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Designing Different Sampling Plans Based on Process Capability Index

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ABSTRACT

Acceptance sampling models have been widely applied in companies for evaluation the raw material as well as the final products. Meanwhile, process capability indices (PCIs) have been used in various industrial environments as capability measures that are obtained based on process departure from a target, process yield, process consistency and process loss. In this research, first a repetitive group sampling (RGS) plan based on process capability index is developed for variables inspection. Then the optimal parameters of proposed RGS plan are determined and also a new multiple dependent state (MDS) sampling plan, a double sampling plan (DSP) and a sampling plan for resubmitted lots are developed and finally, a comparison study is carried out between the proposed sampling plans and the results are elaborated.

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1. INTRODUCTION

Because of the competition between suppliers and its importance, producing a product with good quality is a basic need of any production process. We can increase product quality and reduce waste using statistical techniques. This action can bring customer satisfaction and reduce costs and finally increase sales in market. To produce a product with high quality, it is needed to control and measure the quality of products at all steps of processes. A set of these measurements is defined as process capability analysis. One application of the process capability analysis is to make decision about the acceptance of a lot received from supplier in any industrial environment. Lot acceptance sampling plans provide a method for evaluating the quality of a received lot based on an inspected sample. These sampling plans are used by suppliers, manufacturers, contractors and service providers in a wide range of industrial environments. These plans are applied as a method of quality assurance that reduce the risk of both producers and consumers.

Among the classical methods of acceptance sampling plan, variable sampling plan is applied for quantitative analysis of quality characteristics. Although the use of variable sampling plans is more difficult compared to attribute sampling plan, but it is less risky and has less sample size.

Variable sampling plans are designed for quality characteristics that follow specified probability distribution function in continues scale. One of assumptions of variable sampling plan is the normality of quality characteristics. Process capability index is a continuous variable with a known probability distribution that is used for analyzing the production process thus it can be used for designing sampling plans.

Variable RGS plan is one of the most important methods of sampling. In this method, a specified sample is taken from the lot and analyzed. In the case of non-compliance with tolerance specifications, another sample is taken from the lot and analyzed. This process is repeated the till lot is rejected or accepted.

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Sampling plan has been widely discussed in the previous decades. The basic concepts of variables sampling plans were presented by Jennett and Welch [1] and Moskowitz and Tang [2]. Pearn and Wu [3] designed a variable sampling plan for very low fraction of defectives. Wu [4] proposed a method for estimating process capability under Bayesian approach based on subsamples collected over time from an in-control process. A new system of skip-lot sampling plan defined as SkSP-R was proposed by Balamurali et al. [5] that is efficient to minimize the cost of the lot inspection. A sampling plan based on process capability index was developed by Wu et al. [6] for the situation where resampling is permitted on lots which are not accepted on the main inspection. Suresh and Sangeetha [7] introduced an acceptance sampling plan for construction of Bayesian chain sampling plan (BChSP-1) using quality regions. A double sampling plan (DSP) based on truncated life tests in Rayleigh distribution was proposed by Aslam [8].

Sherman [9] presented a new acceptance sampling plan for the inspection of attributes quality characteristics known as the repetitive group sampling (RGS) plan. In the past years a repetitive mixed sampling plan based on the process capability index was developed by Aslam et al. [10]. Wu et al. [11] proposed a RGS plan based on variables inspection. Also three types of RGSs using the generalized process capability index of multiple characteristics were investigated by Aslam et al. [12]. In another work, Aslam et al. [13] developed a variable RGS plan by considering process loss. Suresh et al. [14] proposed a Bayesian repetitive deferred sampling plan indexed through relative slopes.
A multiple deferred state sampling inspection (MDS sampling plan) was presented by Wortham and Baker [15]. Soundararajan and Vijayaraghavan [16] proposed a method for designing multiple dependent (deferred) state sampling plans. Also Vaerst [17] designed a procedure to construct multiple deferred state sampling plan. Aslam et al. [18] extended the idea of MDS sampling plans based on process capability index when the quality characteristic of the product follows the normal distribution.

