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Monitoring Financial Processes with ARMA-GARCH Model Based on Shewhart Control Chart (Case Study: Tehran Stock Exchange)

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ABSTRACT

Financial surveillance is an interesting area after financial crisis in recent years. In this subject, important financial indices are monitored using control charts. Control chart is a powerful instrument for detecting assignable causes which is considerably developed in industrial and service environments. In this paper, a monitoring procedure based on Shewhart control chart is proposed to monitor financial processes modeled with ARMA-GARCH time series structure. The effect of parameter estimation and power of change detection are evaluated through simulation studies. Then, the proposed method is applied to monitor Tehran Stock Exchange price index (TEPIX) as the main motivation of this research. The obtained results show the ability of the proposed method in comparison with market analysis.

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1. INTRODUCTION

In many financial researches, developing methods for decision support systems (DSSs) is a target research [1]. Most of DSSs are based on the modeling of financial trends. For example, trading strategies in stock market are commonly based on price trends. Meanwhile, making in time decision is too important. There are enormous works which try to improve decision making mechanism in finance [2]. Statistical analysis is a basic approach and there are many references which describe statistical methods in modeling financial processes [3, 4].

In recent years, the application of control charts for monitoring financial processes is attracted many considerations. Frisen [5] stated the general ideas of financial surveillance. In this subject, the main procedure starts with understanding the properties of financial process and models it using time series analysis. Then, process index is monitored by using control chart to detect assignable causes. Finally, decision time arrives when control chart alarms. Researches on this subject could be classified based on the applications [6-8].

Control chart is one of the statistical process control (SPC) tools [9]. It is initially developed for monitoring industrial processes [10]. Afterwards, the application of control chart is extended to service environment, healthcare and finance [11]. The most appropriate time series model to monitor autocorrelated processes in service and industrial areas is autoregressive integrated moving average (ARIMA) model [12, 13]. Since volatility is more interesting in finance, most of the researches in this area are focused on the generalized autoregressive conditional heteroskedasticity (GARCH) model. Severin and Schmid [14] considered both modified and residual approaches to monitor GARCH processes. Sliwa and Schmid [15] proposed two different types of control chart for the surveillance of the multivariate GARCH processes. Golosnoy [16] presented an investment strategy based on monitoring

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minimum variance portfolio weights. In this paper, the asset returns are modeled by GARCH model.

In recent applications of control chart in stock market, more complicated models have been considered. Golosnoy et al. [17] considered intraday volatility and proposed a method to monitor the errors of the model. In this research, the intraday asset price is modeled by a univariate jump-diffusion stochastic process. Cooper and Van Vliet [18] developed a new method called whole-distribution to monitor high frequency trading (HFT) system which outperforms traditional control charts. The process is assumed to follow generalized lambda distribution (GLD). Bondar et al. [19] proposed a statistic to monitor covariance matrix which is robust to changes in the mean vector. In this study, the first moment follows multivariate Gaussian distribution, while the second moment has non-central singular Wishart distribution. Garthoff et al. [20] proposed a method to monitor the mean vector and covariance matrix of multivariate GARCH processes. They concluded that considering covariance dynamics could reduce the number of false alarms in control chart. Golosnoy [8] proposed a method to monitor portfolio betas based on one factor model. They suggested this method as a supplementary tool to rebalance the portfolio weights. Accordingly, most of these researches concluded that early signaling could be effective for investment decisions and market watch.

The most of researches in financial surveillance are dominated by univariate and different types of multivariate GARCH model. This model can well define the autocorrelation in the second moment. While in some cases, the significant autocorrelation exists in both first and second moments. In these cases, the pure GARCH model cannot properly define the time dependency of the process. Therefore, the ARMA model with GARCH volatility model referred to as ARMA-GARCH model should be considered. This model is more general and could well define a wide range of financial applications. To the best of our knowledge, there is no research for monitoring processes with ARMA-GARCH time series model. Hence, in this paper, we develop a Shewhart control chart to monitor ARMA-GARCH processes. Our motivation for this research is monitoring Tehran Stock Exchange price index (TEPIX). To do that a control statistic based on the residuals of the model is proposed to monitor the process.

One of the differences between industrial monitoring and financial surveillance is the model uncertainty. In industrial applications, the control chart is designed under certain conditions. While in financial surveillance, there is not such situation. Although the structure of the model is usually certain, but there is almost uncertainty about the parameters of the model. Therefore, the performance of the proposed methods is evaluated under model uncertainty using simulation studies.

The rest of the paper is organized as follows: in the next section, the problem of monitoring financial processes is generally described. Also, the aim of monitoring TEPIX is deliberated. In Section 3, the ARMA-GARCH model as well as parameter estimation method is explained. Then, the proposed control method is illustrated in Section 4. The effect of parameters estimation as well as the power of change detection is investigated through simulation studies in Section 5. Modeling and monitoring of TEPIX are presented in Section 6. Finally, conclusions are summarized and some advices based on the obtained results are presented for practitioners.

