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Designing an Economic Repetitive Sampling Plan in the Presence of Two Markets

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ABSTRACT

In this paper, we develop an optimization model for the economic design of repetitive sampling plan in the presence of two markets. The process under consideration produces a product with a normally distributed quality characteristic with unknown mean and known variance. The quality characteristic has a lower specification limit. The quality of the product is controlled via lot-by-lot acceptance sampling plan. The objective function used in the model is maximizing profit and product conformity using the Taguchi loss function as a surrogate for product conformity. Risks of producer and consumer in two different markets are considered as constraints. We demonstrate the application of the model using a numerical example. Sensitivity analysis on model parameters shows that the result of the model is sensitive to changes in model parameters.

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1. INTRODUCTION

Acceptance sampling models have been widely applied in companies for inspecting and testing the raw material as well as final products. A number of lots of products are produced in a day in the industries that it might be impossible to inspect/test each item in a single lot. Acceptance sampling plans save the time and cost of the inspection and help the producer send the product to the market at the appropriate time. Thus, the use of acceptance sampling plans not only enhances the reputation of the companies, but also increases the reduced cost of them.

Two different approaches are mainly used in designing acceptance sampling models. First approach is based on the probabilities of the first and second type errors. The optimality of sampling plan in this case is due to designing a sampling plan with minimum sample size that satisfies the constraints of first and second type errors. In the second approach, the objective is to construct an economically optimal sampling plan. Constructing economic models of the acceptance sampling plans has drawn scholars' attention for years. Although adopting this approach is not new, it has led to emergence of insightful results on integrating the continuous loss function with the economic models of the acceptance sampling plan.

There have been several studies proposing various economic acceptance sampling plans. For example, Ferrell and Chhoker [1] presented a method to determine economically optimal acceptance sampling plans. According to Moskowitz and Tang [2], acceptance sampling plans can be optimized based on Taguchi loss function and Bayesian approach. Fallahnezhad and Aslam [3] and Fallahnezhad et al. [4] proposed Bayesian analysis of acceptance sampling problem based on cost analysis. Hsu and Hsu [5] proposed an economic model to determine the optimal sampling plan in a two-stage supply chain that minimizes the producer's and the consumer's total quality cost while satisfying both the producer's and consumer's quality and risk requirements. Balamurali et al. [6] proposed an economic design of the SKSP-R plan for both destructive and non-destructive testing by considering various cost items in order to optimize the plan. Fallahnezhad et al. [7] proposed a mathematical model to design single stage and double stage sampling

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plans that can be used to determine the optimal tolerance limits and sample size. Fallahnezhad and Seifi [8] proposed an optimal double-sampling plan based on process capability index. Asalm et al. [9] Fallahnezhad and Fakhrzad [10] Fallahnezhad and Hosseininasab [11] applied a new control policy for designing sampling plan.

In this research, our aim is to develop a mathematical model to design an optimal repetitive sampling plan while its optimality is resulted by using the expected profit of each decision based on Taguchi's Loss Function and subject to risk constraints in two markets. The probabilities of selecting each decision are determined by employing the Binomial distribution. The risks of producer and consumer are taken into account in order to control the lot quality. FallahNezhad and Jafari Nodoushan [12] proposed an economic rectifying sampling plan in the presence of two markets. Aslam et al. [13] proposed a repetitive mixed sampling plan based on the process capability index. The parameters of their mixed plan are determined by satisfying the given producer's risk and consumer's risk constraints at the same time for the specified acceptable quality level and limiting quality level. Aslam et al [14] proposed a multiple states repetitive group sampling plan by considering the process loss. The optimal plan parameters of their plan are selected such that the constraints of producer's risk and consumer's risk are satisfied simultaneously by minimizing the average sample number. Fallahnezhad and Seifi [15] proposed a repetitive group sampling plan based on the process capability index for the lot acceptance problem. Aslam et al. [16] proposed a different repetitive sampling plan using process capability index of multiple quality characteristics. Aslam et al. [17] proposed a multiple dependent state repetitive group sampling plan for Burr XII distribution.

Modeling the sampling plans can be facilitated by using Markov chain. Markov chain can be efficiently implemented in practical quality control problems [6, 11, 18].

Companies and organizations may require maximization of profit and customer satisfaction simultaneously. Customer satisfaction will be achieved through ensuring product quality by minimizing the deviation of quality characteristic from its target value using Taguchi loss function. The purpose of this paper is to present the economic design of a repetitive acceptance sampling plan using Taguchi loss function in the presence of decision making risks for target markets.

An example for application of such models is the can filling problem. In this problem, cans are produced in lots of size N. The quality characteristic of interest is the volume of liquid put in a can. Depending on whether the liquid volume in the can exceeds the lower specification limit or not, the can is classified as conforming or non-conforming. A sample of size n is drawn from the lot and depending on the number of non-conforming cans found in the sample, the lot is either sold in a primary market, secondary market or reworked. In the juice industry, the vitamin c level is important and the amount of vitamin c must be more than the lower specification limit.

The model can be applied to a wide spectrum of applications in process industry such as food, beverages, petrochemicals, pharmaceuticals, cement, paints, chemicals industry etc. For example, in the juice industry the vitamin C level is important and the amount of vitamin C must be more than the lower specification limit.

The contributions of the proposed approach are as follows:

- An economical model for repetitive group acceptance sampling plan is presented.
- Two markets are considered for product that has not been addressed for designing RGSPs so far.
- Give-away cost per unit of sold excess material is considered in the proposed model.
- Optimal process adjustment problem and acceptance sampling plan is combined in the economical optimization model.
- Process mean and standard deviation are assumed to be an unknown value and its impact is analysed.
- Inspection error is considered and its impact is investigated and analysed.

The rest of the paper is organized as follows: Statement of the problem is presented in Section 2 followed by the model development in Section 3. Section 4 presents an example to demonstrate the application of the model together with sensitivity analysis. Section 5 concludes the paper.

