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Analytical Solution for Free Vibration of a Variable Cross-section Nonlocal Nanobeam

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ABSTRACT

In this article, small scale effects on free vibration analysis of non-uniform nanobeams is discussed. Small scale effects are modelled after Eringen's nonlocal elasticity theory while the non-uniformity is presented by exponentially varying width among the beams length with constant thickness. Analytical solution is achieved for free vibration with different boundary conditions. It is shown that section variation accompanying small scale effects has a noticeable effect on natural frequencies of non-uniform beams at nano scale. First, five natural frequencies of single-layered graphene nanoribbons (GNRs) with various boundary conditions are obtained for different nonlocal and nonuniform parameters which shows a great sensitivity to non-uniformity in different shape modes.

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1. INTRODUCTION

Beams with geometry properties varying along the length with ability to reduce weight or volume as well as to increase strength and stability of structures have engrossed great deal of attention in engineering design. Using non-uniformity advantages beside nanomaterials which have special mechanical, chemical, electrical, optical and electronic properties could be useful in designing many NEMS devices such as oscillators, clocks and sensor devices. In order to make a NEMS device more efficient, nanobeams with non-uniform cross sections should be used. To be able to use non-uniform nanobeams, mechanical behaviors in both static and dynamic conditions should be known. Dynamic studies include vibration analysis which is the key step for many NEMS devices. In last decades, many studies reported the behavior of non-uniform beams in different static and dynamic manners [1-4].

In nanomechanics, there's also a lot of experimental and theoretical studies carried out by the scientific communities. Eringen's nonlocal elasticity theory is one of the well-known nonclassical continuum mechanics represented by Eringen [5] which is widely used by the researchers. Peddieson et al. [6] applied Eringen's nonlocal elasticity theory to formulate a nonlocal version of Euler-Bernoulli beam. The models predicted that MEMS scale devices would not exhibit nonlocal effects while nanoscale devices could. Wang et al. [7] obtained analytical solutions for vibration of nonlocal Euler-Bernoulli and Timoshenko nanobeams while the effects of transverse shear deformation and rotary inertia were accounted for in the latter theory. Sadeghian and Ekhteraei Toussi [8] used the wave method approach for the free vibration analysis of a Timoshenko beam located on an elastic foundation with structural discontinuities in the shape of stepwise cross section change or an open edge crack. Shah-Mohammadi-Azar et al. [9] studied the mechanical behaviour of a fixed-fixed nano-beam based on nonlocal elasticity theory. The nano-beam was sandwiched with two piezoelectric layers through it's upper and lower sides. Khanchehgardan et al. [10] investigated thermo-elastic damping in a nano-beam resonator based on nonlocal elasticity. Golami Bazehhour and Rezaeepazhand [11] presented a relatively simple method for torsional analysis of tubes made of non-

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homogeneous material with continuous variation of shear modulus through the thickness. Loya et al. [12] investigated free transverse vibration of cracked Euler-Bernoulli nanobeams based on nonlocal elasticity model. Tadi Beni et al. [13] studied the transverse vibration of cracked nano-beam based on modified couple stress theory. Bağdatlı [14] presented Non-linear vibration of nanobeams. Exact solutions for the mode shapes and frequencies were obtained for the linear part of the problem and for the non-linear problem, approximate solutions using perturbation technique was applied to the equations of motion.

In the present work, analytical solution for free vibration analysis of a non-uniform nanobeam is investigated for various supports condition using Eringen's nonlocal elasticity theory. It's assumed that the thickness remains constant while the width varies exponentially along the beam. The effects of nonlocal and non-uniformity on the natural frequency of the beam is studied.

2. MATHEMATICAL MODEL

2. 1. Euler Beam According to the Euler beam theory, the displacement relations are given by [15]:

$$\begin{cases} u=-z\dfrac{\partial w(x,t)}{\partial x} \\ v=0 \\ w=w(x,t) \end{cases} \tag{1}$$

Which cause the longitudinal strain as:

$$\varepsilon_{xx}=\frac{du}{dx}=-z\frac{\partial^2 w}{\partial x^2} \tag{2}$$

where u, v and w are the displacement components, x the longitudinal coordinate measured from the left end of the beam, z the coordinate measured from the midplane of the beam and ε_{xx} is the normal strain.
The strain energy U is given by:

$$U=\frac{1}{2}\int_0^L\int_A \sigma_{xx}\varepsilon_{xx}\,dA\,dx \tag{3}$$

In which, A and L are the cross sectional area and length of the beam, σ_{xx} is the axial stress while the strain energy due to the shearing strain is zero. By substituting Equation (2) into Equation (3), the strain energy may be expressed as:

$$U=-\frac{1}{2}\int_0^L\int_A \sigma_{xx} z\frac{\partial^2 w}{\partial x^2}\,dA\,dx=-\frac{1}{2}\int_0^L \mathrm{M}\frac{\partial^2 w}{\partial x^2}\,dx \tag{4}$$

where, M is the bending moment defined as:

$$M=\int_A z\,\sigma_{xx}\,dA \tag{5}$$

Also, by assuming free harmonic motion, the kinetic energy T is given by:

$$T=\frac{1}{2}\int_0^L\left\{\int_A\rho\left[\left(\frac{\partial u}{\partial t}\right)^2+\left(\frac{\partial w}{\partial t}\right)^2\right]dA\right\}dx \tag{6}$$

where, ρ is the mass density of the beam. By excluding the rotary inertia effect, Equation (6) may be expressed as:

$$T=\frac{1}{2}\int_0^L\int_A\rho\left(\frac{\partial w}{\partial t}\right)^2 dA\,dx \tag{7}$$

