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Case Mix Planning using The Technique for Order of Preference by Similarity to Ideal Solution and Robust Estimation: a Case Study

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ABSTRACT

Management of surgery units and operating room (OR) play key roles in optimizing the utilization of hospitals. On this line Case Mix Planning (CMP) is normally applied to long term planning of OR. This refers to allocating OR time to each patient's group. In this paper a mathematical model is applied to optimize the allocation of OR time among surgical groups. In addition, another technique is applied to provide an Order of Preference by Similarity to Ideal Solution (TOPSIS) considering different hospital performance measures. Furthermore, robust estimation approach is used to estimate the models' parameters using real data. The proposed model is solved using GAMS software. The results of the study performed in this paper reveal that the proposed methods results in an increase in total hospital operated "value of patients" by 21.5%. This value is defined according to hospital priority and includes moral and ethical considerations. In addition, for each resource, a sensitivity analysis of the findings to the changes is conducted. The results of the sensitivity analysis indicated that the value of the objective function significantly increased via reallocating OR time to surgical groups and/or enhancing the OR facilities to support more surgical specialties.

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1. INTRODUCTION

According to the statistics of the World Health Organization (WHO) published in 2013, per capita total expenditure on health at average exchange rate was 1038 US$ while contributing about 10% of the gross domestic product. In developing countries, health expenditure per capita has been increased by an average of 4 percent annually from 2000 to 2009 [1]. In addition, among different elements of the health systems, hospitals are known as one of the most important components of this sector. On the other hand, OR is known as the main center of costs and revenues of the hospitals [2]. As reported by "Healthcare Financial Management Association", OR results in an estimated value of about 40% of hospital revenue [3]. Also, it was estimated that about 60% to 70% of hospital admissions require OR [4]. Thus, planning and scheduling of OR is considered as a critical problem in most countries and this has attracted the researcher to study this problem. For further details readers are referred to [5, 6]. In Iran, as addressed in WHO reports, a significant portion of the treatment cost is paid by the government. In 2000, this portion was about 37%, while it has recently been increased by 40%. Thus, the need for a proper planning and scheduling in this area is highly important**.**

Elective surgery planning and scheduling is normally performed in one of the three ways as follows [7]:

1. Block scheduling
2. Open scheduling
3. Modified block scheduling

Review of the researches on OR planning reveals that compared with the open scheduling, block scheduling approach in hospitals has a number of advantages [3, 8]. So, the block-scheduling system is commonly used in hospitals and has also been applied in this paper.

Under a block scheduling system, a set of time blocks is assigned to specific surgical groups in a cyclic schedule. There are various frameworks that describe hospital operations management [5, 9-11]. Hans et al. [5] proposed a generic scheme that comprises multiple

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managerial problems. They classified hospital planning subject into strategic, tactical and operational levels. With respect to the strategic planning, they have considered four areas of planning, namely, medical, resource capacity, material and financial **.**

The aim of this article is to solve resource capacity planning problem in strategic level which includes determining the type and composition of the patients, i.e. CMP, over a period of time. Many review papers discussed the OR planning and scheduling problems [12, 13]. Recently Hof et al. [1] provided the first comprehensive literature review focusing on the CMPP. Even though CMP is a central issue in the strategic planning of a hospital, it has rarely been studied by the researchers in the past. In contrast, the CMPP has been gained increased attention in the last three decades [1].

The CMPP is frequently modeled as linear programing (LP) and solved with standard software [1]. In the basic version, the CMPP has a structure similar to the product mix problem [14]. The main purpose of the CMP can be divided into financial and non-financial objectives [1].

Milsum et al. [15] emphasized that the mix of patients should be considered as an important factor in the patient admissions management. Ma and Demeulemeester [16] proposed a hierarchal integrative approach that consists of three phases, the CMP phase, the MSS phase and the operational phase. Ma et al. [17] developed an integer LP model and deployed a branch-and-price technique to maximize hospital profits under OR blocks and wards' beds capacity constraints. Hobbs [1] applied LP to the CMP of a hospital and presented the idea of introducing an "emphasis" constraint to assure that a certain percentage of patients is treated in designated departments of the hospital. Meyer et al. [18] determined the optimal mix of three cardiac services and showed how the range of the objective function coefficients with the same optimal solution can be identified. They noted that personal judgments have to be incorporated with outcomes of operations planning.

