline n (m^3)
$reft_{p,r}$	Inventory of product p in refinery r during planning horizon (m^3)
$Touch_{p,p'}$	Boolean matrix of possible sequences between products p and p' (m3)
$Demand_{s,n,p}$	Demand of product p of depot s_n (m^3)
$ISPV_{i,n}$	Size of old batch i in the pipeline n at time stf (m^3)
$VSEG_{s,n}$	The volume of segment s_n (m^3)

TABLE 3. List of variables

Continuous Variables	Description
ST_k	Start time of composite pumping run k (h)
$LR_{r,k}$	Activity length of refinery r though composite pumping run k (h)
L_k	Length of composite pumping run k (h)
$Back_{p,s,n}$	Unsatisfied demand of product p in depot s_n (m^3)
$IPV_{i,r,K}$	Size of batch $i \in I_r$ injected into the mainline from the refinery r during composite pumping run k (m^3)
$PPV_{i,p,r,k}$	Size of batch $i \in I_r$ conveying product p injected from the refinery r to mainline during composite pumping run k (m^3).
$IRV_{i,k,n}$	Volume of batch $i \in I_n$ transferred to branching line n during run k (m^3)
$DPV_{i,s,k,n}$	Size of batch $i \in I_n^s$ diverted to depot s_n during composite pumping run k (m^3)
$PDPV_{i,p,s,k,n}$	Size of batch $i \in I_n^s$ covering product p diverted to depot s_n during run k (m^3)
$LPV_{i,k,n}$	Upper coordinate of batch $i \in I_n$ at the end of pumping run k (m^3)
$SPV_{i,k,n}$	Size of batch $i \in I_n$ at the end of pumping run k (m^3)
$SV_{s,k,n}$	stopped volume of segment $s_n$at the end of pumping run k(m^3)
Binary variables	
$\lambda_{i,r,k}$	1 if batch $i \in I_r$ is injected from refinery r to the main line during run k
$u_{i,k,n}$	1 if secondary line n receives batch i during run k
$x_{i,s,k,n}$	1 if batch $i \in I_n^s$ is diverted to depot s_n during run k
$y_{i,p}$	1 if batch i transports product p
$e_{s,k,n}$	1 if segment $s_n$is active during run k
$z_{i,n}$	1 if some portion of batch i exists in branching line n

The activity duration of all active refineries is of the same value and determined the length of a pumping operation, as imposed by Equation (8).

$$\sum_{i \in I_r} \lambda_{i,r,k} \le 1, \qquad \forall k \in 1, r \in R \tag{3}$$

$$LPV_{i,k-1,n_0} \ge \theta_r . \lambda_{i,r,k}, \qquad \forall i \in I_r, k \in K, r \in R \tag{4}$$

$$LPV_{i,k-1,n0} - SPV_{i,k-1,n0} \le \theta_r + \left(PV_{n0} - \theta_r\right)\left(1 - \lambda_{i,r,k}\right), \quad \forall i \in I_r, k \in K, r \in R \tag{5}$$

$$\lambda_{i,r,k} IPV_{r,n0}^{\min} \le IPV_{i,r,k} \le \lambda_{i,r,k} IPV_{r,n0}^{\max}, \quad \forall i \in I_r, k \in K, r \in R \tag{6}$$

$$\frac{\sum_{i \in I_r} IPV_{i,r,k}}{vr_r^{\max}} \le LR_{r,k} \le \frac{\sum_{i \in I_r} IPV_{i,r,k}}{vr_r^{\min}}, \forall k \in K, r \in R \tag{7}$$

$$LR_{r,k} \le L_k, \qquad \forall r \in R, k \in K \tag{8}$$

3. 4. Batch Feature Constraints It should be noted that each batch $i \in I_n$ contains only one product p, Equation (9). If batch i contains product p then the size of the continuous variable $PPV_{i,p,r,k}$ will be equal to $IPV_{i,r,k}$; otherwise, it will be equal to zero, Equations (10) and (11). The volume of product p added to the main line from refinery r during the scheduling should be equal or smaller than $reft_{r,p}$.