2. MONITORING FINANCIAL PROCESSES

In many financial processes, on time decision making is very important when a change happens. For example, when the price of stock starts to increase/decrease, the proper time for buying/selling could affect the overall profit. Therefore, the main issue is decreasing the gap time between an event and a decision as soon as possible. Control chart is a powerful instrument for this aim. However, control chart is originally designed for industrial applications. Therefore, some justifications should be considered for financial applications. For example, in finance, the observations are usually dependent through the time. Therefore, selecting a proper time series model for the observations is very important. Usually, these models are more complicated than industrial applications. In the next section, a control method is proposed to monitor financial processes described by ARMA-GARCH time series model. This model is selected for the sake of monitoring TEPIX which is the original motivation of this research.

TEPIX is the main index of Tehran Stock Exchange. Stock market index is a leading indicator in finance [21]. The leading indicator shows the changes in finance quicker than the others [22]. The existence of information in stock markets and the influence speed of information in market value, make the stock market index as leading indicator. In other words, when a basic decision is made in a firm, it quickly affects the stock market value. However, in the real side of economy, it takes time to make differences. For example, there are several months delay between decision of designing new product and presenting the new product in the market.

According to the importance of the stock market, many initial researches in financial surveillance are emphasized on the monitoring of stock indices [5, 8, 14, 15]. Although, TEPIX is analyzed well enough from many financial aspects [23, 24], to the best of our knowledge, there is no research in monitoring TEPIX using control charts. Therefore, in the next sections a control method is proposed to monitor TEPIX based on the residuals of the ARMA-GARCH model and Shewhart control chart.

3. MODELING THE ARMA-GARCH PROCESS

In the previous section, the financial surveillance problem was defined. Also, it is mentioned that the aim of monitoring TEPIX leads us to the use of ARMA-GARCH time series model as a proper model. In this section, the ARMA-GARCH model is defined and the estimation procedure is explained.

The autoregressive moving average (ARMA) model [25] is a basic model. If $x_1,\dots,x_m$ denote the observations, the ARMA(P,Q) model is presented as Equation (1).

$$x_t = c + \sum_{i=1}^{P} a_i \left(x_{t-i} - c\right) - \sum_{j=1}^{Q} b_j \varepsilon_{t-j} + \varepsilon_t \ , \qquad (1)$$

for $t = 1,2,\dots,m$, where, $\boldsymbol{\xi} = \left(c, a_1,\dots,a_P, b_1,\dots,b_Q\right)^T$ is a vector of parameters and ε_t is the error term which independently follows normal distribution with mean zero and variance σ^2. There are some constraints for stationary and invertible conditions of the ARMA model [26].

Afterwards, Engle [26] presented autoregressive conditional heteroskedasticity (ARCH) model to define volatility in conditional mode. Then, Bollerslev [27] developed Generalized ARCH (GARCH) model as a natural development of AR models to ARMA ones. The GARCH (p,q) model is defined as Equation (2).

$$\varepsilon_t = \sqrt{h_t}\,\eta_t \ , \qquad (2)$$

for $t = 1,2,\dots,m$ in which h_t is defined as Equation (3).

$$h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \ , \qquad (3)$$

and η_t is innovation. The innovation independently and identically follows standard normal distribution. $\boldsymbol{\theta} = \left(\omega, \alpha_1,\dots,\alpha_q, \beta_1,\dots,\beta_p\right)^T$ is the vector of parameters and has the constraints of $\omega > 0, \alpha_i \geq 0, \beta_j \geq 0$ and $\sum_{i=1}^{q}\alpha_i + \sum_{j=1}^{p}\beta_j < 1$.

Therefore, if $x_1,\dots,x_m$ denote the observations of financial process with ARMA (P,Q)-GARCH (p,q) model [28], x_t could be explained as Equation (4).

$$\begin{aligned} & x_t = c + \sum_{i=1}^{P} a_i \left(x_{t-i} - c\right) - \sum_{j=1}^{Q} b_j \varepsilon_{t-j} + \varepsilon_t \ , \quad \varepsilon_t = \sqrt{h_t}\,\eta_t \ , \\ & h_t = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \ , \end{aligned} \qquad (4)$$

for $t = 1,2,\dots,m$ in which $\boldsymbol{\varphi} = \left(\boldsymbol{\xi}^T, \boldsymbol{\theta}^T\right)^T$ is the vector of parameters and η_t is innovation. The constraints of the GARCH model and stationary and invertible assumptions of the ARMA model are considered here as well.