Rifai and Pecenka [19] discussed the combination of the objective of minimizing facilities idle time and maximizing profit in the CMPP using goal programming. They formulated goals of maximizing contribution margin and minimizing idle capacity. Blake and Carter [20] proposed linear goal programming models to allocate resources in Hospital. Their case-mix model identified mix and volume of patients for all surgeons that are economically feasible for both the hospital and its associated providers and comes closest to achieve the surgeons favored mix. Mulholland et al. [21] presented an LP model to optimize financial outcomes for both the hospital and physicians in the department of surgery in the CMPP**.**

Despite the fact that most studies measure the optimal case mix in monetary units, CMP can also be driven by non-financial objectives [17, 22, 23]. E.g., Baligh and Laughhunn [22] maximized "value of patients" in their study. This value is defined according to hospital priority and aim of services and can include moral and ethical considerations, patient mortality rates, or the possibility of being referred to another hospital [1]. According to the governmental nature of Shahid Madani hospital, in this study the most important priority is to cover patients who need the most governmental facilities.

Also, constraints are defined according to Shahid Madani hospital limitations. Furthermore, as OR planning and scheduling decisions affect downstream resources throughout the hospital, it seems to be essential to incorporate these resources, such as ICU or ward beds, in the decision-making process.

With the view of the above-reviewed literature the aim of this study is to solve a case mix planning problem based on the actual data collected from Shahid Madani hospital. The significance of this research is that the developed model consists of more Iranian practical constraints such as sex of patients and gradual minimum capacity for each surgical group in order to meet the medical student training requirements. Furthermore, since the model is based on the actual data collected from Shahid Madani HIS, a robust estimator model is developed for the first time to downweight the effect of outliers and contaminated data**.**

The rest of the paper is organized as follows. Section 2 provides a brief description regarding Shahid Madani hospital. Problem description and the proposed mathematical model for the CMP is presented in Section 3. Section 4 introduces the robust estimator. Section 5 provides the results and sensitivity analysis of the results. Finally, Section 6 provides conclusions and potential areas for future studies.

2. SHAHID MADANI HOSPITAL

Shahid Madani hospital is a general surgery hospital, located in Karaj, Iran. The surgery divisions consist of 46 surgeons and 10 surgical groups. It is affiliated to Alborz University of Medical Sciences and contains 195 beds.

Shahid Madani is referred to as a reference emergency hospital and the only burn center in Alborz province. According to the OR facilities, there are some restrictions using the rooms for specific surgeries. Likewise, there are some surgical groups with specific procedure which can only be executed in special rooms. The hospital characteristics are summarized in Table 1. It contains wards capacity and combination of patients in the wards, available capacity of OR, wards and ICU and the OR corresponding to each surgical group. The constraints and the objectives related to the case were obtained via targeted interviews as well as questionnaires filled in by the medical staff. As a result, the following assumptions and restrictions should be considered in the study.

1- Both elective and non-elective patients are taken into account.

2- Certain procedures can be performed in each special OR depending on the size, equipment constraints and managerial aspects.

3- Duration of the CMP is assumed to be one year.

4- OR personnel or surgeons are not bottleneck.

5- The constrained resources considered in this study are capacity of ward, ICU and OR.

6. Lower limits on the number of patients of each surgical group throughout one year are considered in terms of number of available beds in the wards while respecting the hospital general policy.

Table 1 represents which operated patients can be stayed in which ward. Also, this table shows which OR is related to each surgical group.

TABLE 1. Shahid Madani hospital characteristics

Wards	**Surgical group**										**(number of Beds)**	**Annual elective available bed-day**
	CNS	**ENT**	**Orology**	**Orthopedic**	**Eye**	**Hand**	**Burn**	**Vascular**	**General**	**Maxillofacial**		
Orkideh	F[a]	F	F	F	F	F		F	F	F	30	7717
Ofogh			M[b]			M		M		M	20	5863
Chakavak	M		/								18	6009
Shafagh				M							30	3922
Ghasedak	P[c]	M,P	P	P	P	P		P	P	P	18	4728
Omid							M,F,P				14	4951
Taranom									M		25	7862
Negah					M						10	3639
ICU 1	M,F,P	M,F,P	M,F,P	M,F,P	M,F,P	M,F,P		M,F,P	M,F,P	M,F,P	27	8194
ICU 2							M,F,P				3	1092
OR related to surgical group	1,3	8	1,2,3,4,5,6,7	1,2,3,4	9,10	7	8	1,2,3,4,5,6	1,2,3,4,5,6	8		
Annual elective available OR time **OR**	1	2	3	4	5	6	7	8	9	10		
Time	83667	89600	83667	89600	121199	121199	105909	121855	124491	124491		

[a] *Female*, [b] *Male*, [c] *pediatric*

3. PROBLEM DEFINITION AND CORRESPONDING MATHEMATICAL MODEL

The main purpose of CMP is to determine the optimal patient mix of the hospital. The aim of CMP is to find a resource allocation scheme that induces the optimal case mix. In addition, expensive and scarce resources such as ICU beds shall be included in the solving process.