$$\sum_{p \in P} y_{i,p} = 1, \qquad \forall i \in I \tag{9}$$

$$\sum_{p\in P} PPV_{i,p,r,k} = IPV_{i,r,k}, \qquad \forall i \in I_r, p \in P, r \in R \tag{10}$$

$$\sum_{k\geq 1} PPV_{i,p,r,k} \leq |K|.IPV_{n0}^{\max}.y_{i,p}, \qquad \forall i \in I_r, p \in P, r \in R \tag{11}$$

$$\sum_{k\in K}\sum_{i\in I_r} PPV_{i,p,r,k} \leq reft_{r,p}, \forall r \in R, p \in P \tag{12}$$

3. 5. Operation of Unloading the Main Line and Branch Lines The binary variable $x_{i,s,k,n}$ is equal to 1 if batch $i \in I_s^n$ is sent to depot s_n during the execution of pumping k. A portion of batch $i \in I_n$ can be received by depot s_n only if the coordinates of the batch satisfy Equations (13) and (14). When $x_{i,s,k,n} = 1$, depot s_n will receive a positive portion of batch $i \in I_s^n$, as imposed by Equation (15). If batch $i \in I_s^n$ contains product p the value of the continuous variable $PDPV_{i,p,s,k,n}$ will be equal to $DPV_{i,s,k,n}$; otherwise, it is equal to zero, as imposed by Equations (16) and (17).

$$LPV_{i,k-1,n} \geq \tau_{s,n}.x_{i,s,k,n}, \quad \forall i \in I_s^n, k \in K, s \in S, n \in N \tag{13}$$

$$LPV_{i,k,n} - SPV_{i,k,n} \leq \tau_{s,n} + \left(PV_n - \tau_{s,n}\right).\left(1 - x_{i,s,k,n}\right), \quad \forall i \in I_s^n, k \in K, s \in S_n, n \in N \tag{14}$$

$$x_{i,s,k,n}DPV_{s,n}^{\min} \leq DPV_{i,s,k,n} \leq x_{i,s,k,n}DPV_{s,n}^{\max}, \quad \forall i \in I_s^n, k \in K, s \in S_n, n \in N \tag{15}$$

$$\sum_p PDPV_{i,p,s,k,n} = DPV_{i,s,k,n}, \quad \forall i \in I_s^n, k \in K, s \in S_n, n \in N \tag{16}$$

$$\sum_k PDPV_{i,p,s,k,n} \leq |K|\, DPV_{s,n}^{\max}.y_{i,p}, \quad \forall i \in I_s^n, s \in S_n, n \in N \tag{17}$$

3. 6. Branch Lines Feeding Operation The binary variable $u_{i,k,n} = 1$ shows that batch $i \in I_n$ in pumping k is transferring to the branch line n. A portion of batch i in mainline can be received by branch line n only if the coordinates of the batch satisfy Equations (18) and (19). By Equation, (20), when $u_{i,k,n} = 1$, branch line n will receive a proportion of batch $i \in I_n$.

$$LPV_{i,k-1,n0} \geq \sigma_n u_{i,k,n}, \quad \forall i \in I_n, k \in K, n \in N, n \neq n0 \tag{18}$$

$$LPV_{i,k,n0} - SPV_{i,k,n0} \leq \sigma_n + (PV_{n0} - \sigma_n)(1 - u_{i,k,n}), \quad \forall i \in I_n, k \in K, n \in N, n \neq n0 \tag{19}$$

$$IRV_n^{\min} u_{i,k,n} \leq IRV_{i,k,n} \leq IRV_n^{\max} u_{i,k,n}, \quad \forall i \in I_n, k \in K, n \in N, n \neq n0 \tag{20}$$

3. 7. Mass Balance Constraints The continuous variable $SPV_{i,k,n}$ is the volume of batch i at the end of pumping k in pipeline n. Volume of batch i at the end of pumping k in the main line and branch lines is shown by Equations (21) and (22).

$$SPV_{i,k,n0} = SPV_{i,k-1,n0} + \sum_{r\in R} IPV_{i,r,k} - \sum_{s\in S_{n0}} DPV_{i,s,k,n0} - \sum_{n\in SP} IRV_{i,k,n}, \quad \forall i \in I_{n0}, k \in K, k \geq 1 \tag{21}$$

$$SPV_{i,k,n} = SPV_{i,k-1,n} + IRV_{i,k,n} - \sum_{s\in S_n} DPV_{i,s,k,n}, \quad \forall i \in I_n, k \in K, n \in N, n \neq n_0 \tag{22}$$

In the main line and branch lines, the injected volume is always equal to the output volume, Equations (23) and (24).

