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H_∞ Robust Controller Design and Experimental Analysis of Active Magnetic Bearings with Flexible Rotor System

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ABSTRACT

H_∞ controller for active magnetic bearings (AMBs) with flexible rotor system was designed in this paper. The motion equations of AMBs and flexible rotor system are built based on finite element methods (FEM). Weighting function matrices of H_∞ controller for AMBs are studied for both the sensitivity and the complementary sensitivity of H_∞ control theory. The experiments are completed on a four-degree freedom magnetic bearings-flexible rotor test rig. The experimental results show that the H_∞ control method has a better ability to depress vibration than traditional PID control. H_∞ controller is characterized by the effectiveness of interference immunity and robust stability. The peak to peak vibration amplitudes of flexible rotor are less than 60μm at the first critical speed of flexible rotors. The results indicate that H_∞ controller for the active magnetic bearings with flexible rotor system is stable through the first critical speed of the flexible rotor system.

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1. INTRODUCTION

The magnetic bearings which uses magnetic forces to suspend a shaft was first used as supporting bearings of high-speed ultracentrifuges in 1930 [1]. However, magnetic bearings did not become a practical alternative to rolling element bearings, for that, active magnetic bearing (AMB) is the high sensitivity of the control system to parametric uncertainties and bearing nonlinearities. Control of magnetic bearings has been studied in recent years. The most important modern control methods are used for active magnetic bearings, such as PID controllers [2, 3], 2DOF PID control [4], 2DOF controller [5], adaptive control [6, 7], sliding mode control [8], optimal control [9], fuzzy logic control [10, 11], feedback linearization control [12], time-delay control [13], control by transfer function approach [14] and μ-synthesis control [15]. Despite rapid development of advanced control algorithms for AMBs, the effective control of AMB industrial applications are still being studied by more investigations in the future.

H_∞ control theory applied to AMBs control has made great progress in the past two decades [16, 17]. Schonhoff designed AMBs′ controller using μ synthesis theory, and the success achieved through the first critical speed of the rotor [18]. Zhao designed mixed sensitivity of H_∞ controller for AMBs, and the AMBs-rotor system can be crossed second-order critical speed of the rotor [19]. Although there are many on H_∞ control theory applied magnetic bearing control in research, Due to

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constraints modeling conditions, H_∞ control of the electromagnetic bearing technology research to focus more on theory and simulation, and its practical application has yet to be studied in greater depth and perfection.

This paper studies H_∞ control design for an AMB-flexible rotor system. In first section of the paper, the mathematical models of AMBs are to be presented, and the models of flexible rotor system are constructed based on the finite element methods (FEM). In second section, the obtained mathematical models are used for the H_∞ control system design and numerical evaluation of weighting function matrix. In third section, a short survey of the H_∞ control algorithms used for active magnetic bearing control are shown, and the experiments of four-degree freedom magnetic bearings-flexible rotor test rig for control algorithms evaluation are described. The robustness of the H_∞ control system on dynamics and parameter variations are analyzed by means of simulations and experiments.

2. MODEL OF ACTIVE MAGNETIC BEARINGS AND FLEXIBLE ROTOR SYSTEM

As illustrated in Figures 1 and 2, AMBs combine controller and actuators. The actuators comprise 4 pole pair electromagnets, switching power amplifiers and position sensor. The amplifiers convert the control currents into the electrical currents in the coils. These currents produce the magnetic field in the electromagnet, which produces the corresponding magnetic levitation force. The deviation of rotor *x* and *y* from the x and y sensor are used as feedback signal.

The flexible shaft is constructed in terms of beam-type finite element model as Figure 3. The element of shaft has four degrees of freedom as Figure 4. The elastic element vector *U* of displacement and rotation assigned to element *i* can be arranged by Equation (1),

$$U=\begin{bmatrix} x_i & y_i & \theta_{x,i} & \theta_{y,i} & x_{i+1} & y_{i+1} & \theta_{x,i+1} & \theta_{y,i+1} \end{bmatrix}^T \tag{1}$$