Here, an LP model is described to determine capacity allocation to surgical groups which is a surrogate for CMPP. Sets, list of the variables and parameters required to model CMPP are as follows:

Sets:

I	Set of surgical groups, indexed by i
R	Set of ORs, indexed by r
C	Set of wards, indexed by c
G	Set of patients sex groups (male, female and Pediatrics), indexed by g

Variables:

x_i	OR time which is allocated to surgeon's group i
x_{ir}	Allocated time of OR r to surgical group i
x_{ig}	Allocated time of surgical group i to patient sex group g

Parameters:

Cap_i	The OR time allocated to surgical group *i* in the past year
h_r	Opening hours of OR r for elective surgeries
DY_i	Demand of elective patient's group *i* in the year
R_{ir}	If OR *r* is appropriate for the surgical group *i*; is set to 1; otherwise, 0
α_i	Maximum decrease in the capacity rate of surgical group *i* compared with the capacity of the last year
x'_i	Allocated capacity to surgical group *i* in the last year
w_c	Capacity of Ward c beds
K_{igc}	If ward c is appropriate for the patient sex group g of surgical group *i*; is set to 1; otherwise, 0, $\sum_c k_{igc} = 1$
stw_i	Average LOS of surgical group *i* in the ward
stI_i	Average LOS of surgical group *i* in the ICU
T_i	Average duration of the surgery for group *i*
PN_i	Value of the patients for surgical group *i*

Using these variables and parameters, a mathematical programming formulation of the problem is presented:

$$\sum_i x_{ir} \le h_r \qquad \forall r \tag{1}$$

$$\sum_r x_{ir} = x_i \qquad \forall i \tag{2}$$

$$x_{ir} \le h_r.R_{ir} \qquad \forall i, r \tag{3}$$

$$\sum_{i \ne burn} \frac{x_i}{T_i} \times stI_i \le w_{10} \qquad \forall i \tag{4}$$

$$\frac{x_i}{T_i} \times stI_i \le w_{10} \qquad i\text{=burn} \tag{5}$$

$$\sum_i \sum_g k_{igc} \times \frac{x_{ig}}{T_i} \times stw_i \le w_c \qquad \forall c \tag{6}$$

$$x_i \ge (1-\alpha_i)x'_i \qquad \forall i \tag{7}$$

$$x_i \le DY_i.T_i \qquad \forall i \tag{8}$$

Equation (1) ensures that the OR time allocated to a surgical group is not more than the available time. Equation (2) shows the total allocated time of each surgical group. Constraint (3) guarantees that no surgery can be carried out in an inappropriate OR. Note that the available hours of OR defined excludes the emergency surgeries. After a surgery, all patients except burned patients are moved to the general ICU if necessary. Since the probability of infections of burn patients are high, burned patients are transferred to the ICU stationed in the burn ward when needed. Equations (4) and (5) relate to the capacity constraints of the general ICU and the burn ICU beds, respectively. In Equations (4) and (5), w_9 and w_{10} represent the capacity of general and burn ICU, respectively. Equation (6) ensures that total LOS of each surgical group do not exceed the corresponding ward capacity. Furthermore, the capacities of wards are calculated based on the fact that a certain portion of the capacities are booked for emergency surgeries. Constrains (7) and (8) determine lower and upper limits of the allocated OR time assigned to each surgical group. It is assumed that in each surgical group, minimum amount of OR time shall be allocated. On the other hand, hospital policy is based on gradual reduction of capacity for the surgical groups. Finally, it is assumed that the allocated time of OR to surgical group *i* should not exceed its maximum time required, i.e. the demand of surgical group *i*.

3. OBJECTIVE FUNCTION

In this study, based on the hospital policy, three criteria are considered in calculating "value of patients". Surgical groups demand is one of the major factors affecting "value of patients". Furthermore, covering the most required patients as much as possible is the aim of the hospital, especially those surgeries which are not covered in other hospitals in Alborz province. Thus, the patients who have a less chance to be admitted in the other hospitals should be given more value. On the other hand, since the hospital intends to covers all emergency surgeries, the number of surgeons for each surgical group is affected by the rate of the respective emergency surgeries inherently. Therefore, hospital manager tends to give more portions to the surgical groups with higher emergency rates.

