International Journal of Engineering
Journal Homepage: www.ije.ir

The Reliable Hierarchical Location-allocation Model under Heterogeneous Probabilistic Disruptions

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PAPER INFO

Paper history:
Received 15 June 2016
Received in revised form 03 August 2016
Accepted 25 August 2016

Keywords:
Hierarchical Location-allocation Model
Reliability
Heterogeneous Probabilistic Disruption
Benders Decomposition Algorithm
Accelerating Techniques

ABSTRACT

This paper presents a novel reliable hierarchical location-allocation model where facilities are subject to the risk of disruptions. Based on the relationship between various levels of system, a multi-level multi-flow hierarchy is considered. The heterogeneous probabilistic disruptions are investigated in which the constructed facilities have different site-dependent and independent failure rates. In the occurrence of facility disruptions, to achieve system reliability, the mitigation operation is considered in such a way as to reassign the demand nodes to other operational facilities that can provide services. The problem is modeled from both cost and risk perspectives such that the fixed installation cost as well as the expected costs in normal disruption-free and disruptive conditions are minimized. A Benders decomposition algorithm is developed which seeks to find exact solution of the proposed model. Two efficient accelerating techniques including valid inequalities and knapsack inequalities are also proposed to expedite the convergence of solution procedure. The numerical results illustrate the applicability of the proposed model as well as the efficiency of the designed solution procedure.

***doi**: 10.5829/idosi.ije.2016.29.10a.11*

1. INTRODUCTION

Facility location problems have gained growing importance in the past few decades in a wide range of applications such as supply chain planning, transportation infrastructure design and public service systems. After the seminal paper of Weber [1], various modeling frameworks covering both continuous and discrete spaces have been proposed, including capacitated location-allocation model [2], covering model [3-5], p-median [6], etc. The exhaustive reviews in this area can be found in literature [7, 8].

Generally, the constructed facilities are expected by system designers to remain operational forever. However, the system may become unavailable due to the disruptive events caused by natural disasters or man-made hazards [9, 10]. The disruptive events can significantly deteriorate the system performance and service quality. Recent examples of such disruptions include the SARS outbreak in Toronto, Canada, in the summer of 2003 [11], the massive power outage in 2003 in the Northeast [12], the 2005 Hurricane Katrina in the Gulf Coast region [13] and the 2011 disastrous earthquake in Japan [14]. The aforementioned examples highlight the need of taking into account the reliability issues in the network design such that the system can work properly in both normal and disruptive conditions.

The earliest study of reliable facility location problem dates back to the work of Drezner [15], who formulated both p-median problem (PMP) and (p,q)-center problem under the assumption that one or more facility may become inactive. The reliability-based formulations of PMP and uncapacitated fixed-charge location problem (UFLP) with equal failure probability have been studied by Snyder and Daskin [9]. The median problem with independent failure probabilities under the complete information of customers about the operational status of facilities has been considered by Berman et al. [11]. The UFLP under correlated

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Please cite this article as: N. Zarrinpoor, M. S. Fallahnezhad, M. S. Pishvaee, The Reliable Hierarchical Location-allocation Model under Heterogeneous Probabilistic Disruptions , International Journal of Engineering (IJE), TRANSACTIONS A: Basics Vol. 29, No. 10, (October 2016) 1401-1411

disruptions was addressed by Li and Ouyang [16]. The reliable facility location problem in the presence of random facility disruptions with the option of hardening facilities was proposed by Lim et al. [17]. Cui et al. [18] proposed a model to study the reliable UFLP under site-dependent facility disruptions. Two models for UFLP with unequal failure probabilities have been addressed by Shen et al. [19]. Chen et al. [20] proposed a joint facility location-inventory design framework under equal probabilistic facility failures. Peng et al. [14] used the p-robustness criterion to develop a reliable model for a logistics network design. Li et al. [12] studied interdependent and correlated failures in facility location problem within a supporting structure framework. The impact of misestimating the disruption probability in the reliable facility location model was studied by Lim et al. [21]. Li et al. [22] studied the reliable PMP and UFLP with the facility fortification. Aydin and Murat [13] presented a reliable two-stage stochastic programming model to handle uncertainty associated with disruptive events. Wang and Ouyang [23] proposed game-theoretical reliable facility location models based on continuum approximation approach. An et al. [10] proposed a two-stage robust optimization approach for uncapacitated and capacitated cases of reliable PMP. Alcaraz et al. [24] proposed a set packing formulation of the reliable UFLP and studied certain aspects of its polyhedral properties by identifying a number of clique facets. Farahani et al. [25] proposed a hierarchical maximal covering location model under equal failure and solve it by a hybrid artificial bee colony algorithm. Shishebori [26] proposed a reliable facility location-network design problem in which the failure costs cannot exceed the maximum allowable value. Ghezavati et al. [27] proposed a facility location model with a two-level hierarchical network in a supply chain of disaster relief under uncertainty in order to schedule the customers' services. Zhang et al. [28] addressed a reliable location-inventory model under non-identical disruption probabilities.