And by assuming free harmonic motion:

$$w(x,t)=W(x)e^{i\omega t} \tag{8}$$

According to Equations (4), (7) and (8) the maximum kinetic energy T can be written as:

$$\begin{cases} T_{max}=\dfrac{1}{2}\displaystyle\int_0^L \rho A\omega^2 W\,\delta W\,dx \\ U_{max}=-\dfrac{1}{2}\displaystyle\int_0^L \mathrm{M}\frac{d^2W}{dx^2}\,dx \end{cases} \tag{9}$$

where, ω is the circular frequency of vibration. Governing equation of motion is achieved by Hamilton's principle as:

$$\delta\left(T_{max}-U_{max}\right)=0=\int_0^L \rho A\omega^2 W\,\delta W+\mathrm{M}\frac{d^2\delta W}{dx^2} \tag{10}$$

With integrating by parts, we have:

$$\int_0^L\left(\rho A\omega^2 W+\frac{d^2\mathrm{M}}{dx^2}\right)\delta W\,dx+\left[\mathrm{M}\frac{d\,\delta W}{dx}-\frac{d\,\mathrm{M}}{dx}\delta W\right]_0^L=0 \tag{11}$$

Since δW is arbitrary in $0<x<L$, the governing equation of motion is obtained as:

$$\frac{d^2\mathrm{M}}{dx^2}=-\rho A\omega^2 W \tag{12}$$

2. 2. Nonlocal Elasticity Theory Based on Eringen's nonlocal elastic theory, equations for a linear homogenous nonlocal elastic body without the body forces are given as:

$$\begin{aligned} &\sigma_{ij,j}=0 \\ &\sigma_{ij}(x)=\int\alpha\left(|x-x'|,\tau\right)C_{ijkl}\varepsilon_{kl}(x')\,dV(x') \\ &\varepsilon_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right) \end{aligned} \tag{13}$$

where, σ_{ij} is the stress tensor, C_{ijkl} is the fourth-order elasticity tensor, $|x-x'|$ is the distance in Euclidean form and τ is the nonlocal parameter which is defined as

(e_0a/l) depends on the internal (e.g. lattice parameter, granular distance, distance between C-C bonds) and external (e.g. crack length, wavelength) lengths. Due to the difficulty of solving, the integral constitutive Equation (13) can be simplified to equation of differential form as:

$$\left(1-\tau^2 l^2 \nabla^2\right)\sigma = \boldsymbol{t} \tag{14}$$

For a one dimensional elastic material, the Equation (14) can be simplified as:

$$\left(1-(e_0 a)^2 \frac{\partial^2}{\partial x^2}\right)\sigma_{xx}(x) = E\varepsilon_{xx}(x) \tag{15}$$

where, (e_0a) is the scale coefficient which leads to small scale effect and E is the Young's modulus of the nanobeam. Multiplying Equation (15) by zdA and integrating the result over the area *A* yields:

$$M-(e_0 a)^2 \frac{d^2 M}{dx^2} = EI\frac{d^2 w}{dx^2} \tag{16}$$

where, I is the second moment of area. By substituting Equation (12) into Equation (16), we have:

$$M_{nonlocal} = -EI\frac{\partial^2 \mathrm{w}}{\partial x^2} + (\mathrm{e}_0 a)^2\left[\rho A(x)\omega^2 W\right] \tag{17}$$

Considering an elastic nanobeam with non-uniform variation of cross section while the characteristic height of the cross section or the thickness of the beam is kept constant and the characteristic width of the cross section is assumed to vary exponentially along the length of the beam as:

$$\begin{cases} b(x) = b_0 e^{Nx} \\ b_1 = b_0 e^{Nl} \end{cases} \tag{18}$$

$$\begin{cases} A(x) = b_0 h e^{Nx} \\ A_0 = b_0 h \\ I(x) = \frac{1}{12} b_0 h^3 e^{Nx} \\ I_0 = \frac{1}{12} b_0 h^3 \end{cases} \tag{19}$$

where, N is the non-uniformity parameter, b_0 and b_1 are the width of the beam at the left and right end of the beam shown in Figure 1, I_0 and A_0 are second moment of area and cross section of the beam at the left end and h is the thickness of the beam.

Substituting Equations (17), (18) and (19) into Equation (12), the equation of motion of non-uniform elastic isotropic nanobeam which can be derived as:

$$\begin{aligned} &EI_0 \frac{d^4 w}{dx^4} + 2NEI_0 \frac{d^3 w}{dx^3} + EI_0\left(N^2 + \frac{(e_0 a)^2 \rho A_0 \omega^2}{EI_0}\right)\frac{d^2 w}{dx^2} + \\ &2(e_0 a)^2 \rho N A_0 \omega^2 \frac{dw}{dx} + \left((e_0 a)^2 N^2 - 1\right)\rho A_0 \omega^2 w = 0 \end{aligned} \tag{20}$$

Note that with α=0 the Equation (20) reduces to classical equation of motion of non-uniform elastic