So, the "value of patients" should be calculated based on these three criteria. For this purpose TOPSIS technique is applied. TOPSIS ranks alternatives according to their shortest geometric distance from the positive ideal solution (PIS) and the longest geometric distance from the negative ideal solution (NIS) [24]. For more detail about TOPSIS process the readers are referred to Hwang and Yoon [25]. If PN_i is considered as "value of patients" of surgical group *i*, the objective function can be defined as follows:

$$\max \sum_i \frac{PN_i.x_i}{T_i} \tag{9}$$

4. ROBUST ESTIMATION

In a statistical analysis, it is normally assumed that the data are distributed according to a specified distribution. Based on this assumption, the parameters of interest are usually estimated using classical methods such as maximum likelihood estimation (MLE) [26-28]. In this study, the model parameters are estimated using sample data collected from Shahid Madani HIS. To start with, the content of the data were reviewed. The results revealed that they are contaminated. Thus estimators, called robust estimators, are used to estimate the parameters which are insensitive to the contamination. The details are described below.

4. 1. Maximum Likelihood Estimation of the Parameters

The distributions of the random variables in the problem studied are discrete. Let *Y* denote the random variable with values 1 to *m* with the probabilities of $\theta_1, \theta_2, \dots \theta_m$, respectively. Thus, the probability mass function of Y is defined as follow:

$$\begin{array}{ccccc} y & 1 & 2 & \dots & m \\ P(Y=y) & \Theta_1 & \Theta_2 & \dots & \Theta_m \end{array} \qquad (10)$$

The probability mass function of the random variable Y is defined as follow:

$$P(Y=y) = f_Y(y) = \theta_1^{I(y=1)} \theta_2^{I(y=2)} \dots \theta_m^{I(y=m)} \qquad (11)$$

where I(.) is the indicator function.

Let $y_1, y_2, \dots, y_n$ be a random sample of size n with a probability mass function of $f_Y(y)$. Thus, the likelihood function can be defined as follow:

$$L(\theta_1, \theta_2, \dots, \theta_m) = \theta_1^{\#(y_i=1)} \ \theta_2^{\#(y_i=2)} \dots \theta_m^{\#(y_i=m)} \qquad (12)$$

where #(A) denotes the number of elements of set A.

By defining the sample proportion f_i, we have:

$$f_i = \frac{1}{n} \sum_{j=1}^{n} I(y_j = i) \qquad i=1, 2, \dots, m \qquad (13)$$

Thus:

$$L(\theta_1, \theta_2, \dots, \theta_m) = \theta_1^{nf_1} \theta_2^{nf_2} \dots \theta_m^{nf_m} \qquad (14)$$

By defining $L^* = L^{\frac{1}{n}}$ and $l^* = \log L^*$, we have:

$$L^*(\theta_1, \theta_2, \dots, \theta_m) = \theta_1^{f_1} \theta_2^{f_2} \dots \theta_m^{f_m} \qquad (15)$$

$$l^*(\theta_1, \theta_2, \dots, \theta_m) = f_1 \log \theta_1 + f_2 \log \theta_2 + (1 - f_1 - f_2) \log(1 - \theta_1 - \theta_2) \qquad (16)$$

$$l^*(\theta_1, \theta_2, \dots, \theta_m) = \sum_{i=1}^{m} f_i \log \theta_i \quad \text{s.t:} \ \sum_{i=1}^{m} \theta_i = 1 \qquad (17)$$

$$\begin{aligned} l^*(\theta_1, \theta_2, \dots, \theta_m) &= \sum_{i=1}^{m} \frac{f_i}{\theta_i} \times \theta_i \log(\frac{\theta_i}{f_i} \times f_i) \\ &= \sum_{i=1}^{m} \frac{f_i}{\theta_i} \times \theta_i \left[\log(\frac{\theta_i}{f_i}) + \log f_i \right] \\ &= \sum_{i=1}^{m} \frac{f_i}{\theta_i} \times \theta_i \left[\log(\frac{\theta_i}{f_i}) + \log f_i \right] \\ &= \sum_{i=1}^{m} -\frac{f_i}{\theta_i} \times \theta_i \left[\log(\frac{f_i}{\theta_i}) \right] + \sum_{i=1}^{m} f_i \log f_i \\ l^* &= -\sum_{i=1}^{m} \rho(\frac{f_i}{\theta_i}) \theta_i + A \end{aligned} \qquad (18)$$

where A is the last term which is free of $\theta_1, \theta_2, \dots \theta_m$ and $\rho(x) = x \log x$. So, in order to estimate $\theta_1, \theta_2, \dots \theta_m$ the value of $A\text{-}l^*$ shall be minimized. Thus, the following mathematical programming model must be solved:

$$\begin{aligned} &\min Q = \sum_{i=1}^{m} \rho(\frac{f_i}{\theta_i}) \theta_i \\ &\text{St:} \ \sum_{i=1}^{m} \theta_i = 1, \quad 0 \le \theta_i \qquad i=1, \dots m \end{aligned} \qquad (19)$$