In addition, a new acceptance sampling plan for resubmitted lots was investigated by Govindaraju and Ganesalingam [19]. A variable sampling plan for resubmitted lots based on process capability index was developed by Aslam et al. [20] for normally distributed processes. Also Balamurali et al. [21] proposed a variable RGS plan for minimizing ASN. Balamurali and Jun [22] proposed a RGS procedure for variable sampling plans. An optimal double-sampling plan based on process capability index was proposed by Fallah Nezhad and Seifi [23] in order to reduce the average sample number. Balamurali and Subramani [24] proposed the procedures for designing variables RGS plan indexed by indifference quality level and the relative slope on the operating characteristic curve.

In this research, first we propose a variable RGS plan based on process capability index C_{pm}. Then the optimal parameters of developed RGS plan are obtained with considering the constraints related to the risk of consumer and producer. Also we present various variable sampling plans based on process capability index C_{pm}, then a comparison study is done between the developed sampling plans based on ASN criterion and the results are analyzed.

This research is organized as following. In section 2 the exact probability distribution function of process capability index is introduced. In section 3 a brief introduction of RGS, DSP, MDS and sampling plan for resubmitted lots are presented. A simulation study is presented in section 4. Finally, the results of comparison study and conclusions are presented in section 5 and section 6, respectively.
The main contributions of this research are as follows:
Developing a RGS plan based on exact probability distribution function of C_{pm}.
Comparing the performance of different proposed sampling plans.

2. THE EXACT PROBABILITY DISTRIBUTION FUNCTION OF PROCESS CAPABILITY INDEX

The formula for evaluating process capability index is defined as follows:

$$C_{pm} = \frac{USL - LSL}{6\sigma'}, \tag{1}$$

where $\sigma' = \sqrt{E(X-T)^2}$ and USL, LSL are upper and lower specification limits and T is the target value presented by customers or product designer. The parameter of σ'^2 is usually unknown and have to be estimated; one estimation is as follows (Chan et al. [25]):

$$\hat{\sigma}' = \sqrt{\sum_{i=1}^{n} \frac{(X_i - T)^2}{n-1}}, \tag{2}$$

The resultant estimator is obtained as follows:

$$\hat{C}_{pm} = \frac{(USL - LSL)}{6\hat{\sigma}'}, \tag{3}$$

Since the process measurement arises from a normal distribution, thus the probability $\Pr(\hat{C}_{pmk} \geq C_0)$ is obtained as follows (Chan et al. [25]):

$$\Pr(\hat{C}_{pm} \geq C) = \exp\left(-\frac{\lambda}{2}\right) \sum_{j=1}^{\infty} \frac{\left(\frac{\lambda}{2}\right)^j}{j!\,\Gamma\left(\frac{n}{2}+j\right)} \int_0^{a/2C^2} e^{-w} w^{\frac{n}{2}+j-1} \, \mathrm{dw}, \tag{4}$$

where $a = C_{pm}^2 (1+\lambda/n)(n-1)$ and $\lambda = n(\mu - T)^2/\sigma^2$. Therefore, the statistical properties of process capability index $\hat{C}_{pm}$ can be analyzed for the general cases and we can use this probability function for evaluating process capability index in sampling plans.

3. PROPOSED PLANS

3. 1. Proposed Variables RGS Plan

Variables RGS plan is one of the effective sampling plans and the parameters in the proposed sampling plan are as follows:

n = sample size

k_1 = the lower threshold of process capability index for rejecting the lot based on the single sample

k_2 = the upper threshold of process capability index for accepting the lot based on the single sample

Now we explain procedure of proposed plan with the real example. In many industries, the process capability of systems, materials, and products needs to be compatible with the specified engineering tolerances. In practical case, we consider a company that produce machine tools. This company needs to keep actual production (which includes machine tools) within the desired tolerances. The company engineers define a dimensional tolerance for each portion of tool and estimate process capability index for them. Finally, their decision making about received lot based on the process capability index and RGS procedure will be as follows:

Step 1: Collect a sample with n observation.

Step 2: Accept the lot if $\hat{C}_{pm} \geq k_2$ and reject the lot if $\hat{C}_{pm} \leq k_1$ where $k_2 > k_1$. If $k_1 < \hat{C}_{pm} < k_2$, so repeat steps 1 and 2.