$$\sum_{i\in I_n} IRV_{i,k,n} = \sum_{s\in S_n}\sum_{i\in I_n} DPV_{i,s,k,n}, \quad \forall k \in K, n \in N, n \neq n0 \tag{23}$$

$$\sum_{i\in I_r}\sum_{r\in R} IPV_{i,r,k} = \sum_{s\in S_{n0}}\sum_{i\in I_{n0}} DPV_{i,s,k,n0} + \sum_{n\in N, n\neq n0}\sum_{i\in I_n} IRV_{i,k,n}, \qquad \forall k \in K \tag{24}$$

3. 8. Forbidden Sequences For quality reasons some products should not touch together inside the pipeline. For example in real pipelines, a batch of gasoline (premium, regular, etc.) should never be adjacent to a batch of gas oil. If $Touch_{p,p'} = 1$, product p and p' can touch together inside the pipeline, otherwise their sequence is forbidden. This restriction are controlled by:

$$y_{i,p} + y_{i+1,p'} \leq Touch_{p,p'} + 1, \quad \forall i \in I, p, p' \in P \tag{25}$$

$$z_{i,n} + z_{i',n} \leq \sum z_{j,n} - y_{i,p} - y_{i',p'} + Touch_{p,p'} + 3, \quad \forall i \in I_n^{new}, i' \in I_n \left(i \geq i'\right), p, p' \in P, n \neq n0 \tag{26}$$

where $z_{i,n}$ is a binary variable denoting the existence of batch $i' \in I_n$ in secondary line n and satisfies the following eq $z_{i,n} \leq \sum_k u_{i,k,n} \leq |K| z_{i,n}$, $\forall i \in I_n^{new}, n \in N,, n \neq n0$.

3. 9. Constraints Satisfied Demand In a deterministic mode, the demand of depots is the known parameter $Demand_{s,n,,p}$ and should be satisfied on time. This unsatisfied demand ($Back_{s,n,p}$) will result in backorder costs.

$$\sum_{k \in K} \sum_{i \in I_n^S} PDPV_{i,p,s,k,n} \geq Demand_{s,n,p} - Back_{s,n,p}, \quad \forall p \in P, s \in S_n, n \in N, k \in K \tag{27}$$

3. 10. Identifying Active and Ideal Segments The binary variable $e_{s,k,n}$ determines the status (active or idle) of segment $s \in s_n$ during pumping run k and its value satisfy the following equations:

$$\sum_{s \in S_n} e_{s,k0,n} = 0, \qquad \forall n \in N \tag{28}$$

$$\sum_{i \in I_{r1}} \lambda_{i,r1,k} = e_{s1,k,n0}, \qquad \forall k \in K \tag{29}$$

$$\sum_{i \in I_n^S} x_{i,s,k,n} = e_{s,k,n}, \quad \forall s = last(S_n), k \in K, n \in N \tag{30}$$

$$\sum_{\tau_{s-1,n0} \leq \theta_r \leq \tau_{s,n0}} \sum_{i \in I_r} \lambda_{i,r,k} \leq e_{s,k,n0} \leq \sum_{\tau_{s-1,n0} \leq \theta_r \leq \tau_{s,n0}} \sum_{i \in I_r} \lambda_{i,r,k} + e_{s-1,k,n}, \quad \forall s \in S_{n0}, k \in K \tag{31}$$

$$\sum_{i \in I_{n0}^s} x_{i,s,k,n0} \leq e_{s,k,n0} \leq 1 + \sum_{i \in I_{n0}^s} x_{i,s,k,n0} - \sum_{\tau_{s,n0} \leq \theta_r \leq \tau_{s+1,n0}} \sum_{i \in I_r} \lambda_{i,r,k}, \quad \forall s \in S_{n0}, k \in K \tag{32}$$

$$e_{s,k,n} = \sum_{i \in I_n} u_{i,k,n}, \quad \forall n \neq n0, k \in K, s = first(S_n) \tag{33}$$

$$\sum_{i \in I_n^S} x_{i,s,k,n} \leq e_{s,k,n}, \quad \forall k \in K, s \in S_n, n \in N \tag{34}$$

$$e_{s,k,n0} \geq \sum_{i \in I_n} u_{i,k,n}, \quad \forall n \neq n0, k \in K, s \in S \tag{35}$$