As the related literature shows, the reliable location-allocation models have been mostly studied for single-level systems and hierarchical location-allocation models have gained less attention. In practice, most of service systems in both public and private sectors are hierarchically structured. Some examples of hierarchical systems include health service systems, blood banks, schools, telecommunication area, bank branches, etc. [29-31]. There is often a linkage between different types of interacting facilities at different levels which makes impossible to determine the location of each level separately [29]. Most of papers assumed that facilities fail under equal failure probabilities and heterogeneous probabilistic disruptions have been gained less attention. Although the geographical accessibility of facilities is one of the most important factors in the success of the service network design, no research paper considered it in the modeling. The uncapacitated models have gained more attention in the literature compared with capacitated ones. However, their applications in the practical contexts are limited due to their unrealistic assumptions. Most of the presented research papers developed approximation or meta-heuristic algorithms and few papers presented exact solution algorithms to solve the problems.

With regard to enumerated matters, this paper aims to propose a novel reliable hierarchical location-allocation problem where facilities are subject to the risk of disruptions. We consider a multi-level multi-flow hierarchy based on the relationship between various levels of the concerned service network. The heterogeneous probabilistic disruptions are considered in the model. The geographical accessibility of a service network is considered in terms of the proximity of a facility to the potential customers. The problem is modeled from both cost and risk perspectives such that the fixed installation cost as well as the expected costs in both normal disruption-free and disruptive conditions are minimized. To solve the proposed model, a Benders decomposition algorithm (BDA) enhanced by two efficient accelerating techniques including valid inequalities and knapsack inequalities is proposed.

The rest of this paper is organized as follows. The next section presents the model formulation. In Section 3, an accelerated BDA (ABDA) is developed to solve the model. Section 4 describes some numerical examples to illustrate the applicability of the proposed model. Section 5 ends with some conclusions and possible directions for future research.

2. MODEL FORMULATION

In this section, we present the notation and formulation of reliable hierarchical location-allocation model.

2. 1. Notation The sets, parameters, and decision variables used in the proposed model are defined as follows:

Sets

I Set of demand nodes

J Set of candidate locations for facilities

J_a Set of candidate locations for facilities considering both failable facilities and emergency facility

K Set of service types

L Set of facility levels

R Set of assignment levels

Parameters

f_{jl} Fixed installation cost to establish facility of level l at candidate location j

Tc_{ij} Transportation cost from customer i to facility j

w_{ijk} Unit cost of serving customer residing at demand node i and requiring service type k by facility j

q_j Failure probability of facility j

t_{ij} Shortest traveling time between customer i and facility j

t_{max} Maximum acceptable traveling time for customers to access the service at facilities

h_{ik} Demand for service type k at each demand node i

ca_{jk} Capacity of facility j to provide service type k

Q_l Maximum number of facility of level l that can be established

Decision variables

y_{jl} 1 if a facility of level l is located at node j, 0 otherwise

x_{ijlkr} Portion of customers residing at demand node i and requiring service type k is assigned to facility j of level l at assignment level r

p_{ijlkr} Probability that facility of level l at candidate location j serves customer residing at demand node i and requiring service type k at assignment level r

2. 2. Formulation The system under study is represented as a network where nodes represent either candidate location for facilities or demand concentrations. The facilities are not reliable and due to the disruptive events caused by natural disasters or man-made hazards may become unavailable from a time moment to another one. The risk of facility disruptions are modeled as independent and site-dependent events with probability $0\leq q_j<1$. In the occurrence of disruption, the facility cannot provide any service. In order to achieve system reliability, we consider the mitigation or recourse operation in such a way as to reassign the demand nodes to other operational facilities that can provide service. Therefore, each demand node is assigned to a primary closest facility that can provide service in the normal disruption-free situation. After occurrence of any disruption, each demand node is served by its closest assigned operating facility; if all its assigned facilities have failed, then a penalty cost of Π_i is incurred per unit of unsatisfied demand. Following the work of Cui et al. [18], to consider the penalty cost, Π_i, in the objective function, we consider an "emergency" facility, indexed by a, which has no fixed installation cost, $f_a=0$, failure probability $q_a=0$, and serving and transportation costs $Tc_{ia}+w_{iak}=\Pi_i$ for demand node i. The emergency facility can represent the facility located at the favorable weather areas that remains operational forever. It can also represent an alternative supply source and the penalty cost represents the outsourcing cost [9, 13]. In the proposed model, the emergency facility corresponds to the lost sales or the cost to serve the customer at a competitor's facility.

We assume that each customer can get service from $R\leq|J|$ facilities. For a customer residing at node i, a "level-r" assignment is used when all its assigned facilities at levels $0,\dots,r-1$ have failed and rth facility can provide service for customer. The demand node i must have exactly R assignment levels at the optimal solution, unless demand node i is assigned to the emergency facility at certain assignment level $s\leq R$. To consider the possibility of failure for all R regular facilities, if the demand node i is assigned to the constructed facilities at assignment levels $0,\dots, R-1$, it must be assigned to the emergency facility at the last assignment level, R, at the optimal solution. Note that the emergency facility can provide service of facilities at all levels of hierarchy. Following the work of Cui et al. [18], we define $p_{ijlkr}x_{ijlkr}$ as the probability that facility of level l at candidate location j serves customer i requiring service type k at assignment level r, given its other assigned facilities at assignment levels 0 to $r-1$. Note that p_{ijlkr} is the conditional probability that determines the first, second, ..., $(r-1)$th closest facilities serving demand node i requiring service type k fail, but facility j itself does not fail, when facility j is the rth closest open facility. Note that $p_{ijlk0}x_{ijlk0}$ defines the probability that facility j serves customer i at assignment level 0 and $p_{ijlk0}=1-q_j$. For $1\leq r\leq R$, p_{iulk1} is defined as follows:

$$p_{iulk1}=(1-q_j)\sum_{u\in J}\frac{q_u}{1-q_u}p_{i,u,l,k,0}x_{i,u,l,k,0}\,.$$

For definition p_{iulk2}, we have:

$$p_{iulk2}=(1-q_j)\sum_{u\in J}\frac{q_u}{1-q_u}p_{i,u,l,k,1}x_{i,u,l,k,1}\,.$$

By continuing the same pattern, we obtain:

$$p_{iulkr}=(1-q_j)\sum_{u\in J}\frac{q_u}{1-q_u}p_{i,u,l,k,r-1}x_{i,u,l,k,r-1}\cdot \tag{1}$$

Regarding the aforementioned assumptions and definitions, the formulation of reliable hierarchical location-allocation model under heterogeneous probabilistic disruptions can be stated as follows:

$$Min\sum_{j\in J}\sum_{l\in L}f_{jl}y_{jl}+\sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R}h_{ik}Tc_{ij}\,p_{ijlkr}x_{ijlkr}+\sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R}h_{ik}w_{ijk}\,p_{ijlkr}x_{ijlkr} \tag{2}$$

$$\sum_{j\in J}x_{ijlkr}+\sum_{s=0}^{r}x_{ialks}=1,\ \forall i,l,k,0\leq r\leq R, \tag{3}$$

$$x_{ijlkr}\leq\sum_{l\in L}\sum_{j\in J}y_{jl},\ \forall i,k,0\leq r\leq R-1, \tag{4}$$

$$\sum_{r=0}^{R-1}x_{ijlkr}\leq 1,\ \forall i,j\in J,l,k, \tag{5}$$

$$\sum_{r=0}^{R}x_{ialkr}=1,\ \forall i,l,k, \tag{6}$$

$$p_{ijlk0} = 1 - q_j,\ \forall i, j \in J_a, l, k, \tag{7}$$

$$p_{ijlkr} = (1-q_j)\sum_{u\in J}\frac{q_u}{1-q_u} p_{i,u,l,k,r-1} x_{i,u,l,k,r-1}, \quad \forall i, j \in J_a, l, k, 1 \le r \le R, \tag{8}$$

$$\sum_{j\in J} y_{jl} \le Q_l,\ \forall l, \tag{9}$$

$$\sum_{l\in L} y_{jl} \le 1,\ \forall j \in J, \tag{10}$$

$$\sum_{i\in I}\sum_{r=0}^{R} h_{ik} x_{ijlkr} \le ca_{jk} y_{jl},\ \forall j \in J_a, l, k, \tag{11}$$

$$x_{ijlkr} = 0,\ \forall i, j \in \left\{ j \middle| t_{ij} > t_{\max} \right\} l, k, 0 \le r \le R, \tag{12}$$

$$x_{ijlkr} \ge 0,\ \forall i, j \in J_a, l, k, 0 \le r \le R, \tag{13}$$

$$y_{jl} \in \{0,1\},\ \forall j, l. \tag{14}$$

The Objective function (2) minimizes fixed installation cost, expected traveling cost from customers to the facilities and expected serving cost of customers by constructed facilities. Constraint (3) forces that for each demand node *i*, service type *k*, hierarchial level *l* and assignment level *r*, either demand node *i* is assigned to a regular facility at assignment level *r* or it is assigned to the emergency facility *a* at some assignment levels $s \le r$. Constraint (4) insures that a customer only assigned to the open facilities. Constraint (5) states no customer can be assigned to the same facility at two or more assignment levels. Constraint (6) requires each customer to be assigned to the emergency facility at a certain level. Constraints (7) and (8) are the "transitional probability" equations. Constraint (9) specifies maximum number of facilities at each hierarchical level that can be established. Constraint (10) avoids having different levels of facility at the same location. Constraint (11) ensures that the total demand served by facility of level *l* at candidate location *j* does not exceed its capacity to provide each service type. Constraint (12) ensures that customers should not take more than a maximum acceptable traveling time to access service at facilities. Constraints (13) and (14) enforce the binary and non-negativity constraints on the corresponding decision variables.

Note that for the homogeneous disruptions in which $q_j = q$, the probability that customer *i* requiring service type *k* receives service from its level-r assignment is constant for all *i,j,l* and *k* (e.g. $p_{ijlkr} = (1-q)q^{r-1}$). Thus, the objective function in this case is the following:

$$Min \sum_{j\in J}\sum_{l\in L} f_{jl} y_{jl} + \sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R} h_{ik} Tc_{ij} (1-q) q^{r-1} x_{ijlkr} + \sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R} h_{ik} w_{ijk} (1-q) q^{r-1} x_{ijlkr} \tag{15}$$

s.t. (3)–(6), (9)–(14).

