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Bertrand-nash Equilibrium in the Retail Duopoly Model under Asymmetric Costs

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ABSTRACT

In this paper, the Bertrand's price competition in the retail duopoly with asymmetric costs is analyzed. Retailers sell substitute products in the framework of the classical economic order quantity (EOQ) model with linear demand function. The market potential and competitor price are considered to be the bifurcation parameters of retailers. Levels of the barriers to market penetration depending on the bifurcation parameters are analyzed. The conditions of Bertrand-Nash equilibrium in parametric and trigonometric forms are found.

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1. INTRODUCTION

Integrating operations and marketing decisions are an important objective for retailers in today's competitive environment. An obvious problem common to the retail industry is the joint optimization of the lot size of a product to be stocked and the selling price in order to maximize profit. Pricing and inventory control strategies are closely related to each other. Pricing decisions alter demand forecasts, which are used by inventory control systems.

We note that in the classical economic order quantity model, the demand is assumed as constant. A lot of important results have been obtained in this field of study. There are many studies which modify this condition [1], in particular with linear demand function [2, 3]. Further studies are related to the complexity of market structures, in particular, considering the horizontal competition between retailers [4].

In this article, we developed the obtained results from literature [4] for model duopoly retailers with the Bertrand's price competition in the case of substitute products. Along with trigonometric solution, parametric solution is received as well, where the parameter is return on logistics costs. Parametric solution allows to determine the sufficient conditions for the existence of equilibrium with the asymmetry retailers cost. The market potential and price of competitor are considered to be bifurcation parameters of retailers. Depending on the values of the bifurcation parameters, barriers to entry for retailers are analyzed.

This article is organized as follows. Section 2 reviews the related literature. Section 3 describes the problem statement. In section 4, we analyze barriers to entry depending on the bifurcation parameters. In section 5, we present the analytical solutions of the problem in parametric and trigonometric forms. Section 6 presents the numerical example and sensitivity analysis. Finally, section 7 summarizes the results.

2. LITERATURE REVIEW

Whitin [5] was the first researcher who indicated the fundamental connection between price theory and inventory control. He extended the basic EOQ model by considering the selling price in addition to the order quantity as the decision variables.

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Optimization solutions for pricing and inventory management for the retailer's monopoly are represented in literature [2, 6-12].

The paper of Abad [2] is concerned with finding the optimal price and lot size for a retailer purchasing a product for which the supplier offers all-unit quantity discounts. Demand for the product is assumed to be a decreasing function of price, and a procedure is developed for finding the optimal price and lot size for a class of demand functions. Thomas [6] considers the problem of simultaneously making price and production decisions in dynamic for a single product with a known deterministic demand function. To maximise profit, an efficient algorithm is developed.

Kunreuther and Richard [7] have investigated the interrelationship between the pricing and inventory decisions for a retailer who orders his goods from an outside distributor. Smith et al. [8] have formulated and solved a single-item joint pricing and master planning optimization problem with capacity and inventory constrains.

Tripathi [9] develops an inventory model for deteriorating items with linearly time dependent demand rate under inflation and time discounting over a finite planning horizon.

Parsa et al. [10] have proposed a new model for two-echelon single-manufacturer multi-retailer supply chain under non-consignment Vendor Managed Inventory program by considering time value of money.

Moubed and Mehrjerdi [11] analyzed vendor managed inventory as a collaborative model for reverse supply chains and the optimization problem was modelled in terms of the inventory routing problem. Zareia et al. [12], have explored the issues of the integration of routing, the economic selection of customers aiming at minimizing the transportation, maintenance, discount costs and maximizing the products selling profits.

Study on retailers' competition in the vertical market conditions is presented in [13-18].

Sinha and Sarmah [13] have analyzed the coordination and competition issues in a two-stage supply-chain distribution system where two vendors compete to sell differentiated products through a common retailer in the same market. Huang et al. [14] analyzed the coordination of enterprise decisions such as supplier and component selection, pricing and inventory in a three-level supply chain composed of multiple suppliers, a single manufacturer and multiple retailers. The problem is modelled as a dynamic non-cooperative game.