The vector of parameters ($\boldsymbol{\varphi}$) is usually estimated based on maximum likelihood estimation (MLE) method [28]. Hence, based on normal distribution of the residuals, the likelihood function is written as Equation (5).

$$L(\boldsymbol{\varphi}) = L(\boldsymbol{\varphi}; \varepsilon_1,\dots,\varepsilon_m) = \prod_{t=1}^{m} \frac{1}{\sqrt{2\pi h_t}} \exp\left(-\frac{\varepsilon_t^2}{2h_t}\right). \qquad (5)$$

$\varepsilon_t = b^{-1}(L)\,a(L)\left(x_t - c\right)$ is the residual of the ARMA part in which $b(L) = \left(1 + \sum_{j=1}^{Q} b_j L^i\right)$, $a(L) = \left(1 - \sum_{i=1}^{P} a_i L^i\right)$, and L is the lag operator. The simplified log likelihood function is presented in Equation (6).

$$\log L(\boldsymbol{\varphi}) = -\frac{m}{2}\log(2\pi) - \frac{1}{2}\sum_{t=1}^{m}\left(\log(h_t) + \varepsilon^2 h_t^{-1}\right), \qquad (6)$$

Also, maximizing the log likelihood function in Equation (6) is equal to minimizing Equation (7).

$$\hat{\boldsymbol{\varphi}} = \arg\min I(\boldsymbol{\varphi}) \ , \qquad (7)$$

where, $I(\boldsymbol{\varphi}) = m^{-1}\sum_{t=1}^{m} l_t$ and $l_t = \varepsilon_t^2 / h_t + \log h_t$. Note that there are software packages such as R and MATLAB for optimizing Equation (7).

4. MONITORING PROCEDURE

In this section, a control method is proposed to monitor the AMRA-GARCH financial processes. The control method is based on the residuals of the model.

After estimating the parameters ($\hat{\boldsymbol{\varphi}}$), $\hat{\varepsilon}_t$ and $\hat{h}_t$ could be estimated respectively as Equations (8) and (9) for $t = 1,2,3,\dots$.

$$\hat{\varepsilon}_t = x_t - \hat{c} - \sum_{i=1}^{P} \hat{a}_i \left(x_{t-i} - \hat{c}\right) + \sum_{j=1}^{Q} \hat{b}_j \hat{\varepsilon}_{t-j} \ , \qquad (8)$$

$$\hat{h}_t = \hat{\omega} + \sum_{i=1}^{q} \hat{\alpha}_i \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \hat{\beta}_j \hat{h}_{t-j} \ . \qquad (9)$$

Note that if $q \geq Q$, the following $x_0,\ldots,x_{1-(q-Q)-P},\varepsilon_{-q+Q},\ldots,\varepsilon_{1-q},h_0,\ldots h_{1-P}$ should be predefined or initiated by fixed values for simplicity. When $q < Q$, the initial values are $x_0,\ldots,x_{1-(q-Q)-P},\varepsilon_0,\ldots,\varepsilon_{1-Q},h_0,\ldots h_{1-P}$.

In reference [28], it is proved that under the mentioned assumptions of the ARMA-GARCH model, if *m* limits to infinite in Equation (5), $\hat{\varphi}$ limits to φ as well. Therefore, the proposed control statistic (z_t) in Equation (10) independently follows identical standard normal distribution. However, since in practice *m* is finite, the z_t statistic may fail to be independent but our investigations show that they are at least uncorrelated.

$$z_t = \frac{\hat{\varepsilon}_t}{\sqrt{\hat{h}_t}}. \tag{10}$$

Then, Shewhart control chart is proposed to monitor the process. Therefore, the process assumed to be at the in-control state until Equation (11) is satisfied.

$$\begin{aligned} z_t &< UCL, \\ z_t &> LCL. \end{aligned} \tag{11}$$

The symmetric control limits (UCL and LCL) are determined such that the in-control (IC) average run length (ARL) is equal to the predefined values. In the financial surveillance literature, IC ARL usually is determined as 20, 30, 40, 60, 90, 120 and or 240.

Note that when the financial process goes to out-of-control state, at least one of the model parameters deviates from the in-control state and as a result the proposed control chart alarms.

On the other hand, since, we monitor TEPIX index in each working day, there is only one observation in each day. Hence, we have to design an individual control chart to monitor the process. In this condition, when a sample goes to out-of-control state, the reason is investigated. Then, if there is no evidence for assignable cause, it is considered as a false alarm. Although individual observations face with some weaknesses such as sensitivity to the outliers, however, one can develop robust estimators as a future research to tackle this problem.

In the next section, the robustness of the proposed monitoring method and the parameters estimators is evaluated based on the average of ARL and estimated parameters, respectively. Also, the performance of the proposed method in detecting changes in parameters is evaluated through simulation studies.

5. SIMULATION STUDY

In this section, the performance of the proposed method is evaluated through simulation studies based on two criteria. The first criterion is the robustness when there is model uncertainty. The second one is the power of detecting changes in parameters of the model. Without loss of generality, the order of ARMA-GARCH model is set equal to one. The control limits for Shewhart control chart are set equal to ±2.6383 for IC ARL equal to 120 based on standard normal distribution. The steps of simulation are presented in Appendix section. All simulation studies are performed in MATLAB software.

5. 1. Robustness Analysis In this subsection, the robustness of the proposed method is evaluated under the model uncertainty. The uncertainty is about the value of the parameters, while the model structure is assumed to be known. The performance of the proposed method is evaluated under different values of sample size (*m*). The final aim is finding the proper sample size such that the performance of the proposed method does not deteriorate significantly.