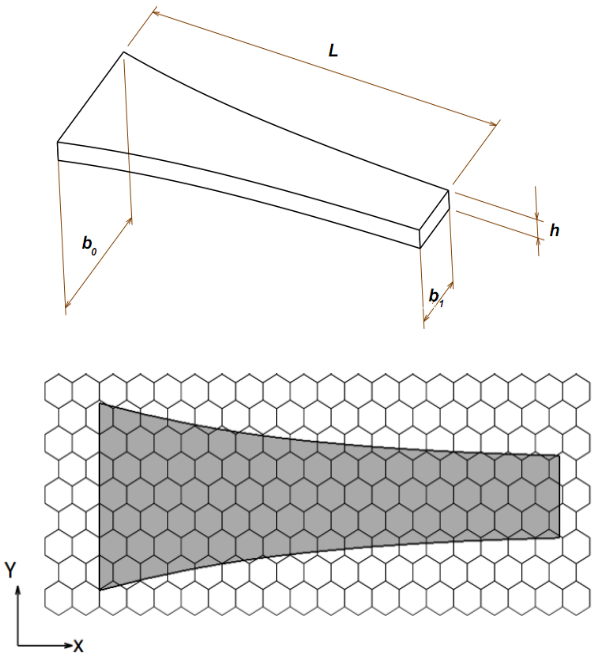


Figure 1. Model of single layered nanobeam with exponentially width variation

beam and with N=0, the Equation (20) reduces to equation of motion of uniform elastic nanobeam and by having both α=0 and N=0 Equation (20) reduces to equation of motion of uniform elastic beam. The dimensionless variables are defined as:

$$\begin{aligned} &X = \frac{x}{L},\ W = \frac{w}{L},\ \alpha = \frac{e_0 a}{L},\ \lambda^2 = \frac{\rho A \omega^2 L^4}{EI}, \\ &\zeta = \frac{L\sqrt{A}}{\sqrt{I}},\ \eta = NL,\ \gamma^2 = \frac{\rho\omega^2 L^2}{E} \end{aligned} \tag{21}$$

where, α denotes the dimensionless nonlocal parameter, λ is the natural frequency parameter, X and W are the dimensionless coordinate measured from the left end of the beam along the length and the dimensionless transverse displacement and η is the dimensionless non-uniformity parameter.

Using these parameters, non-dimensional form of formulation procedures will be defined as:

$$\begin{aligned} &W^{(4)} + 2\eta W''' + \left(\eta^2 + \alpha^2\lambda^2\right)W'' + 2\alpha^2\eta\lambda^2 W' + \\ &\lambda^2\left(\alpha^2\eta^2 - 1\right)W = 0 \end{aligned} \tag{22}$$

In the present study, end of the beams are considered to be simply supported (S) or clamped (C) or free (F). the boundary conditions associated with both ends being simply supported (SS), both ends being clamped (CC) and left end being clamped while the right end being free (CF) may be written in the same order as:

$$\begin{cases} Case\,I \quad (SS): & W(0)=0,\; W''(0)=0, \\ & W(1)=0,\; W''(1)=0 \\ Case\,II \;\; (CC): & W(0)=0,\; W'(0)=0, \\ & W(1)=0,\; W'(1)=0 \\ Case\,III \;\; (CF): & W(0)=0,\; W'(0)=0, \\ & W''(1)=0,\; W'''(1)=0 \end{cases} \tag{23}$$

3. SOLUTION PROCEDURE

Solution of Equation (22) subjected to either boundary conditions given by Equation (23) can be written as:

$$W(X)=C_1e^{(A_1X)}+C_2e^{(A_2X)}+C_3e^{(A_3X)}+e^{(A_4X)} \tag{24}$$

where, $A1$ to $A4$ are depended on natural frequency parameter. Applying boundary conditions in each case yields to an equation for the determination of the natural frequency. The natural frequency equation and the coefficients C_1 to C_3 are given below for each boundary condition case.

Case I: Both ends of the beam are simply supported (SS).

$$W(0)=0: \quad C_3=-1-C_1-C_2 \tag{25}$$

$$W''(0)=0: \quad C_2=-\frac{A_1^2C_1-A_3^2C_1-A_3^2+A_4^2}{A_2^2-A_3^2} \tag{26}$$

$$W''(1)=0: \quad C_1=-L_{11}/L_{12} \tag{27}$$

By using the last boundary condition and applying C1, C2, C3 from the above equations we get:

$$\begin{aligned} W''(1)=0: \; & \frac{e^{A_1}L_{11}}{L_{12}}- \\ & \frac{e^{A_2}}{A_2^2-A_3^2}\left(\frac{\left(A_1^2-A_3^2\right)L_{11}}{L_{12}}+\left(A_4^2-A_3^2\right)\right)+ \\ & e^{A_3}\left(-1-\frac{L_{11}}{L_{12}}+\frac{(A_1^2+A_3^2)}{A_2^2-A_3^2}\left(\frac{L_{11}}{L_{12}}\right)-\frac{A_3^2-A_4^2}{A_2^2-A_3^2}\right)+e^{A_4}=0 \end{aligned} \tag{28}$$

Case II: Both ends of the beam are clamped (CC).

$$W(0)=0: \quad C_3=-1-C_1-C_2 \tag{29}$$

$$W'(0)=0: \quad C_2=-\frac{A_1C_1-A_1C_1-A_3+A_4}{A_2-A_3} \tag{30}$$

$$W'(1)=0: \quad C_1=-L_{21}/L_{22} \tag{31}$$

Applying Equations (29), (30) and (31) into the last boundary condition yields to the equation of natural frequency as:

$$\begin{aligned} W(1)=0: \; & \frac{e^{A_1}L_{21}}{L_{22}}-\frac{e^{A_2}}{A_2-A_3}\left(\frac{\left(A_1-A_3\right)L_{21}}{L_{22}}-A_3+A_4\right)+ \\ & e^{A_3}\left(-1-\frac{L_{21}}{L_{22}}+\frac{A_1-A_3}{A_2-A_3}\left(\frac{L_{21}}{L_{22}}\right)-\frac{A_3-A_3}{A_2-A_3}\right)+e^{A_4}=0 \end{aligned} \tag{32}$$