2. STATEMENT OF THE PROBLEM

The production process produces items that have a quality characteristic y with one sided specification limit LSL and target mean T and known standard deviation σ. A produced item is called conforming if its quality characteristic is more than or equal to LSL ($y \geq LSL$) and is called non-conforming if its quality characteristic falls below LSL ($y < LSL$). Items are produced in lots of size N and a sampling plan is used to decide on the quality of the lots. The sampling plan is described as follows: a sample of size n is drawn from a lot of size N. The sample is inspected and based on the number of the non-conforming items in the sample D, the quality of the lot is evaluated. There are two thresholds d_1 and d_2 for decision making, where $d_1 < d_2$. First, if the number of non-conforming item in the sample is less than or equal to the first threshold, i.e. $D \leq d_1$, then the whole lot is sold in a primary market at a price of $a per item. Second, if the number of non-conforming items in the sample is between the thresholds, $d_1 < D \leq d_2$, then the whole lot is sold in a secondary market at a reduced price $r per item ($a > r$). Finally, if the number of non-conforming items in the sample is more than the second threshold $D > d_2$, then the whole lot is reworked again with a rework cost of $R per item. Note that the rework operation is not perfect and does not ensure the conformance of all products. After rework, the item could be sold in the primary market or the secondary market, or needs to be reworked again. The production cost per item $c and inspection cost per item $I are assumed to be known and constant. The production cost consists of processing and labor costs [19].

3. MODEL DEVELOPMENT

3. 1. Notations

Notations used in the model development are defined in Table 1.

TABLE 1. Notations

PN	Profit per lot produced
E (PN)	The expected profit per lot
c	Production cost per item
I	Inspection cost per item
c_1	The cost of non-conforming item sent to the customer in the primary market
c_2	The cost of replacing a non-conforming item with a conforming item
c_3	The cost of non-conforming item sent to the customer in the secondary market
c_4	The cost of rejecting a conforming item and replacing it
N	The lot size
n	The sample size
D	The number of non-conforming items in a sample of size n
D_e	The number of observed non-conforming items in the sample
T	Target mean
LSL	Lower specification limit
(d_1, d_2)	Inspection plan thresholds
(k_1, k_2)	Quality loss coefficients
$P_1 = LQL_1$	Limiting Quality Level of the primary market
$P_2 = AQL_2$	Acceptable Quality Level of the secondary market
a	Item price at primary market
r	Item price at secondary market
g	Give-away cost per unit of excess material
R	Rework cost
P	Probability of non-conformance
P_e	Probability of non-conformance in the presence of inspection errors
f (y)	Normal distribution function with unknown mean T and known variance σ^2
φ (z)	Standard normal distribution function
Φ (z)	Standard normal cumulative distribution function
β_1	Probability of type-II error in making a decision of primary market
α_2	Probability of type-I error in making a decision of secondary market
e_1	Type I inspection error
e_2	Type II inspection error
p_{ij}	The probability of going from state i to state j in a single step
m_{ij}	The expected number of transitions from a non-absorbing state (i) to another non-absorbing state (j) before absorption occurs
f_{ij}	Long-run probability of going from a non-absorbing state (i) to an absorbing state (j)
p'_{ij}	The probability of going from state i to state j in a single step with consideration of inspection errors
P	The transition Probability Matrix
Q	A square matrix containing transition probabilities of going from a non-absorbing state to another non-absorbing state
R	A matrix containing all probabilities of going from a non-absorbing state to an absorbing state
A	An identity matrix representing the probability of staying in a state
O	A matrix representing the probabilities of escaping from an absorbing state
F	The absorption probability matrix containing the long-run probabilities of the transitions from non-absorbing states to absorbing states
M	The fundamental matrix containing the expected number of transitions from a non-absorbing state to another non-absorbing state before absorption occurs

3. 2. Assumptions

The assumptions made are:

1. The lot is assumed to be large enough to justify the use of the Binomial distribution for the number of non-conforming items in the sample.
2. Costs of processing are assumed to be directly proportional to the values of the product quality characteristics.
3. There is no drift or shift in the means of the processes. The process is under control.
4. Sampling inspection plan is used for lot quality assurance and the inspection is assumed to be error free.
5. Identified non-conforming items are replaced with conforming items.

3. 3. Basic Relationships

Let us first determine the probabilities of classifying the lot to be sold in the primary market, secondary market or to be reworked.

First, the probability that quality characteristic of an item falls below LSL is given by the following:

$$P(y < LSL) = \Phi(\frac{LSL-T}{\sigma}) = P \tag{1}$$

The number of defectives in an incoming lot follows the binomial probability distribution with parameter P.

The lot is sold in the primary market if the number of defectives in the sample is less than or equal to d_1 , this following is obtained:

$$P(D \le d_1) = \sum_{i=0}^{d_1} \binom{n}{i} P^i (1-P)^{n-i} \tag{2}$$

The probability of selling the lot in the secondary market is as following:

$$P(d_1 < D \le d_2) = \sum_{i=0}^{d_2} \binom{n}{i} P^i (1-P)^{n-i} - \sum_{i=0}^{d_1} \binom{n}{i} P^i (1-P)^{n-i} \tag{3}$$

$$P(d_1 < D \le d_2) = \sum_{i=d_1+1}^{d_2} \binom{n}{i} P^i (1-P)^{n-i} \tag{4}$$

Finally, the probability of reworking the whole lot is:

$$P(D > d_2) = \sum_{i=d_{2+1}}^{n} \binom{n}{i} P^i (1-P)^{n-i} = 1 - \sum_{i=0}^{d_2} \binom{n}{i} P^i (1-P)^{n-i} \tag{5}$$

Also, the conditional expected value of the quality characteristic y given that y is more than, or equal to the lower specification limit is determined as following:

$$y' = E(y|y \ge LSL) = \frac{\int_{LSL}^{\infty} y\, f(y)\, dy}{\int_{LSL}^{\infty} f(y)\, dy} \tag{6}$$

Thus $E(y|y \ge LSL) - LSL$ denotes the amount of excess material used in the product.