3. 11. Flow Rate Constraints is Mainline and Branch Lines Segments During any pumping run $k \in K$, the flow rate at each active segment $s \in s_n$ should belong to feasible range $[vs_{s,n}^{\min}, vs_{s,n}^{\max}]$. Equations (36) and (37) control the flow rate limitation on mainline and its branch, respectively.

$$L_k vs_{S_{n0}}^{\min} - (1 - e_{s,k,n0}) \leq \sum_{s' \geq s} \sum_{i \in I_n^S} DPV_{i,s',k,n0} + \sum_{\tau_{s,n0} \leq \sigma_n} \sum_{i \in I_n} IPV_{i,k,n} - \sum_{i \in I_r} \sum_{\tau_{s,0} \leq \theta_r} IRV_{i,r,k} \leq L_k vs_{S_{n0}}^{\max}, \quad \forall s \in S_{n0}, k \in K \tag{36}$$

$$L_k vs_{S_n}^{\min} - (1 - e_{s,k,n}) \leq \sum_{s' \geq s} \sum_{i \in I_n^S} DPV_{i,s',k,n} \leq L_k vs_{S_n}^{\max}, \quad \forall s \in S_n, k \in K, n \in N, n \neq n0 \tag{37}$$

3. 12. Stoppages Volume Continuous variable $SV_{s,k,n}$ is the stoppage volume of segment s of pipeline n during pumping run k. If $e_{s,k-1,n} = 1$ and $e_{s,k,n} = 0$, the stopped volume of the segment s_n will be the volume of segment s_n ($VSEG_{s,n}$), otherwise it will be zero.

$$SV_{s,k,n} \geq VSEG_{s,n}(e_{s,k-1,n} - e_{s,k,n}), \quad \forall k \in K, n \in N, s \in S_n \tag{38}$$

3. 13. Problem Objective Function The problem purpose is to minimize the total tree-like pipeline operating costs including (a) the product pumping cost, (b) backorder demand and (c) pipeline stoppage cost.

$$\min z = \sum_k \sum_r \sum_i \sum_p cp_{p,r}.PIV_{i,r,p,k} + \sum_n \sum_s \sum_p cb_{s,n,p}.Back_{s,n,p} + \sum_n \sum_k \sum_s cs.SV_{s,k,n}$$

4. STOCHASTIC COUNTERPART

Now in this section, we extend the model for the case demand for each product is random variable $\xi_{p,s,n} \cap [a_{p,s,n}, b_{p,s,n}]$. The objective function can be written as a two-stage stochastic model shown below:

Min $z_{-s} = \sum_{k \in K} \sum_{i \in I_r} \sum_{p \in P} \sum_{r \in R} cp_{p,r}. PIV_{i,r,p,k} + + \sum_n \sum_k \sum_s cs. SV_{s,k,n} + \sum_p \sum_s \sum_n E\left(Q(\lambda_{p,s,n}, \xi_{p,s,n})\right)$

s.t.

Equations $(1) - (26), (28) - (38)$

where, $Q(\lambda_{p,s,n}, \xi_{p,s,n})$ is the optimal cost if depot s_n receives $\lambda_{p,s,n} = \sum_{k \in K} \sum_{i \in I} PDPV_{i,p,s,k,n}$ units of product p while demand is random variable $\xi_{p,s,n} \cap [a_{p,s,n}, b_{p,s,n}]$. So, we have:

$Q(\lambda_{p,s,n}, \xi_{p,s,n}) = \min \sum_p cb_p BR_{p,\xi_p}$

s.t.

$Back_{s,n,p} \le \xi_{p,s,n}$

$Back_{s,n,p} \ge \xi_{p,s,n} - \lambda_{p,s,n}$

Cleary the optimal value for $Q(\lambda_{p,s,n}, \xi_{p,s,n})$ is max $\{\xi_{p,s,n} - \lambda_{p,s,n}, 0\}$.