For sensitivity analyses, different values are considered for parameters. Table 1 shows the average IC ARL and the estimated parameters under different values of *m* and parameters settings.

Based on the results of Table 1, it can be concluded that by decreasing the number of sample size (*m*), as expected, in all scenarios, the performance is deteriorated based on average IC ARL. Although, sample sizes more than 1000 satisfy the expectations. However, when the sample size decreases to 500, the average ARL reduction slightly appears. The lowest average IC ARL in all scenarios is 110 and 105 for sample sizes equal to 1000 and 500, respectively. This reduction increases when the sample sizes are less than 500. Although, the performance of the control chart is acceptable for sample sizes equal to 500 and more, but in sample sizes less than 500, the performance gets worse too much. In this condition, for some cases, the average IC ARL becomes less than 100. These values are shown in bold in Table 1.

In the following, performance of parameter estimators is assessed based on the average of estimated parameters. Generally, the performance of estimator is decreased by reducing in sample size.

Table 1 shows that the estimator is not significantly sensitive to the ARMA parameters (c, a_1, b_1). In the other words, the average of estimated parameters of ARMA part is so closed to their actual values in all settings.

In addition, there are two main trends in estimated parameters of the GARCH part (ω, α_1, β_1). In the first trend, when $\alpha_1 < \beta_1$, β_1 is underestimated and ω is overestimated. This error appears from sample sizes less than 5000 in setting equal to (1, 0.05, 0.95). The one asterisk and the two asterisks cells show the underestimation and overestimation situations, respectively.

In the second trend, when α_1 and β_1 are insignificant (near to zero), β_1 is overestimated and ω is underestimated. This error can be seen in (1, 0.05, 0.05) setting for all *m* values. Note that in all settings, α_1 is estimated almost correct.

5. 2. Power of Change Detection In this subsection, the power of change detection in parameters of the ARMA-GARCH process is evaluated. The predefined parameters of the model is set equal to (0, 0.2, 0.2, 0.4, 0.3, 0.3) for $(c, a_1, b_1, \omega, \alpha_1, \beta_1)$. Then, the value of each parameter is altered to the new value through the step shift as $\varphi' = (c', a_1', b_1', \omega', \alpha_1', \beta_1')$. As mentioned in the first section, the performance of the proposed method is evaluated under the model uncertainty. Therefore, all simulations of this subsection are performed based on two different *m* values equal to 1000 and 5000.

Table 2 shows the average of out-of-control ARL (OC ARL) under different step shifts in all parameters of the model to the new value ($\boldsymbol{\varphi}' = (c' \quad a_1' \quad b_1' \quad \omega' \quad \alpha_1' \quad \beta_1')^T$), separately. The first row for each parameter shows new shifted value in out-of-control state.

As seen in Table 2, the proposed method can detect changes in the parameters of the model. Note that changes in the parameters of the ARMA part (c', a_1', b_1') contain both negative and positive shifts. According to the results, the monitoring method is almost symmetric to the changes in these parameters. It means the negative and positive shifts have almost the same OC ARLs. Since, the negative shifts in the parameters of the GARCH part $(\omega', \alpha_1', \beta_1')$ lead to decreasing in variance, the Shewhart control chart cannot detect out-of-control state. Therefore, only the positive shifts are considered.

In spite of this fact that the Shewhart control chart is suitable for detecting changes in the mean parameter, the proposed method can detect changes in the other parameters of the process with reasonable ARL. The results confirm the robustness of the proposed monitoring method in the out-of-control state under different sample sizes as well.

6. CASE STUDY: TEHRAN STOCK EXCHANGE

In this section, the proposed method is employed through a real case. The main motivation of this paper is monitoring TEPIX as the main index of Tehran Stock Exchange. TEPIX shows the general trend of all stocks prices in market. It is started in 1990 (1369 Solar Hijri) with 100 units. However, the dataset starts from 1997 (1376 Solar Hijri) and consists of 4747 daily observations. Note that the dataset is collected from Tehran Stock Exchange official web site. Figure 1 shows the trend of TEPIX.

In financial analysis, it is usual to transform original data into daily log return. Hence, if y_t denotes daily reports of TEPIX index, Equation (12) shows the natural logarithm of daily returns (x_t).

$$x_t = 100 \log\left(\frac{y_t}{y_{t-1}}\right). \tag{12}$$

This transformation provides the stationary assumption for new data. Figure 2 shows daily log returns. Before modeling the process, the stationarity of the log returns is examined through the augmented Dickey-Fuller (ADF) test. The null hypothesis of this test is the presence of unit root in a time series sample. While, the alternative hypothesis is usually considered as stationarity condition [29].

The result of this test is accepting the null hypothesis for original data with p-value equal to 0.9304. Therefore, the original data is not stationary. This test also performed on the log returns data. The result shows rejecting the null hypothesis with p-value of less than 0.01 indicating the stationarity of log returns data.