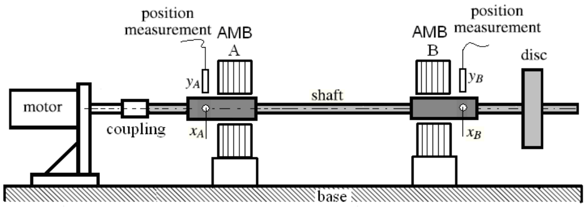

Figure 1. Configuration of active magnetic bearing-rotor system

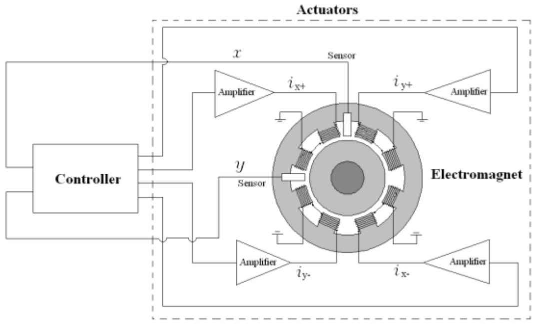

Figure 2. Principle of electromagnetic levitation system

The motion equation of AMBs and rotor system can be described as following [4],

$$[M]\,\ddot{U}+[C]+\Omega[G]\;\dot{U}+[K]\,U=F_{AMB}+F_{mg} \tag{2}$$

Here, ***M*** and ***G*** are the mass matrix and gyroscopic matrix including that of the shaft and the rigid discs. ***C*** is damping matrix, ***K*** is stiffness matrix of the shaft. $\boldsymbol{F_{mg}}$ is the vector of gravity force, and $\boldsymbol{F_{AMB}}$is electromagnetic force of the active magnetic bearing.

The controller provides the control current i_x and i_y in the electromagnet. The electromagnet coils to produce magnetic force $\boldsymbol{F_{AMB}}$ which suspend the rotor. The forces of magnet are to be [18, 19],

$$F_{AMB}=\begin{bmatrix}F_x\\F_y\end{bmatrix}=\begin{bmatrix}k_{sx}&0\\0&k_{sy}\end{bmatrix}\begin{bmatrix}x\\y\end{bmatrix}+\begin{bmatrix}k_{ix}&0\\0&k_{iy}\end{bmatrix}\begin{bmatrix}i_x\\i_y\end{bmatrix}=K_s\begin{bmatrix}x\\y\end{bmatrix}+K_i\begin{bmatrix}i_x\\i_y\end{bmatrix} \tag{3}$$

where, k_{ix} and k_{iy} denote the current stiffness, k_{sx} and k_{sy} are position stiffness, i_x and i_y are the control current , *x* and *y* are the position, respectively.

In which,

$$k_{ix}=\frac{\mu_0 n^2 A i_{x0}}{S^2}cos(\alpha),\quad k_{iy}=\frac{\mu_0 n^2 A i_{y0}}{S^2}cos(\alpha)$$
$$k_{sx}=\frac{\mu_0 n^2 A i_{x0}^2}{S^3}cos(\alpha),\quad k_{sy}=\frac{\mu_0 n^2 A i_{y0}^2}{S^3}cos(\alpha) \tag{4}$$

where, α is the force acting angle, equals to π/8 (half the angle between the poles of an electromagnet); n is the coil turns; A is the air gap, μ_0 is the magnetic permeability of a vacuum, equals to $4\pi\times10^{-7}$Vs/Am; S is the length of air gap. The transfer function of AMBs-flexible rotor system is to be,

$$G(s)=\frac{K_i}{[M]s^2+([C]+\Omega[G])s+([K]-K_s)} \tag{5}$$

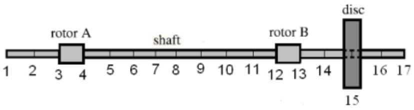

Figure 3. Flexible rotor system

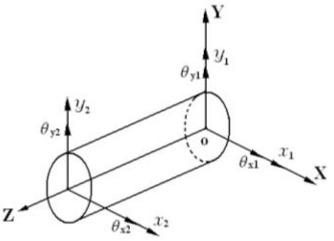

Figure 4. Shaft element

3. H∞ CONTROLLER DESIGN

The dual terminals block diagram of H∞ mixed sensitivity is shown in Figure 5. The transfer function from u_1 to y_1 is given by [20, 21]:

$$T_{y_1u_1}=\begin{bmatrix} W_1\dfrac{1}{1+KG} \\ W_2\dfrac{K}{1+KG} \\ W_3\dfrac{KG}{1+KG} \end{bmatrix}=\begin{bmatrix} W_1S \\ W_2R \\ W_3T \end{bmatrix} \tag{6}$$