Case III: The left end of the beam is clamped while the right end is free (CF).

$$W(0)=0: \quad C_3=-1-C_1-C_2 \tag{33}$$

$$W'(0)=0: \quad C_2=-\frac{A_1C_1-A_1C_1-A_3+A_4}{A_2-A_3} \tag{34}$$

$$W'''(1)=0: \quad C_1=-L_{31}/L_{32} \tag{35}$$

With using the last boundary condition and applying C_1, C_2, C_3 from the above equations we get:

$$\begin{aligned} W''(1)=0: \; & \frac{2e^{A_1}A_1^2L_{31}}{L_{32}}- \\ & \frac{2e^{A_2}A_2^2}{A_2^2-A_3^2}\left(\frac{\left(A_1-A_3\right)L_{31}}{L_{32}}-A_3+A_4\right)+ \\ & 2e^{A_3}A_3^2\left(-1-\frac{L_{11}}{L_{12}}+\frac{A_1-A_3}{A_2-A_3}\left(\frac{L_{11}}{L_{12}}\right)-\frac{A_3-A_4}{A_2-A_3}\right)+2e^{A_4}A_4^2=0 \end{aligned} \tag{36}$$

where, L_{11}, L_{12}, L_{21}, L_{22}, L_{31} and L_{32} are defined in appendix A. By knowing the coefficients C_1, C_2, C_3 andλ, the unsteady transverse vibration of the beam can then be written as:

$$W=\left(C_1e^{\gamma_1X}+C_2e^{\gamma_2X}+C_3e^{\gamma_3X}+e^{\gamma_4X}\right)\left[d_1\cos(\omega t)+d_2\sin(\omega t)\right] \tag{37}$$

4. RESULTS AND DISCUSSIONS

To illustrate the influence of small length scale and non-uniformity cross section on the vibration of single layered graphene nanoribbons (SLGNRs), the non-dimensional natural frequency parameter is presented for different non-locality parameter, non-uniformity parameter and mode numbers. Due to the works done by Wang and Wang [16], it has been shown that the value of e_0a should be smaller than 2.0 nm for carbon nanotubes which the exact value of nonlocal parameter is not exactly known [17]. The external characteristic length is taken as L=5 nm [17] so the nonlocal or scale coefficient parameter is $\alpha = e_0a/L$ =0.0, 0.1, 0.2, 0.3, 0.4 and the non-uniformity parameter is also taken as $|\eta|$=0.2, 0.4, …, 1. It should be noted that due to the symmetric of the boundary conditions the sign of the non-uniformity parameter does not change the results in clamped or simply-supported beam while it has influence on cantilever beams. The analysis presented, describes the nonlocal free vibration of a nanobeam with exponentially varying characteristic width and provides the analytical solutions. The natural frequency for the SS, CC ad CF boundary conditions are obtained by Newton's method numerically solving the implicit Equations (28), (32) and (36), respectively.

4. 1. Validation of Present Computed Results

In order to verify the validation of present solution procedure, the nonlocal parameter α is assumed to be zero to compare the present solution with an isotropic variable cross section beam. In Table 1, the non-dimensional natural frequency parameter of a simply supported beam with exponentially width variation is presented while the non-local parameter is assumed to be zero and the results are compared with those obtained by CemEce et al., [2007]. Same works has been done for clamped and cantilever beams which are presented and compared in Tables 2, 3 and Table 4. Also, to prove the results in nonlocal matter, the non-uniformity parameter N is assumed to be zero to compare the present study with nonlocal Euler beam Wang et al., [2007]. In Table 5, the non-dimensional Natural frequency parameter of a simply-supported beam with various nonlocal parameters are presented while the non-uniformity parameter is to be zero and the results are compared with those obtained for different boundary conditions which are presented by Wang et al., [2007]. Same works has been done and compared in Tables 6 and 7 for clamped and cantilever beam with various scaling effect parameters.

TABLE 1. Natural frequency parameters for a simply supported beam with exponentially width variation

A	Mode Number	Natural frequencies (S-S)			
		|η|=1	[3]	|η|=2	[3]
0	1	3.12617	3.1262	3.08013	3.0801
0	2	6.29050	6.2905	6.31287	6.3129
0	3	9.42477	9.4248	9.45543	9.4554
0	4	12.56638	12.5664	12.59352	12.5935
0	5	15.70801	15.7080	15.73169	15.7317

TABLE 2. Natural frequency parameters for a clamped beams with exponentially width variation

A	Mode Number	Natural frequencies (C-C)			
		|η|=1	[3]	|η|=2	[3]
0	1	4.74464	4.7446	4.78933	4.7893
0	2	7.86509	7.8651	7.90081	7.9008
0	3	11.00491	11.0049	11.03280	11.0328
0	4	14.14476	14.1448	14.16752	14.1675
0	5	17.28516	17.2852	17.30433	17.3043

TABLE 3. Natural frequency parameters for a cantilever beams with exponentiallywidth variation (η>0)

α	Mode Number	Natural frequencies (C-F)			
		η=1	[3]	η=2	[3]
0	1	1.69067	1.6906	1.70557	1.7056
0	2	4.47652	4.4765	4.26325	4.2632
0	3	7.73763	7.7376	7.64125	7.6413
0	4	10.91323	10.9132	10.84860	10.8486
0	5	14.07372	14.0737	14.02506	14.0251