2. 3. Linearization

The resulting proposed model is a mixed integer quadratically-constrained quadratic programming (MIQCQP) problem which is hard to handle in this form. We apply the McCormick inequalities [32], to isolate the nonlinearity caused by bilinear terms containing products of continuous variables. Suppose that m_1 and m_2 are two continuous variables in the intervals $\left[m_1^L, m_1^U\right]$ and $\left[m_2^L, m_2^U\right]$,respectively. To isolate the non-convexity caused by bilinear term of the form $m = m_1 m_2$, we replace the non-convex term *m* with its McCormick convex inequalities as follows [32]:

$$m \ge m_1 m_2^L + m_1^L m_2 - m_1^L m_2^L, \tag{16a}$$

$$m \ge m_1 m_2^U + m_1^U m_2 - m_1^U m_2^U, \tag{16b}$$

$$m \le m_1 m_2^L + m_1^U m_2 - m_1^U m_2^L, \tag{16c}$$

$$m \le m_1 m_2^U + m_1^L m_2 - m_1^L m_2^U. \tag{16d}$$

Therefore, the linearized model can be stated as follows:

$$Min \sum_{j\in J}\sum_{l\in L} f_{jl} y_{jl} + \sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R} h_{ik} Tc_{ij} \varphi_{ijlkr} + \sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R} h_{ik} w_{ijk} \varphi_{ijlkr} \tag{17}$$

s.t. (3)–(7), (9)–(14)

$$p_{iulkr} = (1-q_j)\sum_{u\in J}\frac{q_u}{1-q_u} \varphi_{i,u,l,k,r-1}, \quad \forall i, j \in J_a, l, k, 1 \le r \le R, \tag{18}$$

$$\varphi_{ijlkr} \ge x_{ijlkr} + p_{ijlkr} - 1,\ \forall i, j \in J_a, l, k, 0 \le r \le R, \tag{19}$$

$$\varphi_{ijlkr} \le x_{ijlkr},\ \forall i, j \in J_a, l, k, 0 \le r \le R, \tag{20}$$

$$\varphi_{ijlkr} \le p_{ijlkr},\ \forall i, j \in J_a, l, k, 0 \le r \le R, \tag{21}$$

$$\varphi_{ijlkr} \ge 0,\ \forall i, j \in J_a, l, k, 0 \le r \le R. \tag{22}$$

3. SOLUTION PROCEDURE

After reformulation, the proposed problem is a mixed integer programming (MIP) problem and it can be solved by current state-of-the-art MIP solvers such as CPLEX. The difficulty of solving large-scale instances by such solvers motivates us to develop a BDA. The basic idea of the solution procedure is to decompose the original problem to the master problem, which consists of only complicating variables, and sub-problem. The master problem and sub-problem are solved iteratively by using the solution of one in the other, until the lower (LB) and upper (UB) bounds will converge and an optimal solution can be obtained [33].

3. 1. Benders Decomposition Algorithm To implement the BDA, we formulate the Benders primal sub-problem as follows:

$$Min\sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R}h_{ik}Tc_{ij}\varphi_{ijlkr} + \sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R}h_{ik}w_{ijk}\varphi_{ijlkr} \tag{23}$$

s.t. (3), (5) – (7), (12), (13), (18) – (22)

$$x_{ijlkr} \le \sum_{l\in L}\sum_{j\in J}\hat{y}_{jl},\ \forall i,k,0\le r\le R-1, \tag{24}$$

$$\sum_{i\in I}\sum_{r=0}^{R}h_{ik}x_{ijlkr} \le ca_{jk}\hat{y}_{jl},\ \forall j\in J_a,l,k, \tag{25}$$

Note that the binary variables have been fixed to the given values $\left\{\hat{y}_{jl}=y_{jl}\right\}$. Let ζ_{ilkr} , ϖ_{ijlk} , o_{ilk} , ξ_{ijlk} , ψ_{ijlkr} , ς_{ijlkr} , τ_{ijlkr} , ρ_{ijlkr} , δ_{ijlkr} , Ω_{ikr} and Λ_{jlk} be the vector of dual variables of the constraints (3), (5)–(7), (12), (18)–(21),(24) and (25), respectively. The dual sub-problem can be stated as follows:

$$Max\sum_{i\in I}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R}\zeta_{ilkr} - \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{k\in K}\varpi_{ijlk} + \sum_{i\in I}\sum_{l\in L}\sum_{k\in K}o_{ilk} + \sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\xi_{ijlk}(1-q_j) - \sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\sum_{r=1}^{R}\tau_{ijlkr} - \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R-1}\hat{y}_{jl}\Omega_{ikr} - \sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}ca_{jk}\hat{y}_{jl}\Lambda_{jlk} \tag{26}$$

$$2\zeta_{ilkr} - \varpi_{ijlk} + o_{ilk} + \psi_{ijlkr} - \tau_{ijlkr} + \rho_{ijlkr} - \Omega_{ikr} - h_{ik}\Lambda_{jlk} \le 0,\ \forall i, j\in\left\{j \middle| t_{ij} > t_{\max}\right\} l,k,0\le r\le R, \tag{27a}$$

$$2\zeta_{ilkr} - \varpi_{ijlk} + o_{ilk} - \tau_{ijlkr} + \rho_{ijlkr} - \Omega_{ikr} - h_{ik}\Lambda_{jlk} \le 0,\ \forall i, j\notin\left\{j \middle| t_{ij} > t_{\max}\right\} l,k,0\le r\le R, \tag{27b}$$

$$\xi_{ijlk} + \varsigma_{ijlkr} - \tau_{ijlkr} + \delta_{ijlkr} \le 0,\ \forall i,j\in J_a,l,k,0\le r\le R, \tag{28}$$