H∞ mixed sensitivity optimization method is to find the true function of the controller $\boldsymbol{K}$(s), so that the closed-loop system is stable, and index can be expressed as following:

$$\left\|T_{y_1u_1}\right\|_\infty=\left\|\begin{matrix} W_1S \\ W_2R \\ W_3T \end{matrix}\right\|_\infty \le \gamma \quad \text{, and } \gamma<1 \tag{7}$$

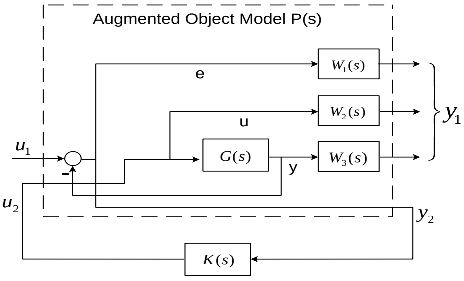

Figure 5. Mixed sensitivity dual terminal block diagram

Select the appropriate weighting function to build augmented object model that can be applied to H∞ control theory to design a controlled object $\boldsymbol{G}$(s) corresponding to the controller. System is shown in Figures 3-5. The augmented object model is given by,

$$P(s)=\begin{bmatrix} W_1 & -W_1G \\ 0 & W_2 \\ 0 & W_3G \\ 1 & -G \end{bmatrix} \tag{8}$$

Here, $\boldsymbol{W}_1$ (s) and $\boldsymbol{W}_3$ (s) are specified by the restraint conditions,

$$\left\|W_1^{-1}\right\|_\infty+\left\|W_3^{-1}\right\|_\infty \ge 1 \tag{9}$$

H∞ control sensitivity depends largely on the quality of the selected $\boldsymbol{W}_1$(s), $\boldsymbol{W}_2$(r) and $\boldsymbol{W}_3$(s) weighting functions. The weighting functions directly reflect the systems' performance, such as dynamic quality, robustness requirements, capability of anti-disturbance. Therefore, it is very important to select optimized weighting functions in the H∞ mixed sensitivity control method design. The weighting functions are selected which depending on the controlled object and the different performance indicators. The optimized weighting functions are given by,

$$W_1=\frac{200}{4.75s+1}, \quad W_2=10^{-8}, \quad W_3=0.001s+1 \tag{10}$$

$\boldsymbol{K}$(s) controller is designed using Matlab/Simulink Robust Control Toolbox of Matlab 2011b.

$$K(s)=\frac{(5.799e010)s^2+(1.828e012)s^2+3.877e011}{s^3+(2.499e008)s^2+(3.905e011)s+8.219e010} \tag{11}$$

Figure 6 shows the step response of $\boldsymbol{K}$(s) controller, and the amplitude-frequency characteristics of sensitivity function ***S***(s), penalty sensitivity function ***T***(s), Weighted function $\boldsymbol{W}_1$(s) and $\boldsymbol{W}_2$(s) are shown in Figure 7. A block schematic of the ***K***(s) controller for single magnet is shown in Figure 8. $\boldsymbol{K}_1$ is the discrete controller ***K***(s) by taking the sampling period T = 0.0001s as following Equation (12).

$$K_1 = \frac{(5.799e010)(\frac{2}{T}\bullet\frac{1-z^{-1}}{1+z^{-1}})^2+(1.828e012)(\frac{2}{T}\bullet\frac{1-z^{-1}}{1+z^{-1}})^2+3.877e011}{(\frac{2}{T}\bullet\frac{1-z^{-1}}{1+z^{-1}})^3+(2.499e008)(\frac{2}{T}\bullet\frac{1-z^{-1}}{1+z^{-1}})^2+(3.905e011)(\frac{2}{T}\bullet\frac{1-z^{-1}}{1+z^{-1}})+8.219e010} \tag{12}$$