Feng and Lu [15] have analyzed the contracting behaviors in a two-tier supply chain system consisting of two competing manufacturers selling to two competing retailers. Alaei et al. [16] have analyzed production – inventory decisions in a decentralized supply chain. A production inventory problem is considered in a two-level supply chain. Modak et al. [17] have explored channel coordination and profit distribution in a two-layer socially responsible supply chain that consists of a manufacturer and two competitive retailers.

Study on retailers' competition in the horizontal market conditions is presented [4, 19, 20].

Otake and Min [4] have analyzed inventory and pricing policies for a duopoly of substitute products. Min [19] has extended the profit maximizing EOQ model to the case of a symmetric oligopoly consisting of sellers of a homogeneous product who complete with each other for the same potential buyers. Sadjadi and Bayati [20] developed generalized network data environment analysis models to examine the efficiency of two-tier suppliers under cooperative and non-cooperative strategies where each tier has its own inputs/outputs and some outputs of the first tier can be fed back to the second tier.

3. PROBLEM STATEMENT

Two retailers periodically buy the finished product from a wholesaler. Products are stored in a warehouse and are evenly sold in the retail network. We assume that every retailer maximizes profit per unit of time on the order size and the price at a current price competitor.

Linear demand function of i-th retailer is presented in Equation (1):

$$D_i = b - k \cdot p_i + \gamma \cdot (p_j - p_i) \tag{1}$$

where

D_i– product demand of i-th retailer per unit time, i=1, 2,

p_i – price of product of i-th retailer per unit,

p_j – unit price of i-th retailer's competitor product, j=3–i,

b – market potential (maximal demand) per unit time,

k > 0 – the own price effect,

$\gamma > 0$ – the cross price effect.

From condition $0<D_i<b$ we obtain the range of acceptable prices: $p_i<(b+\gamma\ p_j)/(k+\gamma)$, $p_i<p_j(k+\gamma)/\gamma$ (Figure 1).

Variables and parameters should be defined as follows:

Q_i – the order size of i-th retailer,

d_i – the ordering cost of i-th retailer,

w_i – the variable cost per unit time of i-th retailer,

l_i – the holding cost per unit per unit time of i-th retailer.

The basic assumptions for traditional EOQ model applied in this article are the following:

– buyer's demand does not change,

– unlimited supply volume,

– no shortage,

– instant delivery.

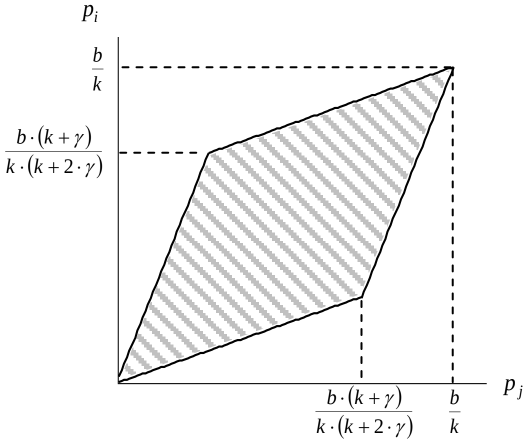

Figure 1. The range of acceptable prices

With reference to the above mentioned, the price competition between retailers on the Bertrand model is analyzed. The objective function of i-th retailer is shown in Equation (2):

$$F_i = GP_i - LC_i = \\ = (p_i - w_i)\cdot D_i - \frac{d_i \cdot D_i}{Q_i} - \frac{Q_i \cdot l_i}{2} \to \max_{p_i, Q_i} \tag{2}$$

where, $GP_i=(p_i-w_i)\cdot D_i$ – gross profit per unit time, $LC_i=d_i\cdot D_i/Q_i+Q_i\cdot l_i/2$ – logistics costs per unit time.
The first-order necessary conditions are depicted in Equations (3)-(6):