The Lagrange function of the above defined mathematical programming model can be defined as fallows:

$$Z = \sum_{i=1}^{m} \theta_i \rho\left(\frac{f_i}{\theta_i}\right) + \lambda\left(\sum_{i=1}^{m} \theta_i - 1\right) \qquad (20)$$

That is equivalent to:

$$\begin{aligned} Z &= \sum_{i=1}^{m} \theta_i \frac{f_i}{\theta_i} log\left(\frac{f_i}{\theta_i}\right) + \lambda\left(\sum_{i=1}^{m} \theta_i - 1\right) = \\ &\sum_{i=1}^{m} f_i log\left(\frac{f_i}{\theta_i}\right) + \lambda\left(\sum_{i=1}^{m} \theta_i - 1\right) \end{aligned} \qquad (21)$$

In order to solve the above addressed mathematical programing model, the gradient shall be calculated:

$$\frac{\partial z}{\partial \theta} = 0 \Rightarrow f_j = \lambda \theta_j \qquad (22)$$

This can also be written as follow:

$$\sum_{j=1}^{m} f_j = \lambda \sum_{j=1}^{m} \theta_j \qquad (23)$$

Since $\sum_{j=1}^{m} f_j = \sum_{j=1}^{m} \theta_j = 1$, therefore according to the Equation (22) the value of λ and the MLE estimator for parameter Θ are defined as it is shown in Equation (25). Note that fi is sample proportion. So, total sum of fi is equal to 1.

$$\lambda = 1 \qquad (24)$$

$$\hat{\theta}_i = f_i \qquad (25)$$

4. 2. Robust Estimation of the Problem Parameters The data used to estimate the model parameters are collected from Shahid Madani HIS. So, it is possible that the collected data includes outliers. Thus, robust estimators may be a good choice to estimate model parameters. Absolutely, the robust estimators of discrete distribution parameters must be used in this study. Ruckstuhl and Welsh in 2001 [29], developed a new class of estimators for the binomial distribution parameter, named as 'E-estimators', and showed that these estimators have a non-standard asymptotic theory which challenges the accepted relationships between robustness concepts. Due to the discreteness nature of the distribution function of the random variables here, this study has been inspired by the methods developed by Ruckstuhl and Welsh [29] to estimate the model parameters robustly.

As shown in Equation (19), the likelihood function of random sample is defined as:

$$Z = \sum_{i=1}^{m} \theta_i \rho\left(\frac{f_i}{\theta_i}\right)$$

where:

$$\rho(x) = x \log x \qquad (26)$$

when the data is not contaminated, it is better to use $\rho(x) = xlogx$. But in presence of contamination, it is wise to use other $\rho(x)$ function. So, this problem can be defined as an optimization problem in which the $\rho(x)$ is a function with special specifications that is not necessarily equal to xlog x.

$min\, Z = \sum_{i=1}^{m} \theta_i \rho\left(\frac{f_i}{\theta_i}\right)$

St:

$\sum_{i=1}^{m} \theta_i = 1$, $\theta_i \geq 0\ \ \forall\, i = 1,2,\dots,m$ which is the same as writing (27)

$\theta_i - s_i^2 = 0 \quad \forall\, i = 1,2,\dots,m$

The Lagrange function of the above problem can be defined as:

$$Z = \sum_{i=1}^{m} \theta_i \rho\left(\frac{f_i}{\theta_i}\right) - \lambda\left(\sum_{i=1}^{m} \theta_i - 1\right) - \sum_{i=1}^{m} \mu_i \left(\theta_i - s_i^2\right) \qquad (28)$$

where all the variables are free and Θ is used as decision variable vector, so:

$$\Theta = (\theta_1, \theta_2, \dots, \theta_m, \lambda, \mu_1, \mu_2, \dots, \mu_m, s_1, s_2, \dots, s_m)' \qquad (29)$$