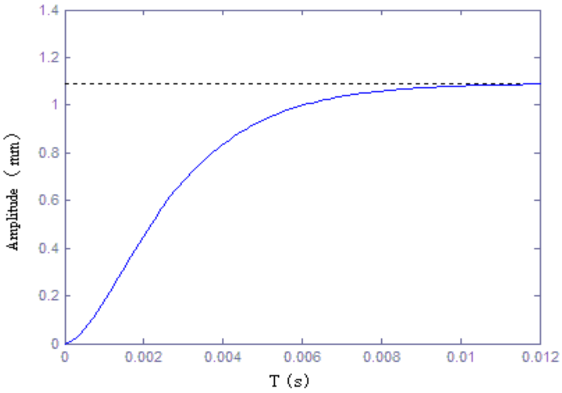

Figure 6. *K*(s) step response

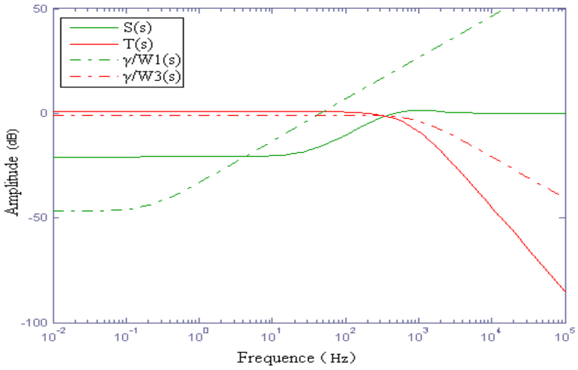

Figure 7. Amplitude-frequency characteristics of parameter $S(s)$, T(s), $\gamma/W_1(s)$, $\gamma/W_3(s)$

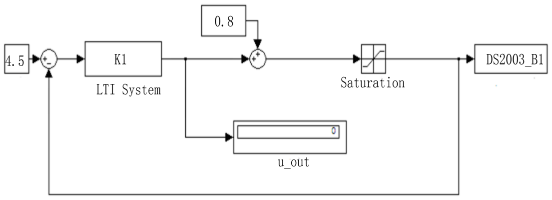

Figure 8. Block diagram for one direction of rotor´ position with AMBs

4. TEST RIG AND IMPLEMENTATION

Figures 9-11 and Tables 1-2 show the detail parameters of shaft, rotor and stator of AMB in test rig. The dSpace DS1103 controller board is used as H_∞ controller for AMBs-flexible rotor system. The position measurements of the two AMBs' rotors are performed by using ZA-GA M8 contactless eddy current displacement sensor. Four ZA-GA M8 contact-less eddy current displacement sensor are used to detect the vibration displacement of two AMBs, one is set in vertical direction of an AMB, and the other is set in horizontal direction of an AMB. DS2003 digital input/output board which is used to input the digital position for AMBs' control.