$$\frac{\partial F_i}{\partial p_i} = b + \gamma\cdot p_j + w_i\cdot(k+\gamma) + \\ + \frac{d_i\cdot(k+\gamma)}{Q_i} - 2\cdot p_i\cdot(k+\gamma) = 0, \tag{3}$$

$$\frac{\partial F_i}{\partial Q_i} = \frac{d_i\cdot D_i}{Q_i^2} - \frac{l_i}{2} = 0, \tag{4}$$

or

$$p_i = w_i + \frac{1}{2}\cdot\left[\frac{b+\gamma\cdot p_j}{k+\gamma} + \frac{d_i}{Q_i} - w_i\right], \tag{5}$$

$$Q_i^2 = \frac{2\cdot d_i\cdot(b - k\cdot p_i + \gamma\cdot(p_j - p_i))}{l_i}. \tag{6}$$

Substituting (6) into (5), we obtain the reaction curves of i-th retailer, $R_i(p_j)$ in implicit form is shown in Equation (7):

$$p_i = w_i + \frac{1}{2}\cdot\left[\frac{b+\gamma\cdot p_j}{k+\gamma} \pm \sqrt{\frac{d_i\cdot l_i}{2\cdot(b - k\cdot p_i + \gamma\cdot(p_j - p_i))}} - w_i\right] \tag{7}$$

To find the stationary points, we substitute (5) into (6) and equate to zero; so, Equation (8) is obtained:

$$Q_i^3 - \frac{d_i\cdot(b + \gamma\cdot p_j - w_i\cdot(k+\gamma))}{l_i}\cdot Q_i + \frac{d_i^2\cdot(k+\gamma)}{l_i} = 0 \tag{8}$$

We have obtained a cubic equation in reduced form. Thus, depending on the parameter values, the number of real stationary points can be 1, 2 or 3.

4. THE ANALYSIS OF BARRIERS TO ENTRY

It should be noted that the key parameter for retailers is a market potential. It is interesting to analyze the dependence of the number of real stationary points of the market potential. For this, the market potential can be presented in Equation (8) as (9):

$$b(Q_i) = \frac{l_i\cdot Q_i^2}{d_i} + \frac{d_i\cdot(k+\gamma)}{Q_i} + (k+\gamma)\cdot w_i - \gamma\cdot p_j \tag{9}$$

The graph of the function $b(Q_i)$ is shown in Figure 2. Figure 2 depicts that the number of real roots of the cubic Equation (8) depends on the level of market potential. When $b < b_i^{bif}$ the Equation (8) has one real negative root q_3, when $b = b_i^{bif}$ – one real negative root q_3 and one real positive two-fold root q_i^{bif}, when $b>b_i^{bif}$ – three real roots ($q_1 > q_2 > q_3$). Thus, Figure 2 shows an imperfect pitchfork bifurcation, where the level of market potential is a bifurcation point [21]. Figure 2 also shows that the positive order size exists only when $b \geq b_i^{bif}$, so the bifurcation point can be considered as a barrier to entry for retailers.

It is easy to notice that the function $b(Q_i)$ at $Q_i = q_i^{bif}$ has a local minimum. From the first order conditions (10)

$$\frac{db}{dQ_i} = \frac{2\cdot l_i\cdot Q_i}{d_i} - \frac{d_i\cdot(k+\gamma)}{Q_i^2} = 0 \tag{10}$$

we find

$$Q_i^{bif} = \sqrt[3]{\frac{d_i^2\cdot(k+\gamma)}{2\cdot l_i}} \tag{11}$$

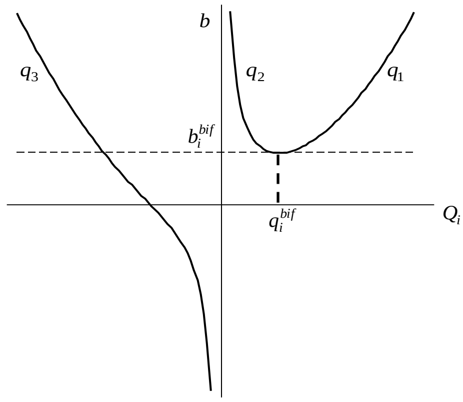

Figure 2. Location of the real roots of Equation (8) depending on the bifurcation parameter ("imperfect pitchfork bifurcation" [21])