TABLE 4. Natural frequency parameters for a cantilever beams with exponentially widthvariation (η<0)

A	Mode Number	Natural frequencies (C-F)			
		η=1	[3]	η=2	[3]
0	1	2.17324	2.1732	2.50175	2.5018
0	2	4.91952	4.9195	5.15593	5.1559
0	3	7.99153	7.9915	8.14705	8.1471
0	4	11.09495	11.0950	11.21092	11.2109
0	5	14.21509	14.2151	14.30718	14.3072

TABLE 5. Natural frequency parameters for a simply supported beams with various scaling effect parameters

α	Mode Number	Natural frequencies (S-S)							
		α=0.1	[7]	α=0.3	[7]	α=0.5	[7]	α=0.7	[7]
0	1	3.06853	3.0685	2.67999	2.6800	2.30223	2.3022	2.02125	2.0212
0	2	5.78167	5.7817	4.30134	4.3013	3.46040	3.4604	2.95858	2.9585
0	3	8.03998	8.0400	5.44225	5.4422	4.29406	4.2941	3.64855	3.6485
0	4	9.91611	9.9161	6.36299	6.3630	4.98200	4.9820	4.22339	4.2234
0	5	11.51114	11.5111	7.15677	7.1568	5.58250	5.5825	4.72734	4.7273

TABLE 6. Natural frequency parameters for a clamped beams with various scaling effect parameters

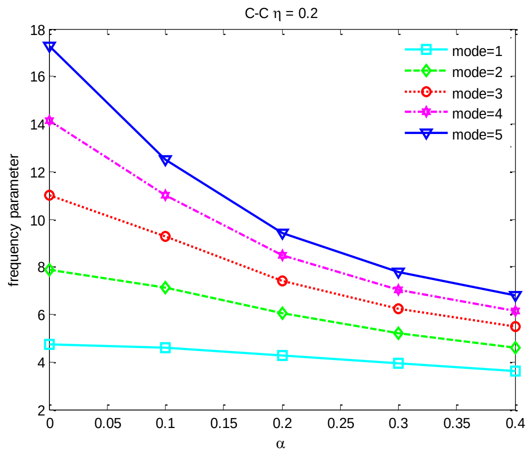
α	Mode Number	Natural frequencies (C-C)							
		α=0.1	[7]	α=0.3	[7]	α=0.5	[7]	α=0.7	[7]
0	1	4.59445	4.5945	3.91836	3.9184	3.31532	3.3153	2.88934	2.8893
0	2	7.14024	7.1402	5.19631	5.1963	4.15608	4.1561	3.54624	3.5462
0	3	9.25831	9.2583	6.23169	6.2317	4.93279	4.9328	4.19964	4.1996
0	4	11.01580	11.016	7.04819	7.0482	5.52128	5.5213	4.68164	4.6816
0	5	12.51960	12.520	7.79554	7.7955	6.09627	6.0963	5.16893	5.1689

TABLE 7. Natural frequency parameters for a cantilever beam with various scaling effect parameters

α	Mode Number	Natural frequencies (C-F)							
		α=0.1	**[7]**	**α=0.3**	**[7]**	**α=0.5**	**[7]**	**α=0.7**	**[7]**
0	1	1.87917	1.8792	1.91537	1.9154	2.02192	2.0219	-	-
0	2	4.54748	4.5475	3.76654	3.7665	2.94327	2.9433	-	-
0	3	7.14589	7.1459	5.29876	5.2988	-	-	-	-
0	4	9.25687	9.2569	6.13853	6.1385	-	-	-	-
0	5	11.01641	11.016	7.14501	7.1450	-	-	-	-

4. 2. Nonlocal Effect on Vibration of Beam with Exponentially Width Variation For various non-local parameters, the variation of natural frequencies are plotted in Figure 2 when the non-uniformity parameter is 0.2 for simply-supported, clamped and cantilever beam. The same analysis has been done for non-uniformity parameter being 0.4 and 1 which is presented in Figures 3 and 4.

Also, in Figure 5, variation of the first mode shape frequency parameter of a beam with exponentially varying width for different nonlocal parameters are presented for simply-supported, clamped and cantilever conditions.



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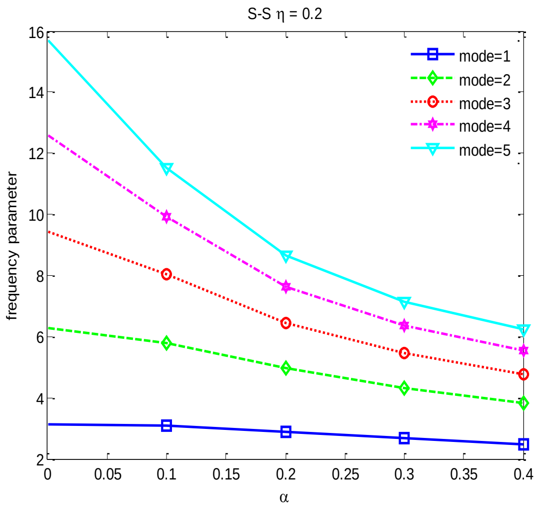