$$-(1-q_j)\sum_{u\in J}\frac{q_u}{1-q_u}\varsigma_{ijlkr} + \tau_{ijlkr} - \rho_{ijlkr} - \delta_{ijlkr} \le h_{ik}Tc_{ij} + h_{ik}w_{ijk},\ \forall i,j\in J_a,l,k,0\le r\le R, \tag{29}$$

$$\varpi_{ijlk},\tau_{ijlkr},\rho_{ijlkr},\delta_{ijlkr},\Omega_{ikr},\Lambda_{jlk} \ge 0,\ \forall i,j\in J_a,l,k,0\le r\le R, \tag{30}$$

$$\zeta_{ilkr},o_{ilk},\xi_{ijlk},\psi_{ijlkr},\varsigma_{ijlkr}\ free,\ \forall i,j\in J_a,l,k,0\le r\le R, \tag{31}$$

According to the solution of dual sub-problem, the master problem, can be written as follows:

$$Min\sum_{j\in J}\sum_{l\in L}f_{jl}y_{jl} + \eta \tag{32}$$

s.t. (9), (10) ,(14)

$$\eta \ge \sum_{i\in I}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R}\bar{\zeta}_{ilkr} - \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{k\in K}\bar{\varpi}_{ijlk} + \sum_{i\in I}\sum_{l\in L}\sum_{k\in K}\bar{o}_{ilk} + \sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\bar{\xi}_{ijlk}(1-q_j) - \sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\sum_{r=1}^{R}\bar{\tau}_{ijlkr} - \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R-1}y_{jl}\bar{\Omega}_{ikr} - \sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}ca_{jk}y_{jl}\bar{\Lambda}_{jlk} \tag{33}$$

$$\eta \ge 0. \tag{34}$$

Constraint (33) represents the optimality cut and $(\bar{\zeta}_{ilkr},\bar{\varpi}_{ijlk},\bar{o}_{ilk},\bar{\xi}_{ijlk},\bar{\psi}_{ijlkr},\bar{\varsigma}_{ijlkr},\bar{\tau}_{ijlkr},\bar{\rho}_{ijlkr},\bar{\delta}_{ijlkr},\bar{\Omega}_{ikr},\bar{\Lambda}_{jlk})$ indicates the extreme point of dual polyhedron obtained by solving the dual sub-problem.

3. 2. Accelerating Techniques The initial experiments show that the standard BDA takes relatively a large amount of time to converge. Therefore, in this sub-section, we develop some accelerating techniques to improve its convergence.

3. 2. 1. Valid Inequalities Appending valid inequalities into the master problem can help to find solutions that are close to the optimal [34, 35]. In this research, we drive two sets of valid inequalities as follows:

$$\sum_{j\in J}\sum_{l\in L}ca_{jk}y_{jl} \ge \sum_{i\in I}h_{ik},\ \forall k, \tag{35}$$

$$\sum_{j\in J}\sum_{l\in L}y_{jl} \ge 1. \tag{36}$$

Constraint (35) ensures that constructed facilities have sufficient capacity to serve the whole demand for each service type. Constraint (36) forces the selection of at least one facility to be open.

3. 2. 2. Knapsack Inequalities Santoso et al. [36] pointed out that adding knapsack inequalities along with optimality cut will result in a good quality solution from the master problem. They also declared that MIP solvers such as CPLEX can derive a variety of valid inequalities from the knapsack inequality. Let UB^t be the current best known upper bound, we have**:**

$$UB^t \geq \sum_{j\in J}\sum_{l\in L} f_{jl} y_{jl} + \eta \tag{37}$$

In iteration *t+1*, we can add the following knapsack inequality as follows:

$$\begin{aligned} &UB^t - \sum_{i\in I}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R} \zeta_{ilkr}{}^t + \sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{k\in K} \varpi_{ijlk}{}^t - \\ &\sum_{i\in I}\sum_{l\in L}\sum_{k\in K} o_{ilk}{}^t - \sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K} \xi_{ijlk}{}^t (1-q_j) + \\ &\sum_{i\in I}\sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K}\sum_{r=1}^{R} \tau_{ijlkr}{}^t \geq \sum_{j\in J}\sum_{l\in L} f_{jl} y_{jl} - \\ &\sum_{i\in I}\sum_{j\in J}\sum_{l\in L}\sum_{k\in K}\sum_{r=0}^{R-1} y_{jl}\Omega_{ikr}{}^t - \sum_{j\in J_a}\sum_{l\in L}\sum_{k\in K} ca_{jk} y_{jl} \Lambda_{jlk} \end{aligned} \tag{38}$$

Let ε be the optimality tolerance. The scheme of the ABDA is presented in Figure 1.

4. COMPUTAIONAL STUDY

In this section, we present some numerical examples to consider the performance of proposed model and the solution procedures. We use the randomly generated, 20-node to 115-node network with a symmetric travel time matrix in which the demands are randomly generated at each node. The solution procedure is coded in GAMS23.4 optimization software and evaluated on a personal computer equipped an INTEL Core 2 CPU with 2.4 GHz clock speed and 2 GB of RAM. The data range for numerical examples is presented in Table 1.