From (11), it follows that the bifurcation value of order size of i-th retailer is invariant with respect to competitor price. Substituting (11) into (9), we find the bifurcation value of market potential of i-th retailer:

$$b_i^{bif} = w_i \cdot (k+\gamma) + \sqrt[3]{\frac{27 \cdot d_i \cdot l_i \cdot (k+\gamma)^2}{4}} - \gamma \cdot p_j \tag{12}$$

The formula (12) shows an inverse relationship between the value of barrier to entry for the i-th retailer and the price of competitor. Thus, to displace a competitor from the market, the retailers can use dumping – reduction of own price to increase the value of the competitor bifurcation point above actual value.

With the help of the interval of existence of competitor's price, $p_j \in (0; b/k)$, it is possible to identify other values of the market potential that are important for the i-th retailer. Substituting lower bound into the formula (12), $p_j = 0$, we find the value of market potential, where the entry into the market for the i-th retailer does not depend on the price of competitor as Equation (13):

$$b_i^{free\ entry} = w_i \cdot (k+\gamma) + \sqrt[3]{\frac{27 \cdot d_i \cdot l_i \cdot (k+\gamma)^2}{4}} \tag{13}$$

Substituting upper bound into the formula (12), $p_j = b/k$, we find the value of market potential, where the entry into the market for i-th retailer is blocked as Equation (14):

$$b_i^{no\ entry} = k \cdot \left(w_i + \sqrt[3]{\frac{27 \cdot d_i \cdot l_i}{4 \cdot (k+\gamma)}} \right) \tag{14}$$

Thus, depending on the market potential for the i-th retailer the following situations are possible (according to the Bain's classification [22]): $b \le b_i^{no\ entry}$ – entry is blocked, competitor monopolizes the market; $b_i^{no\ entry} < b < b_i^{free\ entry}$ – entry is effectively impeded by pricing of competitor, "dumping area"; $b \ge b_i^{free\ entry}$ – free entry.

5. THE ANALYTICAL SOLUTION

5. 1. The Parametric Form Using (b_i^{bif}, Q_i^{bif}), we define stationary points of i-th retailer in parametric form. Consider the functions $b_i^h = w_i \cdot (k+\gamma) + \sqrt[3]{h_i^b \cdot d_i \cdot l_i \cdot (k+\gamma)^2} - \gamma \cdot p_j$ and $Q_i^h = \sqrt[3]{h_i^Q \cdot d_i^2 \cdot (k+\gamma)/l_i}$, where b_i^h, Q_i^h – parametric representation of the roots of the Equation (8), h_i^b, h_i^Q – yet unknown parameters, $h_i^b = f(h_i^Q)$. Substituting b_i^h and Q_i^h into Equation (8), determine the relationship between the parameters: $h_i^b = (h_i^Q+1)^3/h_i^Q$. Let $h_i^Q = h_i$, then $h_i^b = (h_i+1)^3/h_i$, $h_i \neq 0$.

Thus, the real roots of the Equation (8) in the parametric form is obtained as Equation (15):

$$\begin{aligned} b_i^h &= w_i \cdot (k+\gamma) + (h_i+1) \cdot \sqrt[3]{\frac{d_i \cdot l_i \cdot (k+\gamma)^2}{h_i}} - \gamma \cdot p_j, \\ Q_i^h &= \sqrt[3]{\frac{h_i \cdot d_i^2 \cdot (k+\gamma)}{l_i}} \end{aligned} \tag{15}$$

When the $h_i<0$, we obtain the root q_3, when $0<h_i<0.5$ – the root q_2, when the $h_i=0.5$ – bifurcation point q_i^{bif}, when $h_i>0.5$ – the root q_1 (Figure 2).