The OC function of the RGS plan, which includes the proportion of lots that are expected to be accepted for given product quality, (a) is as follows:

$$P_a(C_{pm}) = \frac{P_a}{P_r + P_a}, \tag{5}$$

OC function can be rewritten as follows:

$$P_a(C_{pm}) = \frac{P_a}{P_r + P_a} = \frac{\Pr(\hat{C}_{pm} \geq k_2)}{1 - \Pr(\hat{C}_{pm} \geq k_1) + \Pr(\hat{C}_{pm} \geq k_2)}, \tag{6}$$

Therefore, based on the plan of Balamurali and Jun [22], and considering the producer risk, α and consumer risk, β, model constraints depending on the different values of C_{AQL}, C_{LTPD} can be defined as follows:

$$C_{pm} = C_{AQL} \Rightarrow a = a_1 \Rightarrow \frac{\Pr(\hat{C}_{pm} \geq k_2)}{1 - \Pr(\hat{C}_{pm} \geq k_1) + \Pr(\hat{C}_{pm} \geq k_2)} \geq 1 - \alpha, \tag{7}$$

and:

$$C_{pm} = C_{LTPD} \Rightarrow a = a_2 \Rightarrow \frac{\Pr(\hat{C}_{pm} \geq k_2)}{1 - \Pr(\hat{C}_{pm} \geq k_1) + \Pr(\hat{C}_{pm} \geq k_2)} \leq \beta, \tag{8}$$

where $a_1 = C_{AQL}^2 (1+\lambda/n)(n-1)$, $a_2 = C_{LTPD}^2 (1+\lambda/n)(n-1)$. Also in the first constraint, $\Pr(\hat{C}_{pm} \geq k_2)$ and $\Pr(\hat{C}_{pm} \leq k_1)$ are the probabilities of accepting and rejecting the lot at AQL point based on single sample. In addition, in second constraint, $\Pr(\hat{C}_{pm} \geq k_2)$ and $\Pr(\hat{C}_{pm} \leq k_1)$ are the probabilities of accepting and rejecting the lot at LQL point based on single sample. α is producer risk and β is consumer risk, C_{AQL} is defined as the value of process capability index in the quality level of AQL and C_{LTPD} is defined as the value of process capability index in the quality level of LTPD.

The objective function of model is to minimize the ASN and the number of sampling steps is equal to $\left(\frac{1}{\Pr(\hat{C}_{pm} \leq k_1) + \Pr(\hat{C}_{pm} \geq k_2)}\right)$. In the other words, the number of sampling steps can be defined as the mean value of a geometric distribution which its success probability is equal to $\Pr(\hat{C}_{pm} \leq k_1) + \Pr(\hat{C}_{pm} \geq k_2)$. In each sampling step, the sample size is equal with n. Then the objective function of problem is obtained as follows:

$$Min\, ASN = \frac{n}{\Pr(\hat{C}_{pm} \leq k_1) + \Pr(\hat{C}_{pm} \geq k_2)}, \tag{9}$$

Therefore, by solving an optimization problem with the mentioned constraints and objective function for specified values of C_{AQL}, C_{LTPD}, ξ as well as different values of α and β, we can obtain the optimal values of decision parameters in a RGS plan and the values of n, k_1, k_2, can be determined using computer search procedures.

3. 2. Designing a Double Sampling Plan (DSP)

The parameters of DSP have been defined as follows:

n_1 = sample size of the first sample

n_2 = sample size on the second sample

k_1 = the lower threshold of process capability index for rejecting the lot based on the first sample

k_2 = the upper threshold of process capability index for accepting the lot based on the first sample

k_3 =the upper threshold of process capability index for accepting the lot based on the second sample

The procedure of DSP is as follows:

Step 1: Select n_1 observation from the lot and compute $\hat{C}_{pm}$.

Step 2: Accept the lot if $\hat{C}_{pm} \geq k_2$ else reject the lot if $\hat{C}_{pm} \leq k_1$ where $k_2 > k_1$. If $k_1 < \hat{C}_{pm} < k_2$, then obtain a second sample of n_2 measurements.

Step 3: Compute $\hat{C}_{pm}$ for the n_2 measurement. If $\hat{C}_{pm} \geq k_3$ accept the lot, otherwise reject the lot.