3. 4. Derivation Of Objective

The profit per lot, PN, as given in Equation (7), results from selling lots at the primary market (first part of Equation (7)), selling lots at the secondary market (second part of Equation (7)), and profit from reworked lots (third part of Equation (7)) as following:

$$PN = \begin{cases} aN - g(y' - LSL)N - In - cyN - (N-n)L_{01} - nL_{11} \\ \quad -(N-n)Pc_1 - nPc_2 \quad if\ P(D \le d_1) \\ rN - g(y' - LSL)N - In - cyN - (N-n)L_{02} - nL_{12} \\ \quad -(N-n)Pc_3 - nPc_2 \quad if\ P(d_1 < D \le d_2) \\ \quad E(PN) - RN - In - cyN \quad if\ P(D > d_2) \end{cases} \tag{7}$$

Taguchi's loss function is an effective method for quality engineering. The quality losses occur when the product deviates beyond the specification limit, thereby becoming unacceptable [1]. Taguchi defines quality as 'the loss imported by any product to society after being shipped to a customer, other than any loss caused by intrinsic cost functions of producer [2, 20].

Taguchi's loss function is classified as three types of functions: nominal-is-best characteristics, smaller-is-better characteristics and larger-is-better characteristics.

The quality characteristic y of the items produced by the process under study has no upper specification limit. Hence, the quality level is evaluated by using the loss function approach for the larger -is-better. Since the upper specification limit is absent, thus the larger the value of the quality characteristic, the better it is and the minimum loss is obtained.

The larger-is-better loss function is given by:

$$L(y) = k\frac{1}{y^2}$$

where $L(y)$ is the loss associated with a particular value of quality characteristic y; and k is the average loss coefficient.

The larger-is-better loss functions are employed for evaluating loss for the consumer and are as following [19]:

$$L_{01} = E(Loss) = k_1 \int_{-\infty}^{\infty} \frac{1}{y^2} f(y)\, dy \tag{8}$$

$$L_{11} = E(\{Loss|y > LSL\}) = \frac{k_1 \int_{LSL}^{\infty} \frac{1}{y^2} f(y)\, dy}{\int_{LSL}^{\infty} f(y)\, dy} \tag{9}$$

L_{01} and L_{11} are the expected loss per un-inspected and inspected items of a lot accepted and sold in the primary market, respectively. In Equation (9), we used the conditional expectation that the quality characteristics value is more than LSL, under the condition that the inspected items of a lot accepted and sold in the primary market are assumed to have quality characteristic above LSL.

L_{02} and L_{12} are the expected loss per un-inspected and inspected item of a lot accepted and sold in the secondary market, respectively.

$$L_{02} = E(Loss) = k_2 \int_{-\infty}^{\infty} \frac{1}{y^2} f(y)\, dy \tag{10}$$

$$L_{12} = E(\{Loss|y > LSL\}) = \frac{k_2 \int_{LSL}^{\infty} \frac{1}{y^2} f(y)\, dy}{\int_{LSL}^{\infty} f(y)\, dy} \tag{11}$$

where the terms of objective function are explained in Table 2:

TABLE 2. The terms of objective function

aN	The revenue of selling N items in the primary market
$g(y' - LSL)N$	The give-away cost of excess material above LSL in the lot with N items
In	Cost of inspecting sample with n items
cyN	The production cost of the lot with N items
$(N - n)L_{01}$	The loss for un-inspected items of a lot accepted and sold in the primary market
$n L_{11}$	The loss for inspected items of a lot accepted and sold in the primary market
$(N - n)Pc_1$	The cost of non-conforming items sent to the customer in the primary market
nPc_2	The cost of replacing non-conforming items identified in a sample with conforming items
rN	The revenue of selling N items in the secondary market
$(N - n)L_{02}$	The loss for un-inspected items of a lot accepted and sold in the secondary market
$n L_{12}$	The loss for inspected items of a lot accepted and sold in the secondary market
$(N - n)Pc_3$	The cost of non-conforming items sent to the customer in the secondary market

E(PN)	The expected profit per lot
RN	The rework cost of the lot with N items

Now, the expected profit per lot is given by:

$$E\ (PN) = aN\ .P(D \le d_1) - g(y' - LSL)N\ .P(D \le d_1) - In\ .P(D \le d_1) - cyN\ .P(D \le d_1) - (N-n)L_{01}\ .P(D \le d_1) - nL_{11}\ .P(D \le d_1) \quad - (N-n)Pc_1.P(D \le d_1) - nPc_2\ .P(D \le d_1) \quad + rN\ .P(d_1 < D\ \le\ d_2) \quad - g(y' - LSL)N\ .P(d_1 < D\ \le\ d_2) \quad - In\ .P(d_1 < D\ \le\ d_2) - cyN\ .P(d_1 < D\ \le\ d_2) - (N-n)L_{02}\ .P(d_1 < D\ \le\ d_2) \quad - nL_{12}\ .P(d_1 < D\ \le\ d_2) \quad - (N-n)Pc_3\ .P(d_1 < D\ \le\ d_2) \ - nPc_2.P(d_1 < D\ \le\ d_2) + E(PN)\ .P(D\ > d_2) \ - RN\ .P(D\ > d_2) - In\ .P(D\ > d_2) \quad - cyN\ .P(D\ > d_2) \tag{12}$$