Substituting (15) into (5), we determine the price through the Equation (16)

$$p_i^h = w_i + (h_i+2) \cdot \sqrt[3]{\frac{d_i \cdot l_i}{8 \cdot h_i \cdot (k+\gamma)}} \tag{16}$$

We can give an economic interpretation to the h_i -parameter. The h_i -parameter is expressed through a relative indicator of economic efficiency – return on logistics costs, which is given by: $r_i=(GP_i-LC_i)/LC_i$.

Substitute the parametric solution (15)-(16) into the original profit Equation (17):

$$\begin{aligned} F_i^h &= GP_i - LC_i = \frac{h_i+2}{4} \cdot \sqrt[3]{h_i \cdot d_i^2 \cdot l_i^2 \cdot (k+\gamma)} - \\ &- \sqrt[3]{h_i \cdot d_i^2 \cdot l_i^2 \cdot (k+\gamma)}. \end{aligned} \tag{17}$$

From Equation (17), we determine the return on logistics costs: $r_i=(h_i-2)/4$, where the $h_i=4 \cdot r_i+2$.

Thus, the solution (15)-(16), where the parameter is return on logistics costs, is expressed as Equation (18):

$$\begin{aligned} b_i^r &= w_i \cdot (k+\gamma) + (4 \cdot r_i + 3) \cdot \sqrt[3]{\frac{d_i \cdot l_i \cdot (k+\gamma)^2}{4 \cdot r_i + 2}} - \gamma \cdot p_j \\ Q_i^r &= \sqrt[3]{\frac{d_i^2 \cdot (4 \cdot r_i + 2) \cdot (k+\gamma)}{l_i}} \\ p_i^r &= w_i + 2 \cdot (r_i+1) \cdot \sqrt[3]{\frac{d_i \cdot l_i}{(4 \cdot r_i + 2) \cdot (k+\gamma)}} \end{aligned} \tag{18}$$

At the bifurcation point (h_i=0,5) the return on logistics costs is equal: r_i^{bif}=-37,5%. When h_i>0,5, return on logistics costs will increase, so the retailers are interested only in the root q_1.

Now, it is necessary to analyze the extrema of the Function (2). For this, we define the Hessian matrix as (19):

$$\begin{aligned} H_i &= \begin{bmatrix} \dfrac{\partial^2 F_i}{\partial p_i^2} & \dfrac{\partial^2 F_i}{\partial p_i \partial Q_i} \\ \dfrac{\partial^2 F_i}{\partial Q_i \partial p_i} & \dfrac{\partial^2 F_i}{\partial Q_i^2} \end{bmatrix} = \\ &= \begin{bmatrix} -2 \cdot (k+\gamma) & -\dfrac{d_i \cdot (k+\gamma)}{Q_i^2} \\ -\dfrac{d_i \cdot (k+\gamma)}{Q_i^2} & -\dfrac{2 \cdot d_i \cdot (b - k \cdot p_i + \gamma \cdot (p_j - p_i))}{Q_i^3} \end{bmatrix} \end{aligned} \tag{19}$$

As it is known, the type of extrema of the function depends on the character of definiteness of the Hessian matrix evaluated at the stationary points. The character of definiteness of the Hessian matrix (19) depends on stationary points in Equation (18) and may be different. We are interested in the dependence of the indicator r_i. Since we need the maximum, we will find conditions under which the Hessian matrix is negative definite.