In DSPs, according to the cumulative distribution function of C_{pm}, if we do not use the shortened inspection, the equation of the ASN can be obtained as follows:

$$\min ASN = n_1 + n_2 \left[P(C_{pm} > k_1) - P(C_{pm} > k_2)\right], \tag{10}$$

In addition, the constraint of producer risk and consumer risk are as follows:

$$C_{pm} = C_{AQL} \Rightarrow a = a_1 \Rightarrow Pr(\hat{C}_{pm} \geq k_2 | n = n_1) + Pr(\hat{C}_{pm} \geq k_3 | n = n_2).Pr(k_2 \leq \hat{C}_{pm} \leq k_1 | n = n_1) \geq 1-\alpha, \tag{11}$$

and:

$$C_{pm} = C_{LQL} \Rightarrow a = a_2 \Rightarrow Pr(\hat{C}_{pm} \geq k_1 | n = n_1) + Pr(\hat{C}_{pm} \geq k_3 | n = n_2).Pr(k_2 \leq \hat{C}_{pm} \leq k_1 | n = n_1) \leq \beta, \tag{12}$$

By solving optimization model for given values of C_{AQL}, C_{LTPD}, λ and the different values of α and β, decision parameters of proposed DSP can be obtained.

3. 3. Designing Proposed MDS Sampling Plan

MDS sampling plan is an appropriate plan in which sampling results of past or future lots are considered. This plan belongs to the group of conditional sampling procedures. In these plans, acceptance or rejection of a lot is based not only on the single sample from that lot, but also on sample results from past lots (dependent state sampling) or future lots (deferred state sampling). For application of variable MDS plan, the mentioned assumptions should be valid as follows:

(i) Submitted lots in the order of production from a process having a constant proportion non-conforming.

(ii) The quality characteristic of interest is under a normal distribution.

(iii) The consumer has confidence to supplier and there is no reason to believe that a particular lot is poorer than the preceding lots.

Parameters of MDS sampling plan are defined as follows:

n=sample size m= number of preceding lots

k_1 =the upper threshold of process capability index for accepting the lot based on the first samples

k_2 =the lower threshold of process capability index for rejecting the lot based on the first sample

The decision making about received lot based on the process capability index and MDS procedure will be as follows:

Step 1: take a sample with n observations and calculate process capability index.

Step 2: if $\hat{C}_{pm} \geq k_1$, accept the lot else if $\hat{C}_{pm} \leq k_2$, reject it. If $k_2 \leq \hat{C}_{pm} \leq k_1$, then if m preceding lots on the condition of $\hat{C}_{pm} \geq k_1$ is accepted, then accept the lot else reject it.

The OC function of MDS sampling plan for the specified quality level can be obtained as the follows (Balamurali and Jun [26]):

$$P_a = \Pr\{\hat{C}_{pm} \geq k_1\} + \Pr\{k_2 \leq \hat{C}_{pm} \leq k_1\}.[\Pr\{\hat{C}_{pm} \geq k_1\}]^m, \tag{13}$$

where $\Pr\{\hat{C}_{pm} \geq k_1\}$ is the probability of accepting the lot based on single sample and is defined as follows:

$$P_a = P\left\{\hat{C}_{pm} \geq k_1\right\} = \int_0^{b\sqrt{n}/(1+3k_1)} G\left(\frac{(b\sqrt{n}-t)^2}{9k_1^2} - t^2\right) \times [\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})]\, dt, \tag{14}$$

and $\Pr\{k_2 \leq \hat{C}_{pm} \leq k_1 | p\}.[\Pr\{\hat{C}_{pm} \geq k_1 | p\}]^m$ defined as the probability of accepting the lot based on m preceding lots. Now according to OC function of proposed MDS sampling plan and the probability distribution function of C_{pm}, the required sample size, n can be minimize by solving the following model:

$$\begin{aligned} &Minimize\ n \\ &subject\ to: \\ &a = a_1 \Rightarrow C = C_{AQL} \Rightarrow P_A(C_{AQL}) \geq 1-\alpha \\ &and \\ &a = a_2 \Rightarrow C = C_{LTPD} \Rightarrow P_A(C_{LTPD}) \leq \beta \end{aligned} \tag{15}$$

Thus the above model for given values of α and β can be solved by numerical methods.

We solved the optimization model for the specified values of $m = 1, 2, 3, 4$ to see which one is the minimum ASN in the objective function. A sensitivity analysis is carried out on different values of m to determine which value of m has the better performance. The results are summarized in Table 1.