Rearranging the above equation, following is obtained:

$$E(PN) - E(PN).P(D\ > d_2) = aN\ .P(D \le d_1) - g(y' - LSL)N\ .P(D \le d_1) - In\ .P(D \le d_1) - cyN\ .P(D \le d_1) - (N-n)L_{01}\ .P(D \le d_1) - nL_{11}\ .P(D \le d_1) - (N-n)Pc_1.P(D \le d_1) - nPc_2\ .P(D \le d_1) + \ rN\ .P(d_1 < D\ \le\ d_2) - g(y' - LSL)N\ .P(d_1 < D\ \le\ d_2) - In\ .P(d_1 < D\ \le\ d_2) - cyN\ .P(d_1 < D\ \le\ d_2) - (N-n)L_{02}\ .P(d_1 < D\ \le\ d_2) - nL_{12}\ .P(d_1 < D\ \le\ d_2) - (N-n)Pc_3\ .P(d_1 < D\ \le\ d_2) - nPc_2.P(d_1 < D\ \le\ d_2) - RN\ .P(D\ > d_2) - In\ .P(D\ > d_2) - cyN\ P(D\ > d_2) \tag{13}$$

Note that $[P(D \le d_1) + P(d_1 < D \le d_2\,) + P(D > d_2)] = 1$ and $\mathrm{E}[y\ .P(D \le d_1) + y\ .P(d_1 < D \le d_2\,) + y\ .P(D > d_2)] = T$.

Therefore, Equation (13) is simplified to:

$$E(PN) = \frac{1}{P\ (D{\le}d_2)}\ [aN\ .P(D \le d_1) - g(y' - LSL)N\ .P(D \le d_1) \qquad -In - cTN -(N-n)k_1 \int_{-\infty}^{\infty} \frac{1}{y^2}\ f(y)\, dy.P(D \le d_1) - n\, \frac{k_1 \int_{LSL}^{\infty} \frac{1}{y^2} f(y)\, dy}{\int_{LSL}^{\infty} f(y)\, dy}\ .P(D \le d_1) - (N-n)Pc_1.P(D \le d_1) - nPc_2\ .P(D \le d_1) \quad + rN\ .P(d_1 < D\ \le\ d_2) - g(y' - LSL)N\ .P(d_1 < D\ \le\ d_2) \qquad -(N-n)k_2 \int_{-\infty}^{\infty} \frac{1}{y^2} f(y)\, dy\ .P(d_1 < D\ \le\ d_2) - n\, \frac{k_2 \int_{LSL}^{\infty} \frac{1}{y^2} f(y)\, dy}{\int_{LSL}^{\infty} f(y)\, dy}\ .P(d_1 < D\ \le\ d_2) - (N-n)Pc_3\ .P(d_1 < D\ \le\ d_2) \ -nPc_2.P(d_1 < D\ \le\ d_2) - RN\ .P(D\ > d_2) \tag{14}$$

3. 5. Derivation of Risk Constraints

Consider an incoming lot of N items with a proportion of non-conformities P, of which n items are randomly selected for inspection and depending on the number of non-conforming items found in the sample, the lot is either sold in the primary market, secondary market, or reworked.

A sample of size n is drawn from a lot of size N. The sample is inspected and based on the number of non-conforming items in the sample D, the quality of the lot is decided. There are two thresholds d_1 and d_2, where $d_1 < d_2$.

The states involved in this process can be defined as follows:

State 1: D is more than the second threshold $D > d_2$ then, the whole lot is reworked.

State 2: D is between the thresholds $d_1 < D \le d_2$then, the whole lot is sold in the secondary market.

State 3: D is less than or equal to the first threshold $D \le d_1$, then the whole lot is sold in the primary market.

The transition probabilities among the states can be obtained as follows.

Probability of reworking the whole lot:

$$P_{11} = P\ \{D > d_2\} \tag{15}$$

Probability of selling the whole lot in the secondary market:

$$P_{12} = P\ \{d_1\ < D \le d_2\} \tag{16}$$

Probability of selling the whole lot in the primary market:

$$P_{13} = P\ \{D \le d_1\} \tag{17}$$

where the probabilities can be obtained based on the fact that the number of non-conforming items, D, follows a binomial distribution with parameters n and p. Then, the transition probability matrix is expressed as follows:

$$\mathbf{P} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} \tag{18}$$

As it can be seen, the matrix **P** is an absorbing Markov chain with states 2 and 3 being absorbing and state 1 being transient.

To analyze the above absorbing Markov chain, the transition probability matrix should be rearranged in the following form:

$$\begin{bmatrix} \mathbf{A} & \mathbf{O} \\ \mathbf{R} & \mathbf{Q} \end{bmatrix} \tag{19}$$

Rearranging the **P** matrix yields the following matrix:

$$\begin{array}{c} \\ 2 \\ 3 \\ 1 \end{array} \begin{array}{c} \begin{array}{ccc} 2 & 3 & 1 \end{array} \\ \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ P_{12} & P_{13} & P_{11} \end{bmatrix} \end{array} \tag{20}$$

Then, the fundamental Matrix **M** can be obtained as follows (Bowling et al. [21]):

$$\boldsymbol{M} = \ m_{11} = \ (\boldsymbol{I} - \boldsymbol{Q})^{-1} = \frac{1}{1-P_{11}} = \frac{1}{1-P\ (D>d_2)} \tag{21}$$

where **I** is the identity matrix and m_{11} denotes the expected number of times that the transient state 1 is occupied before absorption occurs (i.e., sold in the secondary or primary market), given that the initial state is 1. The long-run absorption probability matrix, **F**, is calculated as follows (Bowling et al. [21]):

$$\boldsymbol{F} = \boldsymbol{M} * \boldsymbol{R} = \begin{array}{cc} & \begin{array}{cc} 2 & 3 \end{array} \\ 1 & \begin{bmatrix} \frac{P_{12}}{1-P_{11}} & \frac{P_{13}}{1-P_{11}} \end{bmatrix} \end{array} \tag{22}$$

The elements of the **F** matrix, f_{12}, f_{13}, denote the probabilities of the lot being sold in the secondary or primary market, respectively.