Since $\xi_{p,s,n} \cap [a_{p,s,n}, b_{p,s,n}]$, there are three cases:

$$E\left(Q(\lambda_p, \xi_p)\right) = \begin{cases} \frac{(a_{p,s,n}-\lambda_{p,s,n})^2}{2(b_{p,s,n}-a_{p,s,n})}, & \lambda_{p,s,n} \le a_{p,s,n} \\ \frac{(b_{p,s,n}-\lambda_{p,s,n})^2}{2(b_{p,s,n}-a_{p,s,n})}, & a_{p,s,n} \le \lambda_{p,s,n} \le b_{p,s,n} \\ 0, & b_{p,s,n} \le \lambda_{p,s,n} \end{cases}$$

So, the objective function is given by:

$$\text{Min } z_{-s} = \begin{cases} \sum_{k\in K}\sum_{i\in I_r}\sum_{p\in P}\sum_{r\in R} cp_{p,r}.PIV_{i,r,p,k} + \sum_n\sum_k\sum_s cs.SV_{s,k,n} + \sum_{k\in K}\sum_{i\in I}\sum_{p\in P}\sum_s\sum_n cb_{p,s,n}\frac{(a_{p,s,n}-PDPV_{i,p,s,k,n})^2}{2(b_{p,s,n}-a_{p,s,n})}, & \text{Case 1} \\ \sum_{k\in K}\sum_{i\in I_r}\sum_{p\in P}\sum_{r\in R} cp_{p,r}.PIV_{i,r,p,k} + \sum_n\sum_k\sum_s cs.SV_{s,k,n} + \sum_{k\in K}\sum_{i\in I}\sum_{p\in P}\sum_s\sum_n cb_{p,s,n}\frac{(b_{p,s,n}-PDPV_{i,p,s,k,n})^2}{2(b_{p,s,n}-a_{p,s,n})}, & \text{Case 2} \\ \sum_{k\in K}\sum_{i\in I_r}\sum_{p\in P}\sum_{r\in R} cp_{p,r}.PIV_{i,r,p,k} + \sum_n\sum_k\sum_s cs.SV_{s,k,n}, & \text{Case 3} \end{cases}$$

Cases 1 and 3 are not economical ones since Case 1 seeks for the optimal cost in a lower level demand and Case 3 just leads to an infeasible solution in most cases. So, Case 2 is being tackled in the two-stage stochastic model. Note that the stochastic counterpart of the deterministic model is a mixed-integer quadratic programming (MIQP) model.

5. RESULTS AND DISCUSSION

In this section, two case studies are solved. The first one is a simple structure and uses the deterministic data for the demand while the second one uses the industrial data and demands are random variables with a continuous uniform. All MILP models are implemented on GAMS 24.1.2 / CPLEX using an Intel i7-4790K (4.0 GHz) processor and 8 GB of RAM. Note that the stochastic counterpart of the proposed deterministic model was solved using GAMS 24.1.2 / BARON.

5. 1. Case Study 1 This example, which is a small tree network with deterministic demands, deals with a tree-like pipeline network for transporting six products from two refineries to three depots and is a variant of a case study introduced recently by Mostafaei et al. [26]. Note that depot N_1 is considered as a dual purpose node. Maximum and minimum batch sizes injected/diverted to each pipeline/depot though any pumping run, depot and branch point volume coordinates and flow rate range at segments, are shown in Figure 2. Product inventories and pumping cost at refinery and product demands for next 5 days at depots are given in Table 4. Unit backorder cost is \$200/m^3 and unit stoppage cost is \$1/m^3.

Table 5 shows computational results of Case study 1. Compared to the previous work [26], the proposed approach finds the optimal solution in a lower CPU time due to a lower number of pumping operations, having a major impact on the problem size in term of the number of variables and equations.

5. 2. Case Study 2 This example is a real-world problem of the oil pipeline industry, which a variant of the real world case study introduced by Mostafaei et al. [24] that incorporates a dual purpose node on the mainline. The pipeline structure is depicted in Figure 1. The pump rate at refineries should be kept between 300 and 800 m^3/h. The length of planning horizon is $h_{\max} = 192$ h. The maximum injection volume into each pipeline is 15000 m^3 while the minimum one is 500 m^3. The minimum delivery to each depot is also 500 m^3. The inventory of products at refinery and also pumping cost of each product are all listed in Table 6. Unit flow stoppage cost is 1.0 \$/m^3. Demand for each product at pipeline depots is random variable ($\xi_{p,s,n}$) with a continuous uniform distribution in [3000; 10000], given in m^3.