To solve the optimization problem, Equation (28), the gradient must be calculated as follows:

$$F = \frac{\partial Z}{\partial \Theta} \qquad (30)$$

If:

$$\rho(x)' = \Psi \quad \text{and} \quad \Psi(x)' = \xi(x) \qquad (31)$$

Then:

$$F(\Theta)_{3m+1} = \begin{pmatrix} \rho\left(\frac{f_1}{\theta_1}\right) - \frac{f_1}{\theta_1}\Psi\left(\frac{f_1}{\theta_1}\right) - \lambda - \mu_1 \\ \vdots \\ \rho\left(\frac{f_j}{\theta_j}\right) - \frac{f_j}{\theta_j}\Psi\left(\frac{f_j}{\theta_j}\right) - \lambda - \mu_j \\ \vdots \\ \rho\left(\frac{f_m}{\theta_m}\right) - \frac{f_m}{\theta_m}\Psi\left(\frac{f_m}{\theta_m}\right) - \lambda - \mu_m \\ -\left(\sum_{i=1}^{m} \theta_i - 1\right) \\ -\theta_1 + s_1^2 \\ \vdots \\ -\theta_j + s_j^2 \\ \vdots \\ -\theta_m + s_m^2 \\ 2\mu_1 s_1 \\ \vdots \\ 2\mu_j s_j \\ \vdots \\ 2\mu_m s_m \end{pmatrix} \qquad (32)$$

In order to solve the above optimization problem, $F(\Theta) = 0$ shall be solved. According to the non-linearity of some equations in the above system of equations, Newton-Rophson method is applied. Thus:

$$\Theta^N = \Theta^O - J_{\theta_O}^{-1} F(\Theta^O) \qquad (33)$$

where Θ^N is defined as estimation for new value of the root while Θ^O and J represent the current value of the root and Jacobian matrix of system of equations, respectively. To form the Jacobian matrix the gradient of F shall be calculated for each variable. So, the Jacobian matrix can be defined as follows:

$$\begin{bmatrix} \frac{{f_j}^2}{{\theta_j}^3} \xi\left(\frac{f_j}{\theta_j}\right) I_{m\times m} & -1_{m\times 1} & -I_{m\times m} & 0_{m\times m} \\ -1_{1\times m} & 0_{1\times 1} & 0_{1\times m} & 0_{1\times m} \\ -I_{m\times m} & 0_{m\times 1} & 0_{m\times m} & 2s_j I_{m\times m} \\ 0_{m\times m} & 0_{m\times 1} & 2s_j I_{m\times m} & 2\mu_j I_{m\times m} \end{bmatrix} \qquad (34)$$

To define appropriate robust estimator, we use Ruckstuhl and Welsh [29] $\rho(x)$ function which defined as follows:

$$\rho(x) = \begin{cases} (\log(c_1 + 1))x - c_1, & if\ x < c_1 \\ xlog(x), & if\ c_1 \leq x \leq c_2 \\ (\log(c_2 + 1))x - c_2, & if\ x > c_2 \end{cases} \qquad (36)$$

In the above equation, c_1 and c_2 shall be defined in a way that the obtained estimators have the highest robustness. The criteria which is used in this section is the asymptotic bias of estimation. This criteria is defined as the deviation between the values of parameters which are estimated from the estimates. Simulation results show that $c_1=0$ and $c_2=1.7$ lead to appropriate results for the defined robust estimator.

5. RESULTS

To obtain the data required for this study, the data related to 19846 patients for a period of 16 months in the course of March 20, 2014 to July 21, 2015 were collected from Shahid Madani HIS.

The minimum time required for each surgical group is calculated according to the maximum rate of decrease in the capacity of the surgical group i compared with that of the last year's capacity. The "value of patients" of surgical groups are calculated applying TOPSIS method. Table 2 shows the values of the estimated parameters of the model. Also, average duration of the surgery time and average LOS in the wards and the ICU are calculated using E robust estimator.

TABLE 2. Value of the required estimated model parameters

Surgical group	Cap_i	DY_i	α_i	StW_i	StI_i	T_i	Active Hospital	Rate of Emergency
CNS	58965	400	0.2	4.25	2.73	184	5	0.0306
ENT	15871	271	0.2	1.71	1.94	73	5	0.0089
Orology	27945	353	0.2	2.48	0.20	95	7	0.0077
Orthopedic	244634	3398	0.2	3.28	0.20	115	7	0.6211
Eye	50134	813	0.2	0.78	0.01	77	6	0.0032
Hand	71418	748	0.2	2.99	0.16	115	1	0.0659
Burn	9749	114	0.2	8.85	0.13	96	0	0.0023
Vascular	3561	104	0.2	4.23	1.93	78	3	0.0187
General	72565	885	0.2	3.24	0.34	105	9	0.2387
Maxillofacial	7451	104	0.2	3.41	0.23	101	0	0.0027

TOPSIS method requires priority weights of criteria obtained via interviews with the hospital management team and the medical staff. The weight vector of elective demand, number of covering hospital, and emergency rate is equal to 0.516, 0.297, 188, respectively. Table 3 shows the value of parameters corresponding to PIS, NIS and closeness coefficient denoted by d_{iw}, d_{ib} and PN_i, respectively and the ranking of the surgical groups.