The objective function, $E\ (PN)$, should be maximized regarding one constraint on risk of wrong decisions for each market associated with the acceptance sampling plans. Type-II error is the probability of accepting the lot when the non-conforming proportion of the lot is not acceptable. Then, in one hand, if $= P_1 = LQL_1$, the probability of accepting the lot should be less than β_1 for the primary market. Type-I error is the probability of rejecting the lot when the non-conforming proportion of the lot is acceptable. Then, on the other hand, if $= P_2 = AQL_2$, the probability of rejecting the lot should be less than α_2 for the secondary market. Hence:

$$P = P_1 = LQL_1 \ \ ; \ \ \frac{P_{13}}{1-P_{11}} = \frac{P\ (D \le d_1)}{1-P\ (D>d_2)} \le \beta_1 \tag{23}$$

$$P = P_2 = AQL_2 \ \ ; \ \ \frac{P_{12}}{1-P_{11}} = \frac{P\ (d_1<D \le d_2)}{1-P\ (D>d_2)} \ge 1 - \alpha_2 \tag{24}$$

Since consumer risk is more important than producer risk, thus the constraint of consumer risk is considered for the primary market and the constraint of producer risk is considered for the secondary market.

3. 6. Final Model

In short, the optimization model of the problem becomes**:**

$$Max\ E\ (PN) = f(n, d_1, d_2) \tag{25}$$

s.t.

$$1)\ \frac{P\ (D \le d_1)}{1-P\ (D>d_2)} \le \beta_1 \qquad ; P = P_1 = LQL_1$$

$$2)\ \frac{P\ (d_1<D \le d_2)}{1-P\ (D>d_2)} \ge 1-\alpha_2 \qquad ; P = P_2 = AQL_2$$

The optimum values of $n\,, d_1\,, d_2$ among a set of alternative values are determined solving the model given in Equation (25), numerically, where the probabilities are obtained using the binomial distribution. In the next section, a numerical example is given to demonstrate the application of the proposed methodology.

4. SOLUTION

4. 1. Numerical Example

In this section the application of the model is demonstrated via a numerical example. Table 3 provides the specified values for the parameters and their references.

Proposed model is solved using a grid search method and the computer program is written using Matlab Software. For alternative values of $n\,, d_1\,, d_2$ in the assumed intervals for each one, the expected profit per lot is computed, then the maximum value of that is specified for optimal values of $n\,, d_1\,, d_2$.

Table 4 shows the optimal value of objective function and design parameters for above numerical example.

Following decision making framework is obtained:

1. If there is not any defective item in the inspected sample, then the lot should be sold in the primary market.
2. If the number of defective items falls within the set $\{1,2,\ldots,9\}$, then, the lot should be sold in the secondary market.
3. If all items of the sample are defective, then the lot should be reworked.

TABLE 3. Data of the example data

Parameter	Value	Source
LSL	10	Duffuaa et al. [19, 22]
T	10.5	Best guess by authors
σ	0.5	Chen and Lai [23, 24]
N	1000	Best guess by authors
a	80$	Pulak and Al-Sultan [25], Chen and Lai [24]
r	67.5$	Duffuaa and El-Ga'aly [26]
R	4$	Duffuaa and El-Ga'aly [26]
c	6$	Duffuaa and El-Ga'aly [26]
c_1	$15	Best guess by authors
c_2	$10	Best guess by authors
c_3	$12	Best guess by authors
c_4	$11	Best guess by authors
I	1$	Duffuaa and El-Ga'aly [26]
g	2$	Duffuaa and El-Ga'aly [26]
k_1	$400	Best guess by authors
k_2	$300	Best guess by authors
$P_1 = LQL_1$	0.15	Best guess by authors
$P_2 = AQL_2$	0.15	Best guess by authors
β_1	0.20	Best guess by authors
α_2	0.20	Best guess by authors

TABLE 4. The optimal values of objective function and design parameters

Objective function	Design parameters		
f^*	d_1	d_2	n
538.1867	0	9	10

As can be seen, the optimal solution seems to be unbalanced because the lot should be sold in the secondary market in most of the cases. Thus, the values of the parameters are not selected correctly. Therefore, a sensitivity analysis is carried out to investigate the effects of each parameter.

4. 2. Sensitivity Analysis

In this section, sensitivity analysis is conducted to study the impact of the model parameters. The results of ±50% variations for each parameter are denoted in Table 5. All values for objective function are rounded to one decimal digit.

TABLE 5. Sensitivity analysis on the model parameters

Row	Model Parameters																			Design Parameters			Optimal Objective Function	Percent Change of Objective Function
	LSL	**T**	σ	**N**	**a**	**r**	**R**	**c**	c_1	c_2	c_3	**I**	**g**	k_1	k_2	LQL_1	AQL_2	β_1	α_2	d_1	d_2	**n**		
1	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	**538.2**	Original
2	10	10.5	**0.75**	1000	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	-2348.1	-536.3
3	10	10.5	**0.25**	1000	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	4	11	44	11922	2115.1
4	10	10.5	0.5	**1500**	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	809.9	50.5
5	10	10.5	0.5	**500**	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	266.5	-50.5
6	10	10.5	0.5	1000	**120**	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	7647.1	1320.8
7	10	10.5	0.5	1000	80	**33.7**	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	0	7	10	-27214	-5156.3
8	10	10.5	0.5	1000	80	67.5	**6**	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	538.2	0.0
9	10	10.5	0.5	1000	80	67.5	**2**	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	538.2	0.0
10	10	10.5	0.5	1000	80	67.5	4	**9**	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	-30962	-5852.7
11	10	10.5	0.5	1000	80	67.5	4	**3**	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	32038	5852.6
12	10	10.5	0.5	1000	80	67.5	4	6	**23**	10	12	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	314.9	-41.5
13	10	10.5	0.5	1000	80	67.5	4	6	15	**5**	12	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	546.2	1.5
14	10	10.5	0.5	1000	80	67.5	4	6	15	10	**14**	1	2	400	300	0.15	0.15	0.2	0.2	0	9	10	279.9	-48.0
15	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	**1.5**	2	400	300	0.15	0.15	0.2	0.2	0	9	10	533.2	-0.9
16	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	**0.5**	2	400	300	0.15	0.15	0.2	0.2	0	9	10	543.2	0.9
17	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	**3**	400	300	0.15	0.15	0.2	0.2	0	9	10	-105.6	-119.6
18	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	**1**	400	300	0.15	0.15	0.2	0.2	0	9	10	1182	119.6
19	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	**600**	300	0.15	0.15	0.2	0.2	0	9	10	213.7	-60.3
20	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	**150**	0.15	0.15	0.2	0.2	0	9	10	1664.3	209.2
21	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	300	**0.23**	0.15	0.2	0.2	0	9	10	538.2	0.0
22	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	300	**0.08**	0.15	0.2	0.2	0	13	21	-1148.4	-313.4
23	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	300	0.15	**0.23**	0.2	0.2	0	9	10	538.2	0.0
24	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	300	0.15	**0.08**	0.2	0.2	0	13	21	-1148.4	-313.4
25	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	**0.3**	0.2	0	9	10	538.2	0.0
26	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	**0.1**	0.2	2	21	34	-595.3	-210.6
27	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	**0.3**	0	9	10	538.2	0.0
28	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	**0.1**	2	21	34	-595.4	-210.6