TABLE 1. Optimal values of parameters for different values of α , β and m

$\lambda = 0.1, C_{AQL} = 1.7, C_{LTPD} = 1.2$					
α	β	$ASN\ (m=1)$	$ASN\ (m=2)$	$ASN\ (m=3)$	$ASN\ (m=4)$
0.01	0.03	10	10	14	12
	0.05	13	14	17	21
0.03	0.05	4	3	6	9
	0.07	5	4	7	14
0.05	0.03	12	14	15	16
	0.07	14	12	17	16
0.07	0.07	6	4	8	7
	0.09	4	6	9	14
0.09	0.05	15	17	19	22
	0.09	14	16	20	21

It is seen that the results of proposed MDS sampling plan in the cases of $m=1$ and $m=2$ is near to each other but the results of the cases of m=3 , m=4 are not satisfactory. For instance, assuming $\alpha = 0.05, \beta = 0.07$, ASN of MDS sampling plan is equal to 14 and 12 in the cases of $m=1$ or $m=2$ and ASN is equal to 17 and 18, respectively for the cases of $m=3$ and $m=4$.

3. 4. Designing Variable Sampling Plan for Resubmitted Lot

The variable sampling plan for resubmitted lots is one of the important sampling plans. Parameters of the proposed sampling plan are as follows:

m= number of resubmissions n=sample size

k_a =the lower threshold of process capability index for accepting the lot based on the sample

The decision making about received lot based on the process capability index will be as follows:

Step 1: Take a random sample of size n and calculated $\hat{C}_{pm}$.

Step 2: If $\hat{C}_{pm} \ge k_a$ then accept the lot else, after repeating the step 2 and resubmitting the lot for m times, if the lot was not accepted, then reject the lot.

It is noted that when m =1, then mentioned sampling plan would be similar to a single sampling plan (SSP). So sampling plan for resubmitted lot can be considered as a more general form of SSP.

Sampling plan for resubmitted lots is easy to implement. There are some situations that the producer may discard the results of first sample and take the same number units for inspection and investigation under the provision of contract. For real example, in many countries such as India, the tax is paid based on the assessment of the first sample and if the producer does not agree with first inspection results then the second result is obtained under the same sample size as in the first inspection.

The OC function of the sampling plan for resubmitted lots is defined as follows (Govindaraju and Ganesalingam [19]):

$$P_A(C_{pm}) = 1-(1-P_a)^m , \tag{16}$$

where C_{pm} is defined as the quality level of submitted lot and P_a is defined as the acceptance probability in a single stage that is obtained as the follows:

$$P_a = P\left\{\hat{C}_{pm} \ge k_a\right\} = \int_0^{b\sqrt{n}/(1+3k_a)} G\left(\frac{(b\sqrt{n}-t)^2}{9k_a^2} - t^2\right) \times[\phi(t+\xi\sqrt{n}) + \phi(t-\xi\sqrt{n})]\,dt , \tag{17}$$

The ASN of proposed sampling plan for given quality level (C_{pm}) is determined as follows (Govindaraju and Ganesalingam [19]):

$$ASN(C_{pm}) = \frac{n(1-(1-P_a)^m)}{P_a}, \tag{18}$$

Now according to OC function of proposed sampling plan for resubmitted lots and the probability distribution function of C_{pm}, the required ASN can be minimized by solving the following model:

$$Minimize\ ASN\ (a) = \frac{n(1-(1-P_a)^m)}{P_a}$$

subject to:

$$C_{pm} = C_{AQL} \Rightarrow a = a_1 \Rightarrow P_A(C_{AQL}) \geq 1-\alpha$$
and
$$C_{pm} = C_{LQL} \Rightarrow a = a_2 \Rightarrow P_A(C_{LQL}) \leq \beta \quad (19)$$

Also a sensitivity analysis is carried out based on different values of m to determine which value of m has the better performance. The results are presented in Table 2.

It is observed that the ASN of proposed plan in $(m=2)$ is better than other cases. For instance, for specified values of $\alpha = 0.05, \beta = 0.07$, ASN of MDS sampling plan is equal to 19.34 in the case of $m=2$ and in other cases of $m=1,3,4$, ASN is equal to 20.24, 27.08 and 31.67, respectively. Thus we have applied the case $(m=2)$ for comparison study with other plans.

Now, we can design variable RGS plan and then compare variable RGS plan with the proposed DSP, MDS sampling plan and variable sampling plan for resubmitted lot.

Now we present methodology to obtain the proposed RGS plan parameters.

In the case of RGS plan with using a grid search, we can determine the minimum ASN plan searching in the multi-dimensional grid formed setting n=3(1)100, k_1=1.0(0.001)1.5, k_2=1.5(0.001)2.2.