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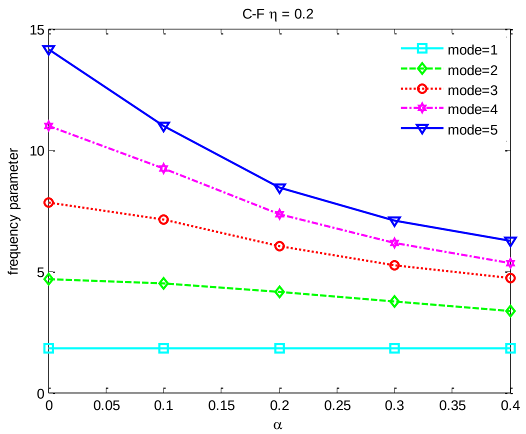
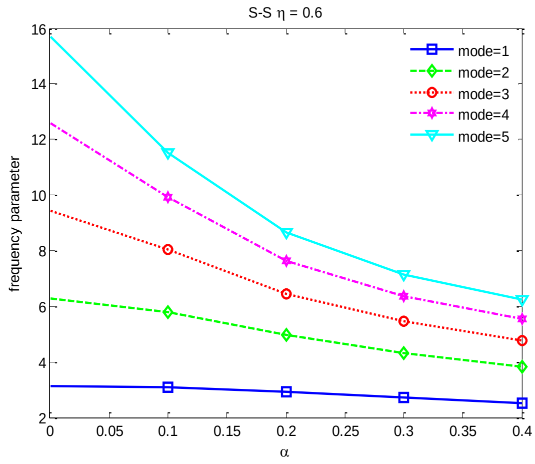


(c)

Figure 2. Variation of frequency parameter of a beam with exponentially varying width (η=0.2) for different nonlocal parameter (a) simply supported (b) Clamped (c) Cantilever



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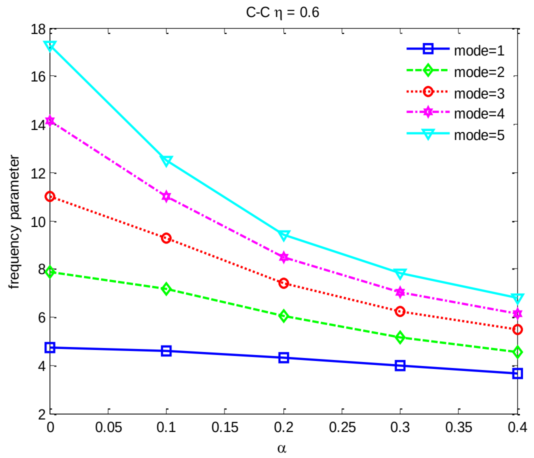

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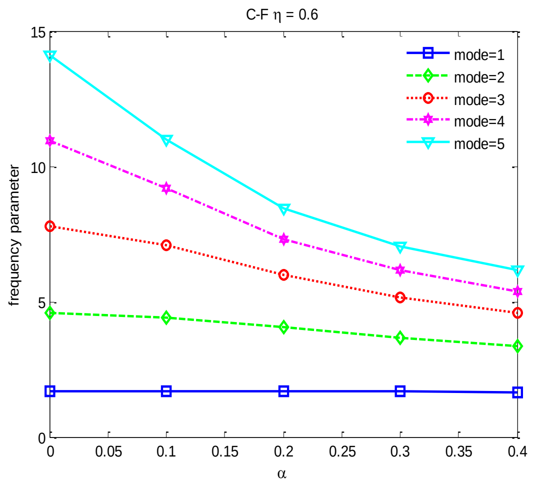
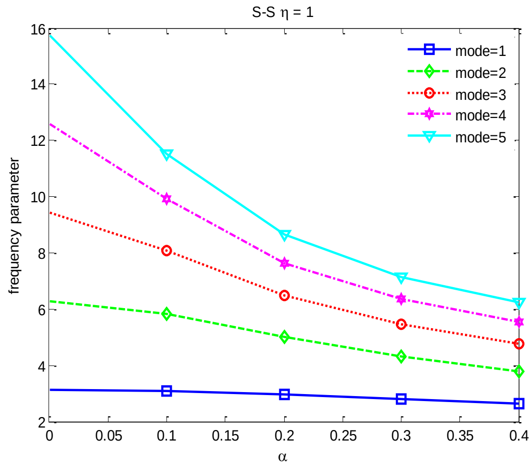
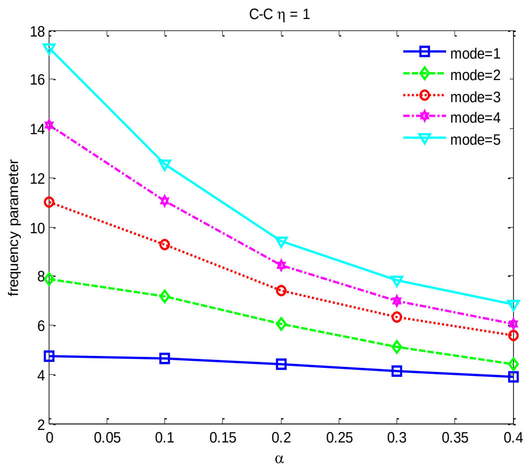


(c)

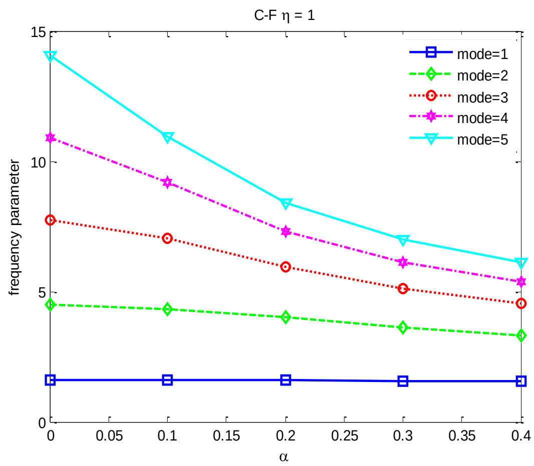
Figure 3. Variation of frequency parameter of a beam with exponentially varying width (η=0.6) for different nonlocal parameter (a) simply supported (b) Clamped (c) Cantilever