The log returns until the end of year 2014 are selected as in-sample data which contain 4149 samples. Table 3 shows the autocorrelation function (ACF) and partial autocorrelation function (PACF) of log returns and squared log returns. Note that confidence intervals for autocorrelations and partial autocorrelations of the both log returns and squared log returns are equal to ±0.0310 and ±0.0311, respectively.

As shown in Table 3, there are significant autocorrelations and partial autocorrelations in the first and second orders of log returns. Therefore, an appropriate model is required which can reduce both autocorrelations in the first and second orders. Hence, in this paper, the ARMA-GARCH model is considered. There are different approaches to set the order of time series models [30]. In order to select the ARMA part order, Akaike information criterion (AIC) and Bayesian information criterion (BIC) are often used. These criteria consider penalties for inefficient orders and should be minimized [30]. The first model we investigated is ARMA (1,1). After that we increase the orders of the model and check AIC and BIC criteria. The best first models with the corresponding criteria are tabulated in Table 4. The results show that ARMA (2,1) is the best model with AIC and BIC equals to 6630.9 and 6649.9, respectively.

Then, the residuals are examined for the existence of the ARCH effect. The null hypothesis of ARCH-LM test is no ARCH effects. This test is rejected for residuals of the ARMA model with 388.46 chi-square statistic and p-value less than 2.2E-16.

TABLE 1. The average of IC ARL as well as estimated parameters

m	**(c,a_1,b_1)** (ω,α_1,β_1)	**(0,0.2,0.2) (1,0.3,0.3)**	**(0,0.2,0.9) (1,0.3,0.3)**	**(0,0.9,0.2) (1,0.3,0.3)**	**(0,0.5,0.5) (1,0.3,0.3)**	**(0,0.5,0.5) (1,0.05,0.05)**	**(0,0.5,0.5) (1,0.05,0.90)**	**(0,0.5,0.5) (1,0.90,0.05)**
120	IC ARL	**92.8684**	**90.6706**	**89.1529**	**94.7459**	104.3838	**87.6937**	**93.4677**
	c	-0.0008	0.0110	-0.0156	0.0011	0.0027	0.0073	0.0096
	a_1	0.1538	0.1893	0.8576	0.4816	0.4811	0.4725	0.4754
	b_1	0.2449	0.9117	0.2097	0.5023	0.5141	0.5200	0.5073
	ω	1.0643	1.0826	1.0216	1.0385	0.6198*	3.5776**	1.0188
	α_1	0.3288	0.2942	0.2990	0.3044	0.1182	0.1267	0.8349
	β_1	0.2275	0.2492	0.2630	0.2702	0.3173**	0.6510*	0.0533
250	IC ARL	**99.6790**	**97.8462**	106.3291	102.9132	111.5845	104.1232	104.5370
	c	-0.0051	0.0188	-0.0085	0.0026	0.0089	-0.0360	0.0087
	a_1	0.1981	0.2005	0.8816	0.4872	0.4923	0.4867	0.4931
	b_1	0.1846	0.9060	0.2065	0.5030	0.5004	0.5046	0.4983
	ω	0.9276	1.0348	1.0998	1.0451	0.6110*	3.7690**	1.0238
	α_1	0.2939	0.3052	0.2867	0.2882	0.0832	0.0878	0.8584
	β_1	0.3249	0.2621	0.2569	0.2739	0.3624**	0.7069*	0.0512
500	IC ARL	109.4133	115.3819	117.4352	112.7462	111.2538	112.0004	105.8187
	c	0.0055	0.0081	0.0031	-0.0002	-0.0014	0.0035	-0.0058
	a_1	0.2063	0.1970	0.8920	0.4997	0.4950	0.4867	0.4959
	b_1	0.2004	0.9002	0.2060	0.5024	0.5046	0.5051	0.5036
	ω	1.0626	1.0766	0.9951	1.0483	0.6640*	2.6827**	1.0097
	α_1	0.3015	0.3031	0.2886	0.2834	0.0600	0.0644	0.8818
	β_1	0.2631	0.2642	0.3124	0.2889	0.3297**	0.7950*	0.0421
1000	IC ARL	115.6687	114.4539	117.4622	115.1945	117.5293	110.7727	110.8455
	c	0.0046	-0.0098	0.0003	0.0063	-0.0064	-0.0044	0.0016
	a_1	0.2004	0.1987	0.8972	0.5038	0.4968	0.4995	0.4958
	b_1	0.1926	0.8974	0.2014	0.4964	0.5020	0.5040	0.4990
	ω	1.0169	1.0016	1.0326	1.0178	0.7072*	1.3267**	1.0029
	α_1	0.2984	0.2877	0.3059	0.2956	0.0529	0.0540	0.8727
	β_1	0.2918	0.3039	0.2817	0.2922	0.3093**	0.8772*	0.0498
5000	IC ARL	119.1654	119.9355	119.7491	120.3388	118.9083	116.2981	116.3679
	c	0.0009	-0.0062	0.0025	-0.0005	0.0017	-0.0032	-0.0001
	a_1	0.2027	0.1960	0.8994	0.5002	0.5016	0.5011	0.5009
	b_1	0.1988	0.9000	0.1993	0.4974	0.4977	0.4975	0.4995
	ω	1.0056	1.0079	1.0162	1.0104	0.9016*	1.0058	0.9938
	α_1	0.3022	0.2992	0.2984	0.3036	0.0503	0.0513	0.8941
	β_1	0.2954	0.2976	0.2942	0.2943	0.1370**	0.8982	0.0503
10000	IC ARL	119.3169	119.6890	119.3459	120.1128	119.0507	120.2305	119.1835
	c	-0.0008	0.0012	-0.0027	0.0014	-0.0020	0.0064	0.0008
	a_1	0.2017	0.2012	0.9000	0.4990	0.4992	0.5013	0.4993
	b_1	0.1988	0.8997	0.1999	0.5008	0.5018	0.5013	0.5003
	ω	0.9982	1.0044	1.0036	1.0035	0.9397*	1.0154	1.0035
	α_1	0.3025	0.3006	0.2994	0.3022	0.0486	0.0494	0.8958
	β_1	0.2985	0.2973	0.2978	0.2975	0.1041**	0.8999	0.0497