In order to find the optimal patient mix of the hospital, the LP model is solved using GAMS software and data derived from the developed robust estimator and TOPSIS. The results are summarized in Table 4. Table 4 shows current situation of OR time allocation compared with optimal solution.

TABLE 3. PIS, NIS, PN_i and the ranking of surgical group

Surgical group	$d_{i\omega}$	d_{ib}	PN_i	Rank
CNS	0.442	0.079	0.151	8
ENT	0.457	0.074	0.139	9
Orology	0.458	0.044	0.088	10
Orthopedic	0.125	0.467	0.788	1
Eye	0.408	0.094	0.188	7
Hand	0.396	0.159	0.287	3
Burn	0.464	0.161	0.258	4
Vascular	0.459	0.108	0.191	6
General	0.284	0.232	0.450	2
Maxillofacial	0.465	0.161	0.257	5

TABLE 4. Optimal value of CMP for Shahid Madani hospital compared with the current situation

Surgical group	Annual required demand time ($DY_i \times T_i$)	Current situation		Optimal solution	
		allocated time	% of deviation from $DY_i \times T_i$	allocated time	% of deviation from $DY_i \times T_i$
CNS	73600	58965	81	52413	71
ENT	19783	15871	80	19783	100
Orology	33535	27945	83	33535	100
Orthopedic	390770	244634	63	294121	75
Eye	62601	50134	80	62601	100
Hand	86020	71418	83	86020	100
Burn	10944	9749	89	10944	100
Vascular	8112	3561	44	8112	100
General	92925	72565	78	92925	100
Maxillofacial	10504	7451	71	10504	100

Furthermore, T_i is calculated using the developed E robust estimation. The results indicate that the proposed model obtains better solution, i.e. 21.5% higher than that of the current solution put in practice by the hospital. In addition the results show that in the majority of cases the optimum solution for each surgical group meets the respective demand. In the optimal solution, only 63% of total OR elective capacity is assigned to the surgical groups while for the case of CNS and orthopedic surgical groups they can only cover 71% and 75% of their elective demands, respectively. The reason behind facing with idle capacity of some of the operating rooms while some surgical groups have uncovered demand, may be attributed to the technical restriction on the OR facilities. In order to study the effect of the OR available time on the objective function the proposed model has been solved in different scenarios in terms of available capacity of elective OR time. The results are presented in Figure 1.

As shown in Figure 1, increasing available elective OR time can improve optimum solution up to 12.9%. On the other hand, based on the results, since underutilization of OR capacity occurrs on about 37% of the optimum solution, this provides an opportunity for hospital managers to facilitate their idle operating rooms in order to support CNS and orthopedic surgeries, if it will be possible.

Figure 2 shows the effect of the changes in the maximum decrease rate of capacity of surgical groups on the objective function.

The results indicate that reducing the lower bound of the allocated capacity to surgical groups leads to an improvement in objective function. When the maximum decrease on the rate of the capacity of surgical groups are equal to 0 (-100% change) the objective function is improved by 10.6%. As a result, it can be concluded that in this situation due to the high priority of orthopedic surgical groups, all of the capacity of operating rooms 1, 2, 3 and 4 are assigned to this surgical group and hence no elective OR time is assigned to CNS surgical group.

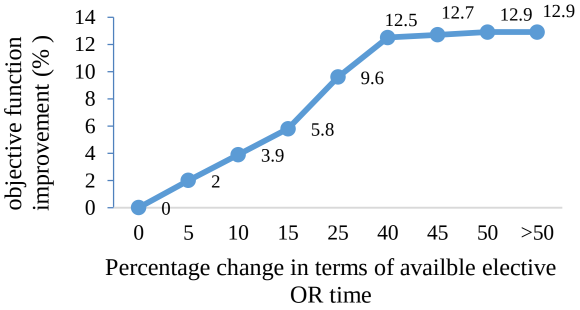

Figure 1. Objective function improvement against the change in the available elective OR time