It is observed that 50% increase in the standard deviation does not change the optimal solution, but the value of the objective function decreases 536.3%. Also, it is observed that 50% decrease in the standard deviation completely changes the optimal solution and the value of objective function increases 2115.1%.

As the lot size changes ±50% , the optimal objective function changes approximately ±50.5%. On the other hand, the lot size impacts on the objective function. Also, it is observed that ±50% change in the lot size does not change the optimal solution. Thus, optimal solution is not very sensitive to the changes in the lot size.

It is observed that 50% increase in the item price at primary market does not change the optimal solution, but the value of the objective function increases 1320.8%. Also, it is observed that 50% decrease in the item price at secondary market does not change d_1 and n but changes d_2 to 7. The value of the objective function decreases 5156.3% due to this change.

The rework cost has been changed by ±50%. The optimal solution and the value of objective function do not change due to this deviation in rework cost. Thus, the objective function and optimal solution are not sensitive to the changes in R.

It is observed that 50% increase in the production cost per item does not change the optimal solution but the value of the objective function decreases 5852.7%. Also, it is observed that 50% decrease in the production cost per item does not change the optimal solution and the value of objective function increases 5852.6%.

It is observed that 50% deviation in c_1, c_2 and c_3 has no effect on the optimal solution. The value of objective function changes slightly due to this change in c_2. The value of objective function decreases 41.5% due to 50% increase in c_1 and 48% due to 50% increase in c_3. Thus, the objective function and optimal solution are not sensitive to the changes in c_2.

It is observed that 50% deviation in inspection cost per item has no effect on the optimal solution and the value of objective function changes slightly due to this change. Thus, the objective function and optimal solution are not sensitive to the changes in inspection cost per item.

It is observed that 50% deviation in give-away cost per unit of excess material does not change the optimal solution and the value of objective function changes ∓119.6%. Thus, the optimal solution is not sensitive to the changes in give-away cost per unit of excess material.

It is observed that 50% increase in the quality loss coefficient in the primary market does not change the optimal solution but the value of the objective function 60.3% decreases. Also, it is observed that 50% decrease in the quality loss coefficient in the secondary market does not change the optimal solution but the value of objective function 209.2% increases. Thus, the optimal solution is not sensitive to the changes in the quality loss coefficients.

It is observed that 50% increase in Limiting Quality Level of the primary market has no effect on the optimal solution and objective function. Also, it is observed that d_2 and n increases but d_1 does not change due to 50% decrease in Limiting Quality Level of the primary market. The value of the objective function 313.4% decreases due to this change.

It is observed that 50% increase in Acceptable Quality Level of the secondary market has no effect on the optimal solution and objective function. Also, it is observed that d_2 and n increases but d_1 does not change due to 50% decrease in Acceptable Quality Level of the secondary market. The value of the objective function 313.4% decreases due to this change.

It is observed that 50% increase in probability of type-II error in making a decision of primary market has no effect on the optimal solution and objective function. Also, it is observed that 50% decrease in probability of type-II error in making a decision of the primary market completely changes the optimal solution and the value of objective function 210.6 % decreases.

It is observed that 50% increase in probability of type-I error in making a decision of the secondary market has no effect on the optimal solution and objective function. Also, it is observed that 50% decrease in probability of type-I error in making a decision of secondary market completely changes the optimal solution and the value of objective function 210.6 % decreases.

4. 3. Comparing with Rectifying Sampling Plan

Our aim is the economic design of a repetitive sampling plan, while products are sold in two different markets. So, we cannot compare the results with double sampling plan because the same procedure is not used in the double sampling plan. But, in Section 4.3, the performance of this repetitive sampling plan is compared with the rectifying sampling plan proposed by FallahNezhad and JafariNodoushan [12] with considering the same constraints. The results of the comparison study are shown in Table 6. From the results of this table, it is concluded that the rectifying sampling plan has better performance in comparison with the repetitive sampling plan in some cases.

4. 4. Processes with Unknown Mean

At this stage, the analysis was with known process mean of 10.5. Now, the process mean (T) is assumed to be unknown and the optimal process mean be found by using the optimization model.