* Underestimated parameter

** Overestimated parameter

TABLE 2. The average of the OC ARL

m	c'	**-4**	**-3**	**-2**	**-1**	**1**	**2**	**3**	**4**
1000	OC ARL	6.4153	10.7063	17.7567	41.1209	42.1515	17.5628	9.9970	6.0694
5000	OC ARL	6.0602	10.2115	17.4723	41.1420	41.4203	17.5194	10.0335	5.9815
m	a_1'	-0.5	-0.3	-0.1	0.1	0.3	0.5	0.7	0.9
1000	OC ARL	53.9449	68.4647	93.9641	116.8721	109.1313	92.1183	69.4252	52.2400
5000	OC ARL	54.7604	71.5379	95.0143	116.2958	115.2279	94.2212	70.2697	54.5097
m	b_1'	-0.5	-0.3	-0.1	0.1	0.3	0.5	0.7	0.9
1000	OC ARL	47.1772	69.5719	90.8090	112.5339	115.5261	86.8204	58.1576	37.0847
5000	OC ARL	49.4317	69.7929	96.4766	116.8796	114.9589	89.5912	58.1576	37.6126
m	ω'	<0.4	0.5	0.6	0.7	0.8	0.9	1	2
1000	OC ARL	>120	66.8974	45.3791	33.1851	26.9393	21.9629	18.9167	8.3619
5000	OC ARL	>120	69.1378	46.5106	34.5967	27.1236	22.6954	19.3488	8.2023
m	α_1'	<0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65
1000	OC ARL	>120	94.4338	76.6545	62.4590	52.8586	45.0224	39.2848	32.3017
5000	OC ARL	>120	97.4064	79.2248	65.7114	53.9958	45.9598	39.0341	33.2049
m	β_1'	<0.3	0.35	0.4	0.45	0.5	0.55	0.6	0.65
1000	OC ARL	>120	85.8980	63.7616	47.6443	35.1898	26.7488	20.2059	16.1615
5000	OC ARL	>120	88.2986	65.9627	48.2128	36.1160	26.8430	20.4335	16.0038

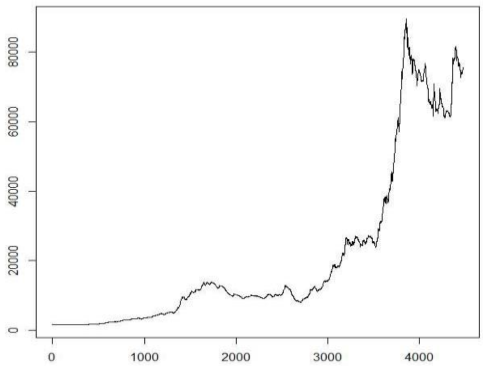

Figure 1. The trend of TEPIX in about 19 years

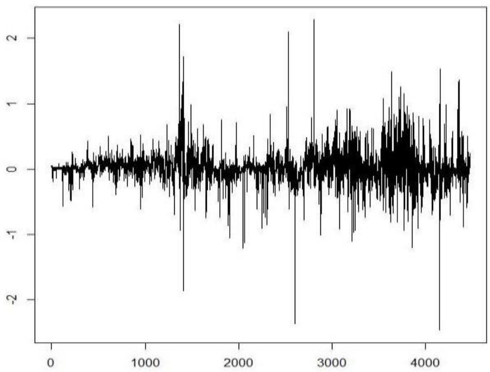

Figure 2. Daily log returns of TEPIX

TABLE 3. ACF and PACF of the log returns and squared log returns

	Log returns		**Squared log returns**	
Lag	ACF	PACF	ACF	PACF
1	0.3777	0.3783	0.2044	0.2046
2	0.2036	0.0706	0.1140	0.0913
3	0.1554	0.0692	0.1394	0.1252
4	0.1427	0.0651	0.0680	0.0145
5	0.1300	0.0496	0.0690	0.0399
6	0.0971	0.0145	0.0560	0.0187
7	0.0931	0.0331	0.0584	0.0352
8	0.1350	0.0838	0.0639	0.0369
9	0.1236	0.0338	0.0749	0.0507
10	0.1487	0.0774	0.0727	0.0388