According to Sylvester's criterion the matrix (19) is negative definite when $|H_i| > 0$ or

$$4 \cdot Q_i \cdot \left(b - k \cdot p_i + \gamma \cdot \left(p_j - p_i\right)\right) - d_i \cdot (k + \gamma) > 0 \tag{20}$$

Substituting the solution (18) into the condition (20), we obtain a sufficient condition for a maximum: $r_i > -37{,}5\%$. Thus, only root q_1 is the point of maximum of the Function (2), $Q_i^*=q_1$.

At the given price of competitor, the parametric solution (18) is an optimal solution of i-th retailer when return of logistics costs r_i^* ensures the equality of potentials: $b_i^r(r_i^*, p_j) = b$.

The Bertrand-Nash equilibrium between retailers will be achieved at r_i^e and r_j^e ensuring equality of the potential: $b_i^r(r_i^e, p_j^r(r_j^e)) = b_j^r(r_j^e, p_i^r(r_i^e)) = b$ and at the point of intersection of the reaction curves: $R_i(p_j^r(r_j^e)) = R_j(p_i^r(r_i^e))$.

5. 2. The Trigonometric Form Let us find the roots of the cubic Equation (8) explicitly. Since the discriminant of the cubic Equation (8) for $b>b_i^{bif}$ is negative, we will seek the trigonometric solution.

Introduce the Function (21)

$$\varphi_i = \arccos\left(-\sqrt{\left(\frac{b_i^{bif} + \gamma \cdot p_j - w_i \cdot (k+\gamma)}{b + \gamma \cdot p_j - w_i \cdot (k+\gamma)}\right)^3}\right) \tag{21}$$

Then the roots of the cubic Equation (5) are equal:

$$\begin{aligned}
z_{i1} &= 2 \cdot \sqrt{\frac{d_i \cdot \left(b + \gamma \cdot p_j - w_i \cdot (k+\gamma)\right)}{3 \cdot l_i}} \cdot \cos\frac{\varphi_i}{3} \\
z_{i2} &= 2 \cdot \sqrt{\frac{d_i \cdot \left(b + \gamma \cdot p_j - w_i \cdot (k+\gamma)\right)}{3 \cdot l_i}} \cdot \cos\frac{\varphi_i - 2\cdot\pi}{3} \\
z_{i3} &= 2 \cdot \sqrt{\frac{d_i \cdot \left(b + \gamma \cdot p_j - w_i \cdot (k+\gamma)\right)}{3 \cdot l_i}} \cdot \cos\frac{\varphi_i + 2\cdot\pi}{3}
\end{aligned} \tag{22}$$

In the analysis of the sufficient conditions it has been found that the optimum order size belongs to the right-hand branch of the function $b(Q_i)$ (Figure 2): $Q_i^*=q_1=\max\{z_{i1},z_{i2},z_{i3}\}$. Compare the roots of (22) with each other. For the $b>b_i^{bif}$ the range of Function (21) is equal: $\varphi_i\in(\pi/2;\pi)$. For $\varphi_i\in(\pi/2;\pi)$ the inequality $\cos(\varphi_i/3)>\cos((\varphi_i-2\cdot\pi)/3)>\cos((\varphi_i+2\cdot\pi)/3)$, therefore $q_1=z_{i1}$, $q_2=z_{i2}$, $q_3=z_{i3}$. The optimal order size: $Q_i^*=z_{i1}$.

The obtained trigonometric solution (22) allows us to represent an implicit Function (7) in the explicit form and build reaction curves of retailers. To express the Function (7) explicitly, we substitute formulas of roots (22) into (5). The graph of Function (7) for $b_i^{no\ entry}<b<b_i^{free\ entry}$ is presented in Figure 3.