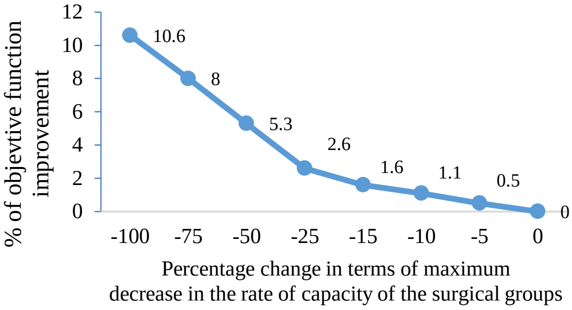

Figure 2. Objective function improvement according to change in maximum decrease in the rate of capacity of surgical groups

The results of the sensitivity analysis also reveal that wards and ICU capacity are not a bottleneck in Shahid Madani hospital, and hence the hospital managers shall focus on available OR elective capacity and their elective covering policy to improve the performance of CMP.

6. CONCOLUSION

This paper studied the problem of allocating OR time to surgical groups, considered as a surrogate for the CMPP. For this purpose an LP model is developed to find the optimum case mix of the patients. In order to make the problem as real as possible, the constraints and the parameters' values are defined based on the data collected from Shahid Madani hospital. According to the hospital staff, three performance measures were considered in the CMPP. To tackle with the multi-criteria nature of objective function of CMP, TOPSIS technique was applied. The values of the parameters of the model were estimated using the data collected from Shahid Madani HIS. In order to overcome contaminations of the data and down weighting the outlier data, a robust estimator was applied to estimate the values of the model parameters. Having estimated the values of the parameters, the developed LP model was solved using GAMS software. The results revealed that compared with the current situation, the proposed methods results in an increase in total hospital operated “value of patients” by 21.5%. In addition, a sensitivity analysis was conducted. Furthermore, the results of the sensitivity analysis performed indicated that as the available elective OR capacity increases, the value of the objective function is increased by 12.9%. Likewise, the objective function is improved by 10.6% when the change on the minimum rate of each surgical group is increased while ICU and ward beds capacity changes did not affect the CMP. As a further study, it is proposed to study the problem in an uncertain environment in terms of LOS in the wards, ICU or operating time. In addition, it is proposed to study on balancing hospital resources while finding optimum solution for the CMPP. Finally, it is supposed to use heuristic or meta-heuristic algorithms such as genetic algorithm [30] or simulated annealing [31] when the problem size will be increased.

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Case Mix Planning using The Technique for Order of Preference by Similarity to Ideal Solution and Robust Estimation: a Case Study

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چکیده

مدیریت بخش‌های جراحی و اتاق‌های عمل نقشی کلیدی در استفاده بهینه از بیمارستانها دارد. در این حوزه مساله “تعیین ترکیب بیماران” معمولا در برنامه‌ریزی بلند مدت اتاق‌های عمل مورد بررسی قرار می‌گیرد. این مساله عبارتست از تخصیص ظرفیت اتاق‌های عمل به گروه‌های مختلف بیماران. در این مقاله از یک مدل برنامه‌ریزی ریاضی جهت تخصیص بهینه ظرفیت اتاق‌های عمل به گروه‌های مختلف جراحی استفاده شده است. به‌علاوه، از روش رتبه بندی TOPSIS به منظور رتبه‌بندی گروه‌های مختلف جراحی بر اساس شاخص‌های مختلف عملکردی بیمارستان استفاده شده است. هچنین، رویکرد برآورد کننده‌های استوار جهت برآورد پارامترهای مساله بر اساس داده‌های واقعی مورد استفاده قرار گرفته است. سپس، مدل توسعه داده شده توسط نرم افزار GAMS حل گردیده است. نتایج به دست آمده حاصل از حل مدل آشکار می‌سازد که شاخص ارزش بیماران جراحی شده نسبت به وضعیت فعلی بیمارستان ۲۱٫۵٪ افزایش می‌یابد. این شاخص بر اساس الویت‌های بیمارستان تعریف شده است و شامل ملاحظات اخلاقی می‌باشد. به علاوه، جهت بررسی حساسیت مدل به تغییر در منابع در دسترس، به تحلیل حساسیت مدل پرداخته شده است. نتایج حاصل از تحلیل حساسیت صورت گرفته نشان می‌دهد که میزان تابع هدف مساله به طور قابل ملاحظه‌ای با افزایش قابلیت اتاق‌های عمل جهت پشتیبانی گروه‌های مختلف جراحی بهبود می‌یابد.

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