TABLE 6. The results of the comparison study

Row	**Model Parameters**																			Optimal Objective Function of Repetitive SP	Optimal Objective Function of Rectifying SP
	LSL	**T**	σ	**N**	**a**	**r**	**R**	**c**	c_1	c_2	c_3	**I**	**g**	k_1	k_2	LQL_1	AQL_2	β_1	α_2		
1	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	**538.2**	**1523.7**
2	10	10.5	**0.25**	1000	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	**11922**	**1194.1**
3	10	10.5	0.5	**1500**	80	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	**809.9**	**2306.7**
4	10	10.5	0.5	1000	**120**	67.5	4	6	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	**7647.1**	**1241.1**
5	10	10.5	0.5	1000	80	67.5	4	**3**	15	10	12	1	2	400	300	0.15	0.15	0.2	0.2	**32038**	**33024**
6	10	10.5	0.5	1000	80	67.5	4	6	15	**5**	12	1	2	400	300	0.15	0.15	0.2	0.2	**546.2**	**1795.4**
7	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	**1.5**	2	400	300	0.15	0.15	0.2	0.2	**533.2**	**1352.4**
8	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	**0.5**	2	400	300	0.15	0.15	0.2	0.2	**543.2**	**1694.9**
9	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	**1**	400	300	0.15	0.15	0.2	0.2	**1182**	**2167.5**
10	10	10.5	0.5	1000	80	67.5	4	6	15	10	12	1	2	400	**150**	0.15	0.15	0.2	0.2	**1664.3**	**2517.8**

The optimal objective function for different values of process mean from 10.2 to 11.4 is presented in Table 7 and plotted in Figure 1. It is seen that the objective function is a concave function of process mean and the optimal process mean is 10.8.

At this point, the optimal objective function is 9064.3 and optimal solution of design parameters (d_1, d_2, n) is (11, 22, 97), respectively.

4. 5. Processes with Unknown Standard Deviation

So far, the analysis was with known process standard deviation of 0.5. In practical situations process standard deviation may be unknown. Now, the process standard deviation (σ) is assumed to be unknown and the optimal objective function and design parameters can be found by using the optimization model.

The optimal objective function for different values of process standard deviation from 0.1 to 0.7 is presented in Table 8 and plotted in Figure 2.

It can be seen that the objective function decreases when process standard deviation increases as we expected. Also, when standard deviation increases to 0.6, it is seen that the objective function becomes negative, thus production of the item will not be economical and must be stopped.

TABLE 7. Process mean and optimal objective function

T	$E(PN)$
10.2	-1550.6
10.3	-1066.3
10.4	-395.9
10.5	538.2
10.6	3095.3
10.7	7520
10.8	9064.3
10.9	8715.2
11	8221.4
11.1	7644.4
11.2	7009.7
11.3	6334.5
11.4	5629.8

4. 6. Considering Inspection Errors

The probability that a produced item is categorized as non-

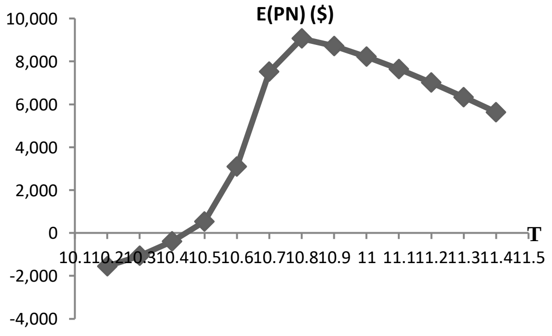

Figure 1. The optimal objective function for different values of process mean

TABLE 8. Process standard deviation and optimal objective function

σ	$E(PN)$
0.1	12361.0
0.2	12233.0
0.3	11485.0
0.4	6875.9
0.5	538.2
0.6	-959.5
0.7	-1957.5

conforming under the presence of inspection errors (observed probability of unconformity for one item) is given by:

$$P_e = P * (1 - e_2) + (1 - P) * e_1 \tag{26}$$

where e_1 is the probability of Type I error and e_2 is the probability of Type II error. It is worth noting that the apparent probability of unconformity for one item is affected by the inspection errors.

Thus, the probability of selling the lot in the primary and secondary markets and reworking are as following, respectively:

$$P(D_e \leq d_1) = \sum_{i=0}^{d_1} \binom{n}{i} {P_e}^i (1 - P_e)^{n-i} \tag{27}$$

$$P(d_1 < D_e \leq d_2) = \sum_{i=d_1+1}^{d_2} \binom{n}{i} {P_e}^i (1 - P_e)^{n-i} \tag{28}$$

$$P(D_e > d_2) = \sum_{i=d_{2+1}}^{n} \binom{n}{i} {P_e}^i (1 - P_e)^{n-i} = 1 - \sum_{i=0}^{d_2} \binom{n}{i} {P_e}^i (1 - P_e)^{n-i} \tag{29}$$

Under the presence of inspection errors, the expected profit per lot is given by:

$$PN = \begin{cases} aN - g(y' - LSL)N - In - cyN - (N-n)L_{01} - nL_{11} - (N-n)Pc_1 \\ \quad -n(1-P)e_1c_4 - nP(1-e_2)c_2 - nPe_2c_1 \qquad if\ P(D_e \leq d_1) \\ rN - g(y' - LSL)N - In - cyN - (N-n)L_{02} - nL_{12} - (N-n)Pc_3 \\ -n(1-P)e_1c_4 - nP(1-e_2)c_2 - nPe_2c_3 \qquad if\ P(d_1 < D_e \leq d_2) \\ \quad E(PN) - RN - In - cyN \qquad if\ P(D_e > d_2) \end{cases} \tag{30}$$

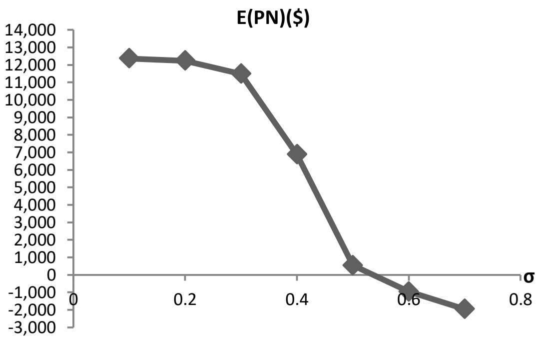

Figure 2. The optimal objective function for different amounts of process standard deviation

The new terms of objective function are explained in Table 9:

Now, the expected profit per lot is given by:

$$E(PN) = \frac{1}{P\ (D_e \leq d_2)} [aN\ .P(D_e \leq d_1) - g(y' - LSL)N\ .P(D_e \leq d_1) - In - cTN \quad -(N - n)k_1 \int_{-\infty}^{\infty} \frac{1}{y^2}\ f(y)\ dy . P(D_e \leq d_1) -n\ \frac{k_1 \int_{LSL}^{\infty} \frac{1}{y^2} f(y)\ dy}{\int_{LSL}^{\infty} f(y)\ dy}\ .P(D_e \leq d_1) - (N - n)Pc_1 . P(D_e \leq d_1) - n(1-P)e_1c_4\ .P(D_e \leq d_1) - nP(1 - e_2)c_2 . P(D_e \leq d_1) + rN\ .P(d_1 < D_e \leq\ d_2) - g(y' - LSL)N\ .P(d_1 < D_e \leq\ d_2) - (N - n)k_2 \int_{-\infty}^{\infty} \frac{1}{y^2} f(y)\ dy\ .P(d_1 < D_e \leq\ d_2) -n \frac{k_2 \int_{LSL}^{\infty} \frac{1}{y^2} f(y)\ dy}{\int_{LSL}^{\infty} f(y)\ dy}\ .P(d_1 < D_e \leq\ d_2) \qquad -(N - n)Pc_3\ .P(d_1 < D_e \leq\ d_2) -n(1-P)e_1c_4 . P(d_1 < D_e \leq\ d_2) \quad -nP(1 - e_2)c_2 . P(d_1 < D_e \leq\ d_2) - nPe_2c_3 . P(d_1 < D_e \leq\ d_2) - RN\ .P(D_e > d_2) \tag{31}$$

With considering inspection errors, risk constraints are as following:

$$P = P_1 = LQL_1 \ \ ; \ \ \frac{P_{13}}{1 - P_{11}} = \frac{P\ (D_e \leq d_1)}{1 - P\ (D_e > d_2)} \leq \beta_1 \tag{32}$$

$$P = P_2 = AQL_2 \ \ ; \ \ \frac{P_{12}}{1 - P_{11}} = \frac{P\ (d_1 < D_e \leq d_2)}{1 - P\ (D_e > d_2)} \geq 1 - \alpha_2 \tag{33}$$

The optimal objective function and design parameters for different pairs of inspection errors are presented in Table 10. It can be seen that with $(e_1, e_2) = (0.05, 0.05)$, the value of optimal objective function is 563.4.

If we increase e_2 to 0.10, the optimal objective function decreases to 506.5 but if we increase e_1 to 0.10, the optimal objective function decreases to 491.7.

TABLE 9. The new terms of objective function

$nP(1 - e_2)c_2$	The cost of replacing non-conforming items identified in a sample with conforming items
$n(1 - P)e_1c_4$	The cost of replacing rejected conforming items in a sample
nPe_2c_1	The cost of non-conforming items sent to the customer in the primary market
nPe_2c_3	The cost of non-conforming items sent to the customer in the secondary market

TABLE 10. Impact of inspection errors on design parameters and optimal objective function

Senario	(e_1, e_2)	**Design Parameters**			**Optimal Objective Function**	**Percent Change of Objective Function**
		d_1	d_2	n		
1	(0.02,0.05)	1	17	18	448.6	-16.6
2	(0.02,0.10)	0	9	10	515.8	-4.2
3	(0.05,0.05)	0	7	8	563.4	4.7
4	(0.05,0.10)	1	15	16	506.5	-5.9
5	(0.10.0.05)	3	22	23	491.7	-8.7

Thus, increase in inspection error of type I leads to more reduction in the objective function.

5. CONCLUSION

In this paper, we have developed an optimization model for the economic design of repetitive group sampling plan while its optimality is resulted by using the expected profit of each decision based on the Taguchi's Loss Function and subject to risk constraints in target markets. In this approach, the required probabilities of each decision were determined employing the Binomial Distribution. The product quality was controlled using a lot-by-lot sampling plan. The application of the model had been demonstrated using a numerical example. Sensitivity analysis has been conducted to assess the impact of changes in model parameters on the optimal value of objective function and design parameters. It was observed that with $\pm 50\%$ change in the standard deviation, the expected profit per lot changed dramatically, e.g. due to 50% decrease in σ, the expected profit per lot decreased 536.3%. It could be seen massive change in the expected profit per lot due to $\pm 50\%$ change in production cost per unit of quality characteristic (c). However, the expected profit per lot changed slightly due to $\pm 50\%$ change in cost parameters of c_1, c_2, c_3. There had been 119.6% change in the expected profit per lot due to $\pm 50\%$ change in the give-away cost per unit of excess material (g).

Also, we have performed an analysis for the processes with unknown mean and the importance of combining the sampling plan and optimal process adjustment was investigated. The effects of inspection errors are also investigated.

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Designing an Economic Repetitive Sampling Plan in the Presence of Two Markets

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چکیده

در این مقاله، ما به توسعه یک مدل بهینه سازی برای طراحی اقتصادی طرح نمونه گیری تکراری در حضور دو بازار می‌پردازیم. فرایند مورد نظر ما یک محصول یک مشخصه کیفی دارای توزیع نرمال با میانگین نامعلوم و واریانس معلوم تولید می کند. مشخصه کیفی دارای حد مشخصه فنی پایین است. کیفیت محصول از طریق طرح نمونه گیری پذیرشی دسته به دسته، کنترل می–شود. تابع هدف مورد استفاده در مدل، حداکثر کردن سود و انطباق محصول با استفاده از تابع زیان تاگوچی به عنوان یک جانشین برای انطباق محصول می‌باشد. ریسک تولید کننده و مصرف کننده در دو بازار مختلف به عنوان محدودیت در نظر گرفته شده‌اند. کاربرد مدل را با استفاده از یک مثال عددی نشان می‌دهیم. تحلیل حساسیت بر روی پارامترهای مدل، نشان میدهد که نتیجه مدل نسبت به تغییرات در پارامترهای مدل حساس است.

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