(a)

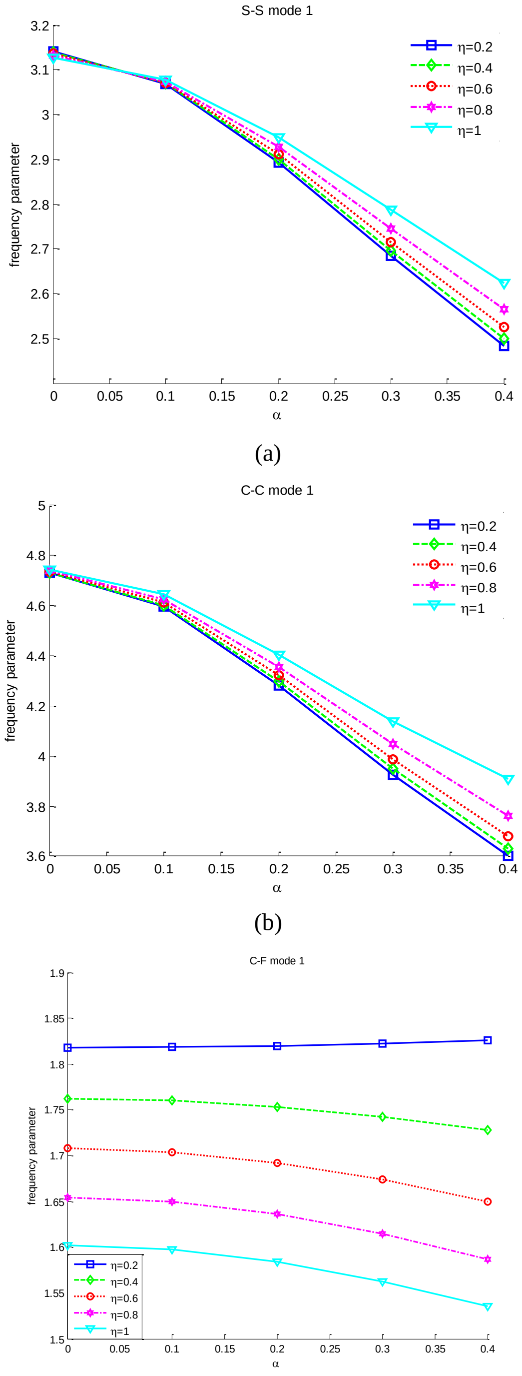


(b)



(c)

Figure 4. Variation of frequency parameter of a beam with exponentially varying width (η=1) for different nonlocal parameter (a) simply supported (b) Clamped (c) Cantilever



(a) (b) (c)

Figure 5. Variation of first mode shape frequency parameter of a beam with exponentially varying width for different nonlocal parameter (a) simply supported (b) Clamped (c) Cantilever

5. CONCLUSION

A general analytical solution, based on the Eringen's nonlocal elasticity theory is formulated for non-uniform nanobeams to describe the free vibration of single-layered graphene nanoribbons (GNRs) with different boundary conditions. Results showed that the frequency ratio of the non-uniform nanobeam is sensitive to the non-uniformity and nonlocal effect. When the nonlocal effect was taken into account without consideration non-uniformity, the frequency ratio of the beam decreased by increasing the nonlocal parameter except for the first frequency mode of cantilever nanobeam. Also, for the time when non-uniformity was taken into account without consideration nonlocal effect, the frequency ratio of the beam changes differently with an increase in non-uniformity parameter depending on the boundary conditions. It was also shown that by increasing the non-uniformity parameter to a special amount in cantilever beams, the first mode of vibration will also decrease by increasing the nonlocal parameter. Non-uniformity has the most effect on the first mode of vibration while the dependency decreases in higher modes for all boundary conditions and nonlocal parameters.

6. APPENDIX A

L_{11}, L_{12}, L_{21}, L_{22}, L_{31} and L_{32} are defined as

$$\begin{cases} L_{11} = A_2^2 A_3^2\left(e^{A_2}-e^{A_3}\right)+A_2^2 A_4^2\left(e^{A_4}-e^{A_2}\right)+A_3^2 A_4^2\left(e^{A_3}-e^{A_4}\right) \\ L_{12} = A_1^2 A_2^2\left(e^{A_1}-e^{A_2}\right)+A_1^2 A_3^2\left(e^{A_3}-e^{A_1}\right)+A_2^2 A_3^2\left(e^{A_2}-e^{A_3}\right) \\ L_{21} = A_2 A_3\left(e^{A_2}-e^{A_3}\right)+A_2 A_4\left(e^{A_4}-e^{A_2}\right)+A_3 A_4\left(e^{A_3}-e^{A_4}\right) \\ L_{22} = A_1 A_2\left(e^{A_1}-e^{A_2}\right)+A_1 A_3\left(e^{A_3}-e^{A_1}\right)+A_2 A_3\left(e^{A_2}-e^{A_3}\right) \\ L_{31} = A_2 A_3\left(A_2^2 e^{A_2}-A_3^2 e^{A_3}\right)+A_2 A_4\left(A_4^2 e^{A_4}-A_2^2 e^{A_2}\right) \\ \quad +A_3 A_4\left(A_3^2 e^{A_3}-A_4^2 e^{A_4}\right) \\ L_{32} = A_1 A_2\left(A_1^2 e^{A_1}-A_2^2 e^{A_2}\right)+A_1 A_3\left(A_3^2 e^{A_3}-A_1^2 e^{A_1}\right) \\ \quad +A_2 A_3\left(A_2^2 e^{A_2}-A_3^2 e^{A_3}\right) \end{cases} \tag{A.1}$$