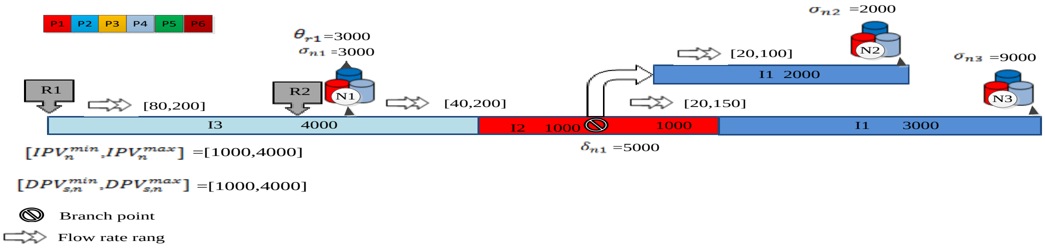

Figure 2. Pipeline network of Case study 1

TABLE 4. Related data for Example 1

	Inventory (m^3)		umping cost ($/$m^3$)		Demand (m^3)		
P	R_1	R_2	R_1	R_2	N_1	N_2	N_3
P1	9000		12	0	2000	1000	3000
P2	-	12000	5.5	4	0	2000	2000
P3	10000		10	0	3000	0	0
P4	5000		12.5	0	2000	0	3000
P5	8000		8	0	2000	0	0
P6	5000	7500	5.5	5	0	0	0

TABLE 5. Computational results of Case 1

Case	#Pump runs	CPU (s)	Con. Var	Bin. Var	Eqs.	Back order (%)	Pump cost ($)	Stop cost ($)	Obj. Fun. ($)
Our approach	6	24	2047	318	3884	0	182000	6000	188000
Mostafaei et al. [26]	10	141	2597	399	4486	0	248500	8000	256500

TABLE 6. Inventory and pumping cost

	Inventory (m^3)		Pumping cost ($/$m^3$)	
P	R_1	R_2	R_1	R_2
P1	36000	-	8	-
P2	25000	18000	7.5	6
P3	10000	18000	8	5
P4	42000	-	7	-

Other data related to this example (i.e., the size of old batches, the coordinate of refineries, depots and branch points, the volume of segments and flow rate limitation) can be found in Figure 1. To solve the problem formulation, we set the number of new batches to be injected into the mainline equal to 4 (i.e., $I_n^{new} = \{\mathrm{I5}, \mathrm{I6}, \mathrm{I7}, \mathrm{I8}\}$). The optimal solution includes nine pumping operations and is found in 2730 s of CPU. At the optimum, the new batches I_5, I6, I7 and I8 convey products P_3, P_2, P_4 and P_2, respectively. Such product sequences inputted in the pipeline lead to an optimal cost of $534233.

6. CONCLUSION

This paper has presented a new optimization framework for the detailed scheduling of tree-like pipelines featuring multiple refineries and depots. The proposed approach has overcome the drawback of previous contributions that only considered a single refinery at the origin of the mainline and deterministic data for depot requirements. In the first stage, a deterministic mixed-integer linear programming (MILP) model has been developed, and then the deterministic model has been extended to a two-stage stochastic programing model. The proposed formulation has been tested with a real-case study. The results have shown that the proposed approach was capable of solving the detailed scheduling of large-scale problems in a quite lower CPU time. For Future work will involve extending the proposed approach by considering production scheduling at refineries, and also the proposed approach will be extended to handle mesh structure pipeline networks.

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Detailed Scheduling of Tree-like Pipeline Networks with Multiple Refineries

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چکیده

در زنجیره تامین نفت، فرآورده‌های نفتی به وسیله‌ی روش‌های مختلفی از قبیل راه آهن، جاده، کشتی و خط لوله انتقال می‌یابند. خطوط لوله یکی از امن‌ترین و ارزان‌ترین راه‌های ارتباط مناطق تولید به بازارهای محلی را فراهم می‌آورد. در این مقاله، زمان‌بندی عملیاتی یک خط لوله درختی چند محصولی که چند پالایشگاه را به چند ترمینال توزیع با تقاضای غیر قطعی وصل می‌کند، مورد بررسی قرار می‌گیرد. ابتدا، یک مدل برنامه‌ریزی عدد صحیح آمیخته (MILP) قطعی ارائه، و سپس یک مدل دومرحله‌ای غیرقطعی پیشنهاد می‌شود. هدف این مدل، تامین تقاضای انبارها (مراکز توزیع) با حداقل کردن کل هزینه‌ها شامل هزینه‌های پمپاژ و توقفات است. کارایی و عملکرد مدل پیشنهادی با دو مثال عددی که یکی از آن‌ها از داده‌های واقعی و صنعتی استفاده می‌کند، نشان داده می‌شود.

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