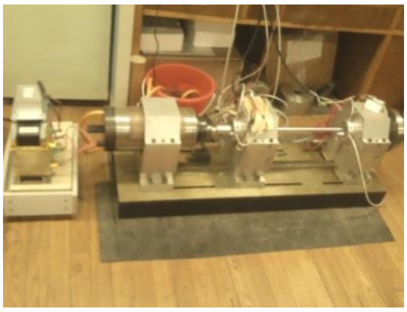

Figure 9. Test rig of AMBs-flexible rotor system

Figure 10. Shaft and rotor of AMBs

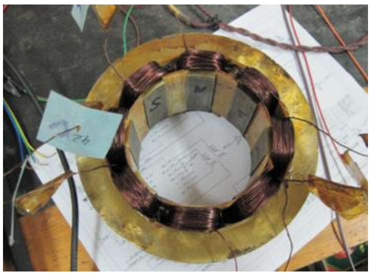

Figure 11. Stator of AMB

5. EXPERIMENTAL ANALYSIS

This section presents the experiment of the H_∞ controller of AMBs-flexible rotor test rig. The mathematical model of H_∞ controller are given by Equations (5) to (12), and the H_∞ controller is done using MATLAB/Simulink toolbox of Matlab 2011b. Figure (12) is the H_∞ control program set for the AMBs of test rig. Figures (13) and (14) illustrate the experiment of H_∞ control and PID control for the test rig at 1500rpm speed. The results show the vibrations of flexible rotor in test rig. The H_∞ control in Figure 13 depicts that the rotor displacement is -40μm<Y<35μm, and PID control in Figure 14 shows that the displacement is -55μm<Y<140μm. The H_∞ controller has a better ability to depress the vibrations of AMBs' rotor in test rig. The experimental results of the flexible rotor at speed 2400rpm, 3000rpm and 4200rpm are illustrated in Figures15, 16 and 17. The responses of flexible rotor show the horizontal direction vibration in test rig as depicted in Figures 15-17. The flexible rotor's first-order bending critical speed is about 3000rpm, and the results in Figure 16 confirms that the observed shaft displacement is -55μm<X<60μm, -70μm<Y<60μm, obtained from the position sensors. The H_∞ controller ensures the flexible rotor system smoothly through rotor's first-order bending critical speed.

TABLE 1. Detail of rotor in test rig

Parameters	Physical dimension
Diameter of shaft	20 mm
Length of shaft	850mm
Outer diameter of AMB's rotor	95 mm
Inner diameter of AMB's rotor	20 mm
Thickness of AMB's rotor	60 mm
Outer diameter of disc	200mm
Inner diameter of disc	20 mm
Thickness of disc	30 mm
Damping coefficients C_x	500N/m/s
Damping coefficients C_y	500N/m/s

TABLE 2. Detail of AMB

Parameters	Physical dimension
Inner diameter of AMB's stator	95.6 mm
Outer diameter of AMB's stator	170mm
Thicker of AMB's stator	95 mm
Area magnetic pole	1147.12 mm^2
Coil turns	110
Air gap	0.3mm
Angle of magnetic pole	22.5°
Coil resistance	2.5Ω
Bias current	2.5A
Saturation magnetic dense	1.2T
Force-current coefficient of AMB	716.2N/A
Force-displacement coefficient of AMB	2.205KN/mm