The computational results are illustrated in Table 2. Note that TC, FC, TRC and SC represent total cost, fixed installation cost, traveling cost and serving cost, respectively. We see that the total system cost increases when the potential demands for system increases. For example, the system in 80-node network (e.g. |J|=50, |I|=30) will incur 53.74% much more cost than the one in 35-node network (e.g. |J|=15, |I|=20) for the case |K|=3 and |L|=2.

$UB^n \leftarrow +\infty, LB^n \leftarrow -\infty, t \leftarrow 1, U \leftarrow 0, \{y_{jl}{}^0\}_{j\in J, l\in L}$

terminate ← **false**

while (terminate=**false**)**do**

Use $\{y_{jl}{}^0\}_{j\in J, l\in L}$ to solve dual sub-problem and to obtain its extreme point

$U^{n+1} = U^n \cup$ $\{(\zeta_{ilkr}{}^t, \varpi_{ijlk}{}^t, o_{ilk}{}^t, \xi_{ijlk}{}^t, \psi_{ijlkr}{}^t, \varsigma_{ijlkr}{}^t, \tau_{ijlkr}{}^t, \rho_{ijlkr}{}^t, \delta_{ijlkr}{}^t, \Omega_{ikr}{}^t, \Lambda_{jlk}{}^t)\}$

Add (35), (36) and (38) to master problem. Solve master problem to obtain $\{y_{jl}{}^t\}_{j\in J, l\in L}$

if $((\sum_{j\in J}\sum_{l\in L} f_{jl} y_{jl}{}^t + \eta^t) > LB^n)$ **then**

Update $LB^n \leftarrow (\sum_{j\in J}\sum_{l\in L} f_{jl} y_{jl}{}^t + \eta^t)$

end if

For fixed $\{y_{jl}{}^t\}_{j\in J, l\in L}$, solve dual sub-problem to obtain its optimal value θ^t

if $((\sum_{j\in J}\sum_{l\in L} f_{jl} y_{jl}{}^t + \theta^t) < UB^n)$ **then**

$UB^n \leftarrow (\sum_{j\in J}\sum_{l\in L} f_{jl} y_{jl}{}^t + \theta^t)$

end if

if $UB^n = LB^n \left(UB^n - LB^n \leq \varepsilon\right)$ **then**

terminate ← **true**

end if

$t \leftarrow t+1$

end while

Figure 1. The scheme of ABDA

TABLE 1. Data ranges for numerical examples

Parameters	Values
f_{jl}	U[1000,7000]
Tc_{ij}	U[100,200]
w_{ijk}	U[50,100]
q_j	U[0,1]
t_{ij}	U[0,100]
t_{max}	50
h_{ik}	U[0,10]
μ_{jk}	U[75,150]
Q_l	U[3,15]

We compare the results from heterogeneous and homogeneous probabilistic disruptions for 35-node and 75-node networks, as shown in Table 3. The *q* values for homogeneous pattern are calculated by taking the average over different q_j values which are applied in heterogeneous pattern.

Note that HTF, HMF and *DF* are the heterogeneous failure, homogeneous failure and percentage of improvement in system cost.

As it can be seen, on average, the system with homogeneous failure pattern incurs 11.12% more than the one with heterogeneous failure pattern. Note that solving the instances with heterogeneous pattern are more complicated than homogeneous one. The heterogeneous pattern is also more practical in real world problems since the failure probabilities are considered site-dependent.

TABLE 2. Computational results

|J|	|I|	|K|	|L|	TC	Cost components		
					FC	TRC	SC
20	15	3	2	24625.1	7435	12126.65	5063.45
			4	23597.19	8013	10355.17	5229.02
		5	3	36030.47	9214	17875.29	8941.18
			5	35443.82	9467	17121.87	8854.95
30	20	3	2	30994.39	8954	14389.06	7651.33
			4	30154.89	10187	12324.74	7643.15
		5	3	48223.07	12812	22654.55	12756.52
			5	46439.92	14430	19876.13	12133.79
40	25	3	2	42419.39	13015	18654.33	10750.06
			4	40219.63	14653	16002.42	9564.21
		5	3	59215.1	17327	27125.81	14762.29
			5	58203.54	17001	26654.9	14547.64
50	30	3	2	53234.45	19765	22344.31	11125.14
			4	52932.14	22387	20113.71	10431.43
		5	3	75125.92	23498	33741.43	17886.49
			5	74822.81	23987	33081.55	17754.26
60	35	3	2	62472.52	25174	24809.21	12489.31
			4	61982.17	26790	23041.07	12151.1
		5	3	86834.55	27330	37603.54	21901.01
			5	85741.08	27561	37198.11	20981.97

TABLE 3. The impact of failure patterns on the system

|J|	|I|	|K|	|L|	TC		DF
				HTF	HMF	
25	10	3	2	19734.12	21542.75	8.4
			4	18830.74	23298.13	19.17
		5	3	28721.91	32109.41	10.55
			5	26544.02	29765.56	10.82
50	25	3	2	41654.11	45376.82	8.2
			4	40342.43	47654.59	15.34
		5	3	56002.29	60432.1	7.33
			5	54378.83	59876.77	9.18

The impact of reliability on the system is presented in Table 4. Note that the minimum cost solution indicates the results based on the hierarchical location-allocation model presented in Section 2.2 without consideration of risk of disruptions. Although the system cost increases 3.92% by considering disruptions in the model, the system will be protected against a higher degree of uncertainty due to the probabilistic disruptions consideration.