TABLE 4. AIC and BIC for the 5 best first models

	ARMA (1,1)	ARMA (2,1)	ARMA (1,2)	ARMA (2,2)	ARMA (3,1)
AIC	6712.1	6630.9	6641.0	6632.7	6632.7
BIC	6724.7	6649.9	6660.0	6658.0	6658.0

These results confirm the necessity of using ARMA-GARCH model. The most usual order for GARCH model is GARCH(1,1). This order can well define many processes in practice. Hansen and Lunde [31] showed there is no model better than GARCH(1,1) through vast empirical studies. Therefore, the parameters of ARMA(2,1)-GARCH(1,1) model are estimated as $\hat{\varphi} = \left(\hat{c}, \hat{a}_1, \hat{a}_2, \hat{b}_1, \hat{\omega}, \hat{\alpha}_1, \hat{\beta}_1\right)^T =$

$= (0, 1.3282, -0.3321, -0.9430, 0.0068, 0.1827, 0.8128)^T$.

The ACF and PACF of the first and second orders of the residuals (not reported here) confirm the elimination of autocorrelation in both orders. The average of the residuals is equal to 0.0459 which is so close to zero. But, the Jarque-Bera and Sharpio-Wilk normality tests are rejected for residuals. Although, the normality assumption of residuals is rejected, as shown in Figure 3 the histogram of the residuals is so close to the normal distribution.

After estimating parameters ($\hat{\varphi}$) of the ARMA-GARCH model, the control statistic (z_t) is computed based on Equations (8) to (10). Note that the initial values are set equal to $x_0 = 0, \varepsilon_0 = 0, h_0 = 1$.

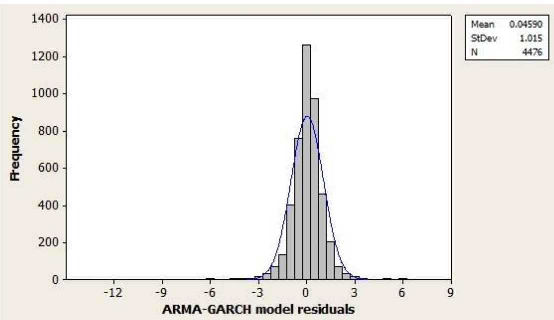

Figure 3. Histogram of ARMA-GARCH model residuals

Figure 4 shows the variation of z_t through the time. The horizontal lines in this figure show the control limits.

For monitoring the process, a desired IC ARL is set equal to 120. Then, control limits are set equals to ±2.6383 and the process is monitored based on Equation (11). For this level of control limits, on average 3 false alarms are expected yearly which lead to 37 signals totally. In this case, control chart is alarmed 93 times. Explanations about the out-of-control signals and the source of variations are interpreted based on fundamental analyses which are reported by Securities and Exchange News Agencies in the cite[2]. For some reasons, these analyses cover the last 8 years from 2008 to 2015. First, this period consists of many interesting events which can better show the performance of the proposed method. Second, the growth of market value in this period is meaningful. Third, it is near to the present time. Finally, this time is less than half of the full time, but contains more than half of the signals. Table 5 shows the source of variations as well as the number of signals for separated period of times.

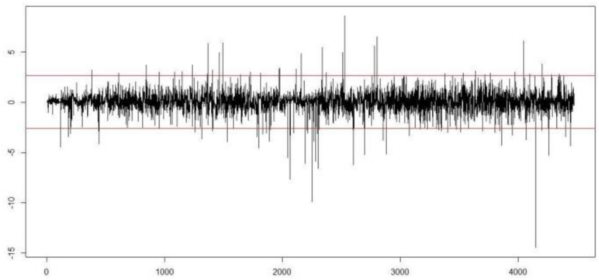
Figure 4. The control statistic with control limits

2 https://goo.gl/AeYOau

TABLE 5. The sources of variation and related signals

3 signals in 2008	**3 signals in the last 6 months of 2013**
Global financial crisis	The first round of Iran nuclear talks
Metal and oil price decrease	Geneva agreement
Profit decrease in stock market values of oil and metal industries	Emotionally increase in TEPIX (50 percent)
TEPIX decrease from 10000 to 8000	The peak of 89500 units for TEPIX
8 signals from 2009-2010	TEPIX increase from 63000 to 79000
Escape from global crisis	**6 signals in 2014**
Sugar world production issue and import troubles	Recession in internal economy
Stock value increase in sugar industries	Interest rate increase
TEPIX increase from 8000 to 24000	Surplus IPOs
5 signals in 2011	Problem in Iran export to Iraq due to ISIS attack
The United Nations and United States of America sanctions	50 percent fall in oil price and different metals decrease
Recession in the most of industries	Export decrease due to recession in Euro zone and China