7. REFERENCES

1. Mirzabeigy, A., "Semi-analytical approach for free vibration analysis of variable cross-section beams resting on elastic foundation and under axial force", ***International Journal of Engineering, Transactions C: Aspects***, Vol. 27, No. 3, (2014), 455-463.
2. Chaudhari, T. and Maiti, S., "Modelling of transverse vibration of beam of linearly variable depth with edge crack", ***Engineering Fracture Mechanics***, Vol. 63, No. 4, (1999), 425-445.
3. Ece, M.C., Aydogdu, M. and Taskin, V., "Vibration of a variable cross-section beam", ***Mechanics Research Communications***, Vol. 34, No. 1, (2007), 78-84.
4. Attarnejad, R. and Shahba, A., "Dynamic basic displacement functions in free vibration analysis of centrifugally stiffened tapered beams; a mechanical solution", ***Meccanica***, Vol. 46, No. 6, (2011), 1267-1281.

5. Eringen,A.C., Nonlocal continuum field theories, ***New York (NY): Springer***, (2002), 148-159.

6. Peddieson, J., Buchanan, G.R. and McNitt, R.P., "Application of nonlocal continuum models to nanotechnology", ***International Journal of Engineering Science***, Vol. 41, No. 3, (2003), 305-312.

7. Wang, C., Zhang, Y. and He, X., "Vibration of nonlocal timoshenko beams", ***Nanotechnology***, Vol. 18, No. 10, (2007), 105-113.

8. Sadeghian, M. and Ekhteraei Toussi, H., "Frequency analysis for a timoshenko beam located on an elastic foundation", ***International Journal of Engineering***, Vol. 24, No.1, (2011), 87-105.

9. Shah-Mohammadi-Azar, A., Khanchehgardan, A., Rezazadeh, G. and Shabani, R., "Mechanical response of a piezoelectrically sandwiched nano-beam based on the nonlocaltheory", ***International Journal of Engineering***, Vol. 26, No. 12, (2013), 1515-1524.

10. Khanchehgardan, A., Shah-Mohammadi-Azar, A., Rezazadeh, G. and Shabani, R., "Thermo-elastic damping in nano-beam resonators based on nonlocal theory", ***International Journal of Engineering-Transactions C: Aspects***, Vol. 26, No. 12, (2013), 1505-1513.

11. Bazehhour, B.G. and Rezaeepazhand, J., "Simplified approach for torsional analysis of non-homogenous tubes with non-circular cross-sections", ***International Journal of Engineering***, Vol. 25, No. 3, (2012), 277-287.

12. Loya, J., Lopez-Puente, J., Zaera, R. and Fernández-Sáez, J., "Free transverse vibrations of cracked nanobeams using a nonlocal elasticity model", ***Journal of Applied Physics***, Vol. 105, No. 4, (2009),78-86.

13. Beni, Y.T., Jafaria, A. and Razavi, H., "Size effect on free transverse vibration of cracked nano-beams using couple stress theory", ***International Journal of Engineering-Transactions B: Applications***, Vol. 28, No. 2, (2014), 296-305.

14. Bağdatlı, S.M., "Non-linear vibration of nanobeams with various boundary condition based on nonlocal elasticity theory", ***Composites Part B: Engineering***, Vol. 80, (2015), 43-52.

15. Rao, S.S., "Vibration of continuous systems, John Wiley & Sons, (2007), 69-75.

16. Wang, Q. and Wang, C., "The constitutive relation and small scale parameter of nonlocal continuum mechanics for modelling carbon nanotubes", ***Nanotechnology***, Vol. 18, No. 7, (2007),756-764.

17. Pradhan, S. and Murmu, T., "Small scale effect on the buckling of single-layered graphene sheets under biaxial compression via nonlocal continuum mechanics", ***Computational materials science***, Vol. 47, No. 1, (2009), 268-274.

Analytical Solution for Free Vibration of a Variable Cross-section Nonlocal Nanobeam

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P A P E R I N F O

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چکیده

در این مقاله رفتار مکانیکی دسته ای از تیرهای سطح مقطع متغیر با در نظر گیری اثرات غیرمحلی با توجه به تئوری غیرمحلی ارینگن بررسی شده است. تغییرات سطح مقطع به شکل نمایی در راستای طولی با ضخامت ثابت فرض شده است. معادلات ارتعاشات آزاد این دسته از تیرها برای شرایط مرزی مختلف گام به گام به شکل تحلیلی بدست آمده است. نتایج بدست آمده نشان از تاثیر بالای استفاده از سطح مقطع متغیر در کنار اثرات غیرمحلی است. برای نمایش میزان حساسیت فرکانس ارتعاشی بر تغییر شکل سطح مقطع تیر، پنج مود اول ارتعاشی نانوریبون ها با تکیه گاه های مختلف با داشتن طیف گسترده ای از پارامترهای غیرمحلی و غیریکنواختی مورد بررسی قرار گرفتند.

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