4. SIMULATION STUDIES

Tables 3 and 4 denote the optimal parameters of n, k_1, k_2 for specified values of C_{AQL}, C_{LTPD}, λ and different values of α and β. Since T is the target value and $M = \frac{USL+LSL}{2}$ is the midpoint of the specification limits, according to the formula $\lambda = n(\mu - T)^2/\sigma^2$, we assumed data comes from standard normal distribution thus $\lambda = n(T)^2$ and T values can be obtained based on simulated values of λ.

For example, if, $\lambda = 0.05$, $C_{AQL} = 1.7$, $C_{LTPD} = 1.2$ and $\alpha = 0.05, \beta = 0.03$, then the optimal solution is $n = 45, k_1 = 1.441, k_2 = 1.624$ and the procedure of variable RGS plan will be as follows:

Step 1: Collect a sample with $n = 45$ observations.

Step 2: Accept the lot if $\hat{C}_{pm} \geq 1.624$ and reject the lot if $\hat{C}_{pm} \leq 1.441$. If $1.441 < \hat{C}_{pm} < 1.624$, then repeat steps 1 and 2.

TABLE 2. Optimal values of parameters for different values of α, β and m

$\lambda = 0.1, C_{AQL} = 1.7, C_{LTPD} = 1.2$					
α	β	$ASN\ (m=1)$	$ASN\ (m=2)$	$ASN\ (m=3)$	$ASN\ (m=4)$
0.01	0.03	19.65	17.16	21.46	26.12
	0.05	63.41	63.07	74.17	75.43
0.03	0.05	30.26	19.47	93.75	115.93
	0.07	26.79	21.76	98.44	116.88
0.05	0.03	25.55	25.49	24.62	38.86
	0.07	20.24	19.34	27.08	31.67
0.07	0.07	65.37	61.20	82.49	129.07
	0.09	61.46	59.82	112.53	131.82
0.09	0.05	30.51	24.59	107.51	119.64
	0.09	10.07	9.79	28.67	17.49

TABLE 3. Values of optimal parameters for different values α , β

$\lambda=0.05, C_{AQL}=1.7, C_{LTPD}=1.2$						$\lambda=0.1, C_{AQL}=1.7, C_{LTPD}=1.2$					
α	β	n	k_1	k_2	ASN	α	β	n	k_1	k_2	ASN
0.01	0.01	30	1.225	1.997	57.65	0.01	0.01	34	1.356	1.738	55.22
	0.03	28	1.248	1.865	55.89		0.03	25	1.221	1.879	54.53
	0.05	26	1.295	1.778	54.28		0.05	21	1.235	1.774	46.50
	0.07	24	1.225	1.755	53.55		0.07	20	1.287	1.756	42.31
	0.09	20	1.248	1.777	50.23		0.09	36	1.362	1.512	41.56
	0.10	24	1.363	1.768	47.07		0.10	34	1.319	1.535	34.588
0.05	0.01	32	1.364	1.878	48.18	0.05	0.01	28	1.185	1.965	38.08
	0.03	45	1.441	1.624	45.69		0.03	24	1.155	1.848	44.79
	0.05	25	1.354	1.715	38.47		0.05	20	1.387	1.747	33.67
	0.07	44	1.487	1.555	46.95		0.07	24	1.135	1.721	34.93
	0.09	17	1.226	1.884	38.78		0.09	31	1.436	1.529	27.22
	0.10	22	1.349	1.619	37.33		0.10	26	1.462	1.589	25.06
0.07	0.01	45	1.475	1.797	45.84	0.07	0.01	41	1.215	1.794	48.04
	0.03	34	1.442	1.698	36.78		0.03	35	1.222	1.635	35.58
	0.05	21	1.364	1.776	34.19		0.05	27	1.187	1.746	31.48
	0.07	22	1.358	1.716	32.69		0.07	19	1.392	1.712	26.96
	0.09	36	1.459	1.520	35.68		0.09	30	1.268	1.537	34.66
	0.10	35	1.487	1.511	38.15		0.10	27	1.458	1.568	25.65
0.09	0.01	32	1.424	1.779	44.22	0.09	0.01	36	1.216	1.765	36.77
	0.03	27	1.324	1.848	36.95		0.03	20	1.334	1.534	27.59
	0.05	24	1.365	1.879	37.09		0.05	27	1.428	1.649	29.14
	0.07	20	1.389	1.768	31.15		0.07	21	1.155	1.757	25.60
	0.09	19	1.341	1.747	27.08		0.09	17	1.367	1.721	25.39
	0.10	28	1.432	1.546	35.06		0.10	15	149	1.808	27.45
0.10	0.01	37	1.448	1.747	37.69	0.10	0.01	33	1.222	1.737	37.74
	0.03	25	1.395	1.868	33.18		0.03	22	1.135	1.869	36.25
	0.05	31	1.454	1.679	31.67		0.05	17	1.319	1.861	23.67
	0.07	29	1.426	1.654	31.55		0.07	19	1.112	1.730	20.99
	0.09	16	1.268	1.867	24.32		0.09	14	1387	1.719	21.88
	0.10	14	1.389	1.857	24.17		0.10	18	1.383	1.708	24.36