Note that Figure 3 shows an imperfect pitchfork bifurcation, where the bifurcation parameter is the competitor's price. Thus, in the "dumping area" ($b_i^{no\ entry}<b<b_i^{free\ entry}$) the price of competitor is the second bifurcation parameter for retailers. Entry into the market for the i-th retailer is possible only at $p_j^{bif}<p_j<b/k$, where p_j^{bif} is determined from Equation (12):

$$p_j^{bif} = max\left\{0;\ \frac{w_i \cdot (k+\gamma)}{\gamma} + \sqrt[3]{\frac{27 \cdot d_i \cdot l_i \cdot (k+\gamma)^2}{4 \cdot \gamma^3}} - b\right\} \tag{23}$$

Reaction curve of the i-th retailer corresponds to the average branch of the function (7): $R_i(p_j)=p_i^*(q_1(p_j))$.

Reaction curves of the i-th retailer for different values of market potential are presented in Figure 4.

From Figure 4 we can see that with the growth of the market potential, the reaction curves are shifted to the left and up.

6. NUMERICAL EXAMPLE AND SENSITIVITY ANALISIS

We illustrate the obtained results on the numerical example, using data from [4]. Data are presented in Table 1.

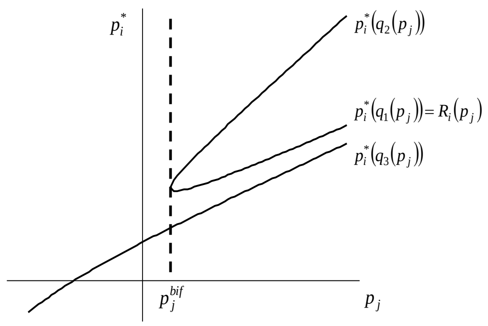

Figure 3. The graph of implicit function (7) for $b_i^{no\ entry}<b<b_i^{free\ entry}$

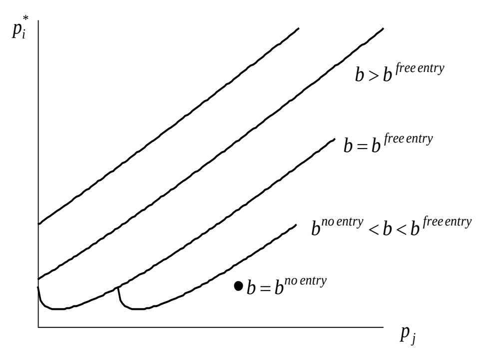

Figure 4. The reaction curves of the i-th retailer

TABLE 1. Initial numerical example data

b	k	γ	w_i	w_j	l_i	l_j	d_i	d_j
100	1,0	0,5	20	15	4,0	3,0	1000	750

The levels of barriers to entry are shown in Table 2. From Table 2, we see that due to the cost advantages j-th retailer has lower barrier to entry. When b<36,6, retailers are not able enter to the market, when 36,6≤b<46,2, the j-th retailer is a monopolist, when b≥46,2 there is a duopoly market. Due to the high value of the market potential, retailers can not use dumping: $p_j^{bif}=p_i^{bif}=0$.

The equilibrium variables are presented in Table 3. Profit function of the i-th retailer at $p_j = p_j^e$ is shown in Figure 5.

Now, we study the impact of changes in the values of the key parameters b, k, γ, w_i, d_i, l_i on the equilibrium profit of the i-th retailer. We change one parameter at a time, keeping the other parameters unchanged. The results are summarized in Table 4 (Figure 6).

TABLE 2. Levels of barriers to entry

$b_i^{no\ entry}$	$b_j^{no\ entry}$	$b_i^{free\ entry}$	$b_j^{free\ entry}$	p_j^{bif}	p_i^{bif}
46,2	36,6	69,3	55,0	0	0

TABLE 3. Equilibrium variables

p_i^e	p_j^e	Q_i^e	Q_j^e	F_i^e	F_j^e	r_i^e	r_j^e
55,5	52,5	147	157	940	1368	1,6	2,9