The impact of geographical accessibility on the system cost is illustrated in Figure 2 for a 40-node network with |J|=20, |I|=20, |K|=3 and |L|=3. When maximum acceptable traveling time increases, the constructed facilities can serve customers in the much further locations, and thus, we expect that TRC and SC have the increasing trends. However, these cost components decrease since the customers are not assigned to the emergency facility with a higher traveling and serving costs. Note that the FC has a relatively steady trend due to the consideration of the maximum number of opened facilities in the model. We see that the total cost has a decreasing trend as maximum acceptable traveling time grows. Therefore, to design an efficient service network, the geographical accessibility improvement and cost minimization must be considered concurrently.

TABLE 4. Impact of reliability on the system cost

|J|	|I|	|K|	|L|	Reliable solution	Minimum cost solution
35	25	3	2	43654.87	41496.64
			4	41313.39	38872.26
		5	3	60274.85	57125.47
			5	58961.44	54610.35
55	35	3	2	63149.62	60564.81
			4	60784.73	58753.37
		5	3	86190.04	84562.34
			5	84287.27	83105.59

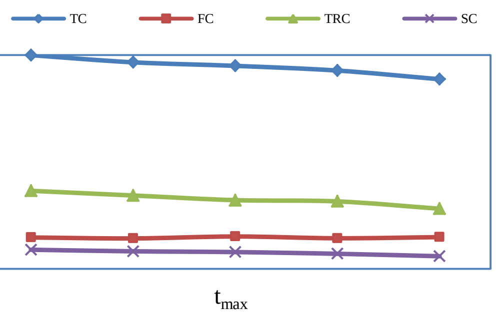

Figure 2. The impact of t_{max} on the system cost

To evaluate the performance of proposed ABDA, different experiments are conducted. Table 5 illustrates the size of test problems. In this Table, SP, MP and LP represent small-size, medium-size and large-size problems, respectively.

The comparative results of solution procedures are given in Table 6. Note that *iter* represents the iteration numbers of solution procedures. The results illustrate that CPLEX emerges as a good tool to solve the small-size examples. However, as the size of examples increases, CPLEX cannot solve the problems and the efficiency of the ABDA becomes considerably conspicuous. On average, the ABDA is 3.05, 6.11 and 3.34 times faster than standard BDA in small-size, medium-size and large-size problems, respectively. Regarding the examples that can be solved by CPLEX, on average, the proposed ABDA is 5.16 times faster than CPLEX. In addition, the proposed ABDA is capable to solve all the examples. The obtained results indicate the proposed ABDA is significantly more time efficient compared to CPLEX and standard BDA. It should be noted that the proposed ABDA can achieve the exact optimal solution in a reasonable time and its application for the concerned problem is quite acceptable. The results also show the applied acceleration methods play important role in rapid convergence of the proposed ABDA.

TABLE 5. The size of test problem

problem	|J|	|I|	|K|	|L|	problem	|J|	|I|	|K|	|L|
SP1	4	3	2	2	MP7	25	10	5	3
SP2	4	3	2	3	MP8	25	10		5
SP3	4	3	3	4	MP9	35	20	3	2
SP4	4	3	3	5	MP10	35	20		4
SP5	6	5	2	2	MP11	35	20	5	3
SP6	6	5	2	3	MP12	35	20		5
SP7	6	5	3	4	LP1	45	30	6	3
SP8	6	5		5	LP2	45	30		5
SP9	8	9	2	2	LP3	45	30	8	4
SP10	8	9		3	LP4	45	30		6
SP11	8	9	3	4	LP5	55	40	6	3
SP12	8	9		5	LP6	55	40		5
MP1	15	5	3	2	LP7	55	40	8	4
MP2	15	5		4	LP8	55	40		6
MP3	15	5	5	3	LP9	65	50	6	3
MP4	15	5		5	LP10	65	50		5
MP5	25	10	3	2	LP11	65	50	8	4
MP6	25	10		4	LP12	65	50		6

TABLE 6. Comparative results of solution procedures

Problem	BDA		ABDA		CPLEX
	Iter	Time(s)	Iter	Time(s)	Time(s)
SP1	5	0.73	3	0.51	0.18
SP2	5	0.98	2	0.43	0.26
SP3	7	2.16	3	0.74	0.46
SP4	8	2.47	4	0.32	0.21
SP5	7	1.95	3	0.83	0.63
SP6	6	3.51	5	0.94	0.74
SP7	7	5.64	4	0.45	0.34
SP8	7	7.37	5	1.12	0.85
SP9	8	6.56	6	2.32	1.72
SP10	6	7.22	5	3.08	1.91
SP11	10	9.57	7	4.31	3.14
SP12	7	7.82	5	3.33	2.07
MP1	8	11.42	5	5.21	3.18
MP2	10	10.41	5	5.43	3.91
MP3	9	14.25	7	6.74	5.96
MP4	9	15.84	8	7.32	6.01
MP5	21	142.09	7	32.11	139.16
MP6	32	276.17	10	46.51	252.87
MP7	45	480.53	19	54.45	471.34
MP8	37	400.87	17	67.12	391.85
MP9	41	775.91	18	121.32	787.72
MP10	56	1168.65	15	134.78	1070.51
MP11	35	889.43	12	180.31	876.54
MP12	38	935.52	15	176.33	912.07
LP1	35	1582.75	15	354.82	1617.96
LP2	51	1892.92	20	443.13	1983.14
LP3	39	2171.04	19	509.57	-
LP4	53	2065.63	26	470.14	-
LP5	59	2444.37	34	762.68	-
LP6	48	2381.41	30	686.11	-
LP7	52	2762.64	37	903.59	-
LP8	64	2859.12	44	922.45	-
LP9	59	2970.27	42	965.15	-
LP10	73	3251.78	50	1054.55	-
LP11	67	3490.31	45	1173.23	-
LP12	65	3774.33	48	1241.87	-