TABLE 4. Optimal values of parameters for different values of α , β

$\lambda=0.2, C_{AQL}=1.7, C_{LTPD}=1.2$						$\lambda=0.3, C_{AQL}=1.9, C_{LTPD}=1.3$					
α	β	n	k_1	k_2	ASN	α	β	n	k_1	k_2	ASN
0.01	0.01	18	1.265	1.935	37.26	0.01	0.01	22	1.462	1.963	29.65
	0.03	17	1.248	1.849	32.53		0.03	14	1.435	1.987	26.48
	0.05	29	1.357	1.662	31.57		0.05	19	1.448	1.549	24.76
	0.07	16	1.269	1.767	29.39		0.07	18	1.359	1.928	23.18
	0.09	28	1.362	1.566	24.99		0.09	14	1.564	1.803	22.79
	0.10	21	1.337	1.565	22.79		0.10	19	1.465	1.741	21.65
0.05	0.01	17	1.378	1.828	26.27	0.05	0.01	15	1.323	1.875	23.22
	0.03	14	1.385	1.779	29.30		0.03	17	1.316	1.795	29.58
	0.05	16	1.226	1.991	21.72		0.05	18	1.358	1.765	20.79
	0.07	23	1.449	1.565	25.07		0.07	16	1.279	1.924	16.68
	0.09	12	1.249	1.845	19.35		0.09	15	1.221	1.835	17.57
	0.10	15	1.368	1.616	19.40		0.10	11	1.237	1.804	16.89
0.07	0.01	17	1.375	1.918	26.49	0.07	0.01	28	1.444	1.763	27.07
	0.03	19	1.342	1.835	22.24		0.03	15	1.316	1.898	19.13
	0.05	28	1.456	1.616	26.19		0.05	22	1.459	1.615	26.98
	0.07	16	1.234	1.937	17.77		0.07	17	1.286	1.930	17.68
	0.09	24	1.418	1.565	21.88		0.09	25	1.472	1.586	21.24
	0.10	17	1.575	1.544	20.81		0.10	14	1.465	1.507	13.89
0.09	0.01	16	1.393	1.913	17.32	0.09	0.01	23	1.488	1.793	23.66
	0.03	15	1.365	1.833	19.44		0.03	18	1.355	1.86901	17.75
	0.05	17	1.449	1.666	20.83		0.05	19	1.443	1.671	20.98
	0.07	19	1.247	1.985	12.61		0.07	15	1.327	1.855	15.88
	0.09	9	1.252	1.997	15.54		0.09	11	1.286	1.948	14.74
	0.10	18	1.334	1.763	11.24		0.10	12	1.359	1.735	13.78
0.10	0.01	18	1.368	1.928	22.53	0.10	0.01	15	1.348	1.941	22.59
	0.03	29	1.449	1.776	23.57		0.03	37	1.588	1.595	34.23
	0.05	18	1.355	1.145	18.00		0.05	13	1.346	1.879	16.43
	0.07	19	1.419	1.655	19.73		0.07	13	1.468	1.678	19.73
	0.09	17	1.376	1.765	15.95		0.09	15	1.341	1.768	13.59
	0.10	15	1.368	1.798	14.46		0.10	15	1.334	1.729	12.37

TABLE 5. The optimal parameters for proposed sampling plan ($T \neq \mu$)