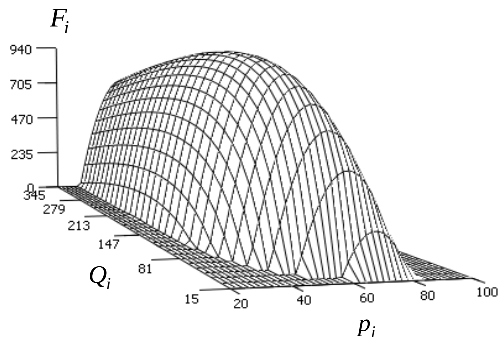

Figure 5. Profit function of the i-th retailer

TABLE 4. The impact of changes in the values of the key parameters on the equilibrium profit of the i-th retailer

changes %	b	k	γ	w_i	d_i	l_i
– 25	248	1627	972	1158	1017	1017
– 20	360	1456	966	1113	1001	1001
– 15	485	1305	959	1068	985	985
– 10	624	1170	953	1025	969	969
– 5	775	1049	946	982	954	954
+ 5	1117	841	933	898	925	925
+ 10	1306	752	926	858	911	911
+ 15	1508	671	920	818	898	898
+ 20	1723	597	913	779	885	885
+ 25	1950	530	906	740	872	872

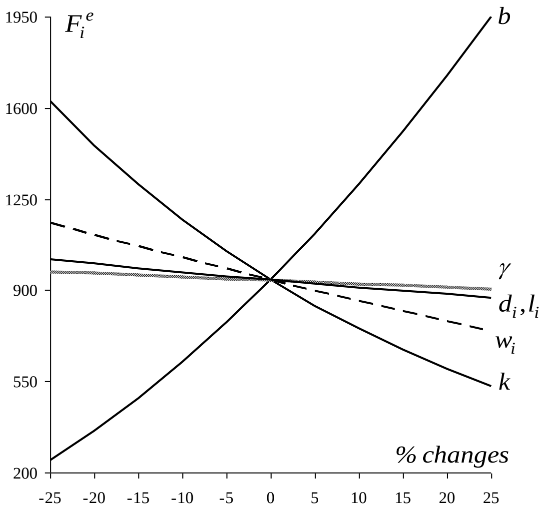

Figure 6. Sensitivity analysis of equilibrium profit of the i-th retailer

Based on the results of Table 4, the following observation can be made:

1. A higher value of market potential b results in higher values of equilibrium profit of the i-th retailer. Additionally, we find that equilibrium profit of the i-th retailer is highly sensitive to changes in b.
2. A higher value of other parameters k, γ, w_i, d_i, l_i results in lower values of equilibrium profit of the i-th retailer. Additionally, we find that equilibrium profit of the i-th retailer is highly sensitive to changes in k.

Also, we have got that ordering and holding costs equally influence on the equilibrium profit of the i-th retailer.

7. CONCLUSIONS

In this article, we have analyzed the Bertrand's price competition in retail duopoly. The market potential and price of competitor are considered to be the bifurcation parameters of retailers. The values of the market potential at which dumping from the competitor's side is possible are defined. The necessary and sufficient conditions for the existence of extremum are analysed.

The optimal solution in parametric form, where the parameter is the return on logistics costs, is found. The solution is determined in explicit form, taking into account the bifurcation point. The theoretical results are illustrated by a numerical example. Based on the results obtained, retailers can plan the level of profitability of logistics costs and assess the level of the entry barrier to the market.

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Bertrand-nash Equilibrium in the Retail Duopoly Model under Asymmetric Costs

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چکیده

در این مقاله، رقابت قیمت برتراند در انحصار خرده فروشی با هزینه های نامتقارن مورد تحلیل قرار گرفت. خرده فروشان محصولات جایگزین را در چارچوب مدل EOQ کلاسیک با تابع تقاضای خطی می فروشند. پتانسیل بازار و قیمت رقیب پارامترهای انشعاب از خرده فروشان در نظر گرفته می شود. سطوح موانع نفوذ در بازار بسته به پارامترهای انشعاب تجزیه و تحلیل می شوند. شرایط تعادل برتراند-نش در اشکال پارامتری و مثلثاتی یافت می شوند.

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