The convergence plot of the ABDA is shown in Figure 3 for the 50-node network with $|J|=30$, $|I|=20$, $|K|=4$ and $|L|=4$.

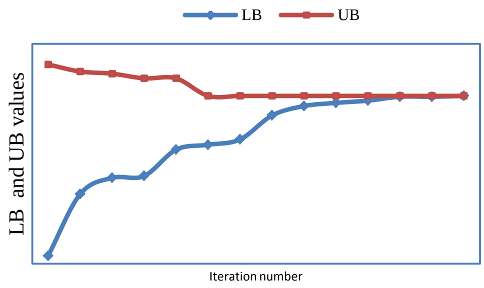

Figure 3. The convergence plot of ABDA

5. CONCLUSIONS

This paper presents a novel reliable hierarchical location-allocation problem where facilities are subject to the risk of disruptions. We consider a multi-level multi-flow hierarchy based on the relationship between various levels of the concerned service network. The heterogeneous probabilistic disruptions are considered in the model in which the constructed facilities have different site-dependent and independent failure rates. The geographical accessibility of a service network is considered in terms of the proximity of a facility to the potential demands. The problem is modeled from both cost and risk perspectives in such a way as to minimize the fixed installation cost as well as the expected costs in normal disruption-free and disruptive conditions. To solve the proposed model, a BDA enhanced by two efficient accelerating techniques including valid and knapsack inequalities is proposed. Based on the numerical results, we observe that (i) by considering the risk of probabilistic disruptions in the model, the reliability of the designed system will improve significantly such that the system will hedge against a high degree of uncertainty without significant increase in the total cost; (ii) the heterogeneous probabilistic disruption pattern plays a significant role in reducing the total cost and ignoring it would lead to significant overestimation of the system cost; (iii) to obtain a more reliable system design, the geographical accessibility improvement and cost minimization must be considered concurrently; (iv) a significant reduction in system cost will be obtained when the number of levels of the hierarchy increases, since there is an effective coordination of services provided at different levels in hierarchical network; and (v) the ABDA is significantly more time efficient compared to CPLEX and standard BDA.

As future research, the proposed model can be developed over multiple periods to capture the dynamic situations. It would be interesting to consider the capacity of each facility as an endogenous factor. The proposed model can be extended in the congested situations in which the systems are expected to satisfy large and heavy demands and it is not capable to serve all the simultaneous requests for service.

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The Reliable Hierarchical Location-allocation Model under Heterogeneous Probabilistic Disruptions

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PAPER INFO

Paper history:
Received 15 June 2016
Received in revised form 03 August 2016
Accepted 25 August 2016

Keywords:
Hierarchical Location-allocation Model
Reliability
Heterogeneous Probabilistic Disruption
Benders Decomposition Algorithm
Accelerating Techniques

چکیده

این مقاله یک مدل جدید مکان‌یابی-تخصیص سلسله‌مراتبی قابل اطمینان را با در نظر گرفتن ریسک اختلالات تسهیلات معرفی می‌کند. بر اساس رابطه‌ی بین سطوح مختلف سیستم، یک سلسله مراتب چندسطحی با چندین جریان بررسی شده است. در مدل پیشنهادی، الگوی اختلالات احتمالی غیرمشابه در نظر گرفته شده است که بر اساس آن تسهیلات استقراریافته، نرخ اختلال متفاوت وابسته به محل و مستقلی دارند. در هنگام وقوع اختلالات تسهیل، به منظور دستیابی به قابلیت اطمینان سیستم، عملیات کاهش ریسک در نظر گرفته می‌شود، به گونه‌ای که گره‌های تقاضا به دیگر تسهیلات عملیاتی که توانایی ارائه سرویس موردنظر را دارند، دوباره تخصیص می‌یابند. مسئله بر اساس هر دو دیدگاه هزینه و ریسک مدل‌سازی شده است به طوری که هزینه‌ی ثابت استقرار و هزینه‌های مورد انتظار سیستم در موقعیت‌های نرمال و اختلال کمینه‌سازی شود. جهت حل مدل پیشنهادی، یک الگوریتم بندرز پیشنهاد شده است که در پی یافتن جواب‌های بهینه‌ی دقیق است. دو روش تسریع‌سازی شامل نامعادلات معتبر و نامعادلات نپسک نیز جهت تسریع هم‌گرایی روش حل معرفی شده است. نتایج حل مدل، قابلیت کاربرد مدل پیشنهادی و همچنین کارایی الگوریتم بندرز تسریع‌سازی شده را شرح می‌دهند.

***doi**: 10.5829/idosi.ije.2016.29.10a.11*