	$\lambda = 0.05, C_{AQL} = 1.7, C_{LTPD} = 1.2$			
	n	k_1	k_2	ASN
$T = \mu - 0.5\sigma$	42	1.417	1.587	41.61
$T = \mu - 0.4\sigma$	44	1.428	1.596	42.85
$T = \mu - 0.3\sigma$	44	1.431	1.612	44.97
$T = \mu - 0.2\sigma$	46	1.428	1.610	43.46
$T = \mu - 0.1\sigma$	45	1.437	1.615	45.93
$T = \mu$	45	1.441	1.624	45.69
$T = \mu + 0.1\sigma$	45	1.445	1.634	46.46
$T = \mu + 0.2\sigma$	45	1.443	1.631	46.74
$T = \mu + 0.3\sigma$	47	1.458	1.637	47.46
$T = \mu + 0.4\sigma$	46	1.461	1.642	48.20
$T = \mu + 0.5\sigma$	48	1.463	1.748	49.76

TABLE 6. Results of comparison study under different approaches

α	β	DSP	MDS sampling plans	RGS plan	Sampling plan for resubmitted lots	RGS plan (Wu et al. [11])
0.01	0.025	50.62	19	33.62	38.64	80
0.01	0.075	44.24	17	29.74	32.17	71
0.05	0.01	63.89	16	26.27	28.35	62
0.075	0.025	37.46	10	22.66	23.77	50
0.075	0.10	44.97	14	20.54	34.42	37
0.10	0.05	26.03	13	18.36	21.22	40
0.10	0.075	32.85	9	18.49	19.23	36
0.10	0.10	20.45	8	14.52	23.57	33

Now with regards to the target value (T) and process mean (μ), with assuming $T \neq \mu$, we evaluate the process capability index $\hat{C}_{pm}$. To analyze the behavior of proposed plan in the case of $T \neq \mu$, we compared the ASN of proposed sampling plan for different values of T and $\alpha = 0.05, \beta = 0.03$. The results are denoted in Table 5.

It is seen that when values of parameter *T* increase then ASN of RGS plan increases too. In addition, it is seen that decision thresholds of process capability index often increase by increasing the values of parameter *T*.

5. COMPARISON STUDY AMONG DEVELOPED SAMPLING PLANS

Simulation results under different values of α, β and specified value of $\lambda = 0.1, C_{AQL} = 1.8, C_{LTPD} = 1.1$ for probability distribution function are presented in Table 6. It is observed that MDS sampling plan has the least values of ASN and is the best method. RGS plan performs better than sampling plan for resubmitted lots and DSP. DSP has the worst performance in comparison with other sampling plans.

6. CONCLUSION

In this paper, optimization models were developed for designing acceptance sampling plans like RGS, DSP, MDS and sampling plan for resubmitted lots considering consumer risk, producer risk as the constraints and process capability index as the performance measure. The proposed plan was based on exact probability distribution of process capability index. In addition, we presented a procedure to obtain the required sample size, and the thresholds of process capability index to make decision about the lot.

It is observed that MDS sampling plan has the least values of ASN and is the best method. RGS plan performs better than sampling plan for resubmitted lots and DSP. DSP has the worst performance in comparison with other sampling plans.

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Designing Different Sampling Plans Based on Process Capability Index

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چکیده

مدل‌های نمونه برداری پذیرش به طور گسترده‌ای در شرکت‌ها برای ارزیابی مواد خام و همچنین محصولات نهایی استفاده شده است. در همین حال، شاخص قابلیت فرآیند (PCIS) در محیط‌های صنعتی مختلف به عنوان معیار توانایی استفاده شده است که بر مبنای خروج فرایند از یک هدف، بازده فرآیند، ثبات فرایند و اتلاف فرآیند به دست می‌آید. در این پژوهش، ابتدا طرح نمونه گروه تکراری (RGS) بر اساس شاخص قابلیت فرآیند برای متغیرهای بازرسی توسعه یافته است. سپس پارامترهای بهینه‌ی طرح RGS پیشنهادی تعیین شده و همچنین طرح نمونه برداری حالت‌های وابسته چندگانه جدید (MDS)، یک طرح نمونه‌برداری دوگانه (DSP) و یک طرح نمونه‌برداری برای گروه‌های تکراری توسعه یافته و در نهایت، یک مطالعه‌ی مقایسه‌ای بین نمونه طرح‌های پیشنهادی انجام می‌شود و نتایج توضیح داده می‌شوند.

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