TEPIX remains around 25000	Disagreement in Iran nuclear talks
4 signals in 2012	TEPIX decrease from 79000 to 63000
USD/IRR exchange rate increases almost 3 times	**7 signals in 2015**
Share profits (IRR) due to the conversion of assets value (USD)	Superficial optimism repetition in several times
Extra profit in oil, chemistry, medicinal, metal, banks and automotive industries	Nuclear Lausanne agreement
Initial public offering (IPO) of the oil refinery firms	Oil price fall (60 percent of market value depends on oil)
Expansionary policy of the government	Negative reports from firms
TEPIX increase from 25000 to 38000	Astonish increase in TEPIX like the other market indices
3 signals in the first 6 months of 2013	International policy and financial improvement
President election result	Swift sanction elimination
Emotionally increase in TEPIX (40 percent)	Better future prediction for internal economic
TEPIX increase from 39000 to 62000	TEPIX increase from 63000 to 80000

7. CONCLUSIONS

Monitoring financial process requires some justifications in control chart. Since financial processes are usually time dependent, control chart should be adjusted with more complicated time series models than industrial applications. The main motivation of this paper was monitoring TEPIX as the main index of Tehran Stock Exchange, using control charts. For this aim, the ARMA-GARCH time series model was selected as an appropriate one. Note that this model could also explain time dependency in many other financial processes.

Then, a control statistic was proposed based on the residuals of the model. According to the type of shifts in Tehran Stock Exchange trends, Shewhart control chart was proposed for monitoring TEPIX. Simulation studies revealed the robustness and the change detection power of the proposed monitoring method. Finally, the proposed method was applied to monitor TEPIX. The received signals from control chart were interpreted by market analysis for the last 8 years. According to the results, it was concluded that control chart can be trusted for decision making along with the other instruments.

8. ACKNOWLEDGEMENT

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9. APPENDIX

Simulation steps for the robustness analysis are as follows:

A. Set $\boldsymbol{\varphi}$ as well as UCL and LCL

B. For i = 1 … 100 do

B.1. Set the initial values $x_0 = 0, \varepsilon_0 = 0, h_0 = 1$

B.2. For t = 1 … m do

B.2.1. Simulate x_t based on the model (4)

End for t

B.3. Estimate $\hat{\boldsymbol{\varphi}}$ based on Equation (7)

B.4. For j = 1 … 5000 do

B.4.1. Set the initial values $x_0 = 0, \varepsilon_0 = 0, h_0 = 1$

B.4.2. For t = 1 … 20 do

B.4.2.1. Simulate x_t based on the model (4)

B.4.2.2. Estimate $\hat{\varepsilon}_t$ and $\hat{h}_t$ based on Equations (8) and (9)

End for t

B.4.3. Count = 0

B.4.4. Do

B.4.4.1. Count = Count+1

B.4.4.2. $t = t+1$

B.4.4.3. Simulate x_t based on the model (4)

B.4.4.4. Estimate $\hat{\varepsilon}_t$ and $\hat{h}_t$ based on Equations (8) and (9)

B.4.4.5. Estimate z_t based on Equation (10)

While Equation (11) satisfies

B.4.5. RL*j* = Count

End for j

B.5. ARL*i* = Average (RL*j*)

End for i

C. Average ARL = Average (ARL*i*)

D. Average Parameters = Average ($\hat{\boldsymbol{\varphi}}$'s)

Note that Steps [B.4.2], [B.4.2.1] and [B.4.2.2] are performed to reduce the effect of initial dataset in time series simulation.

Simulation steps for the power of change detection are the same as the robustness analysis except Steps [B.4.4.3] and [D]. In this analysis, Step [B.4.4.3] should be changed to "B.4.4.3. Simulate x_t based on model (4) with new value for each parameter after step shift $\boldsymbol{\varphi}'$ " and Step [D] is eliminated.

Monitoring Financial Processes with ARMA-GARCH Model Based on Shewhart Control Chart (Case Study: Tehran Stock Exchange)

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چکیده

پایش فرآیندهای مالی یکی از موضوعات مورد توجه پس از بحران‌های مالی در سال‌های اخیر می‌باشد. در تحقیقات این حوزه، شاخص‌های مهم فرآیندهای مالی با استفاده از نمودارهای کنترل پایش می‌شوند. نمودارهای کنترل یکی از ابزارهای قدرتمند در کشف تغییرات معنادار می‌باشد که در بخش‌های صنعت و خدمات به میزان قابل توجهی توسعه داده شده است. در این مقاله، یک روش کنترل بر اساس نمودار کنترل شوهارت برای پایش فرآیندهای مالی با مدل سری زمانی آرما – گارچ پیشنهاد می‌شود. اثر تخمین پارامترها و توانایی کشف تغییرات با استفاده از مطالعات شبیه سازی ارزیابی می‌شود. سپس، روش پیشنهادی برای پایش شاخص قیمت بورس اوراق بهاردار تهران (تپیکس) به عنوان انگیزه اصلی این تحقیق، بکار برده می شود. نتایج به دست آمده نشان دهنده توانایی روش پیشنهادی در مقایسه با تحلیل بازار می‌باشد.

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