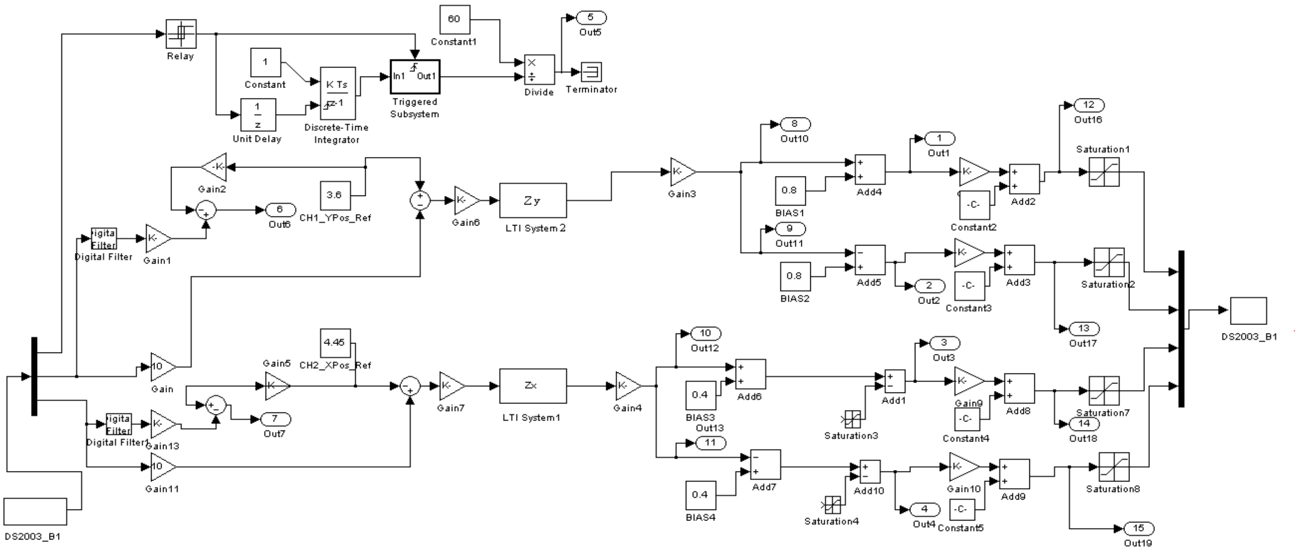

Figure 12. H_∞ control program for 4-DOF AMBs

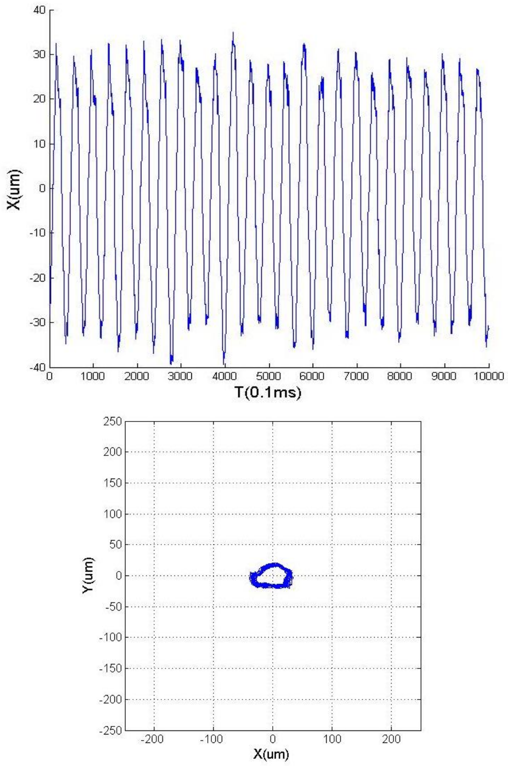

Figure 13. H_∞ control AMB, vibration of horizontal direction and orbit of the rotor at speed n=1500rpm.

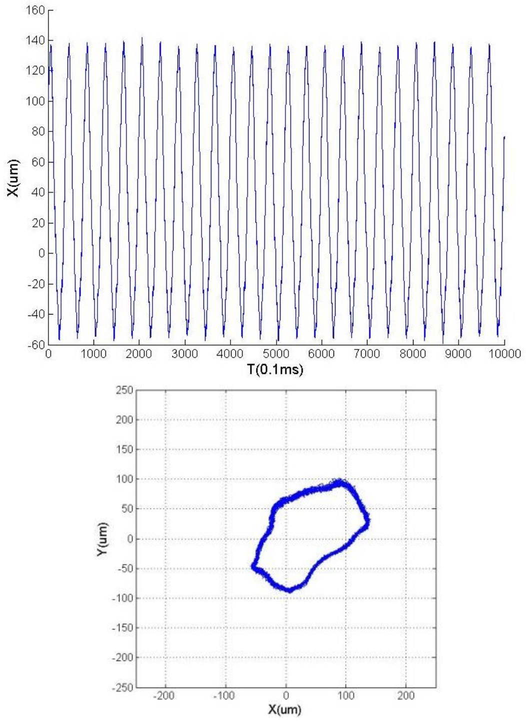

Figure 14. PID control of AMB, vibration of horizontal direction and orbit of the rotor at speed n=1500rpm.

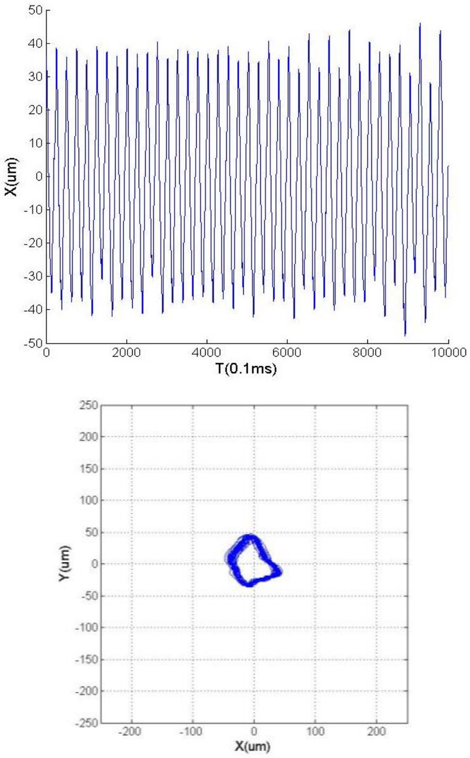

Figure 15. H_∞ control AMB, vibration of horizontal direction and orbit of the rotor at speed n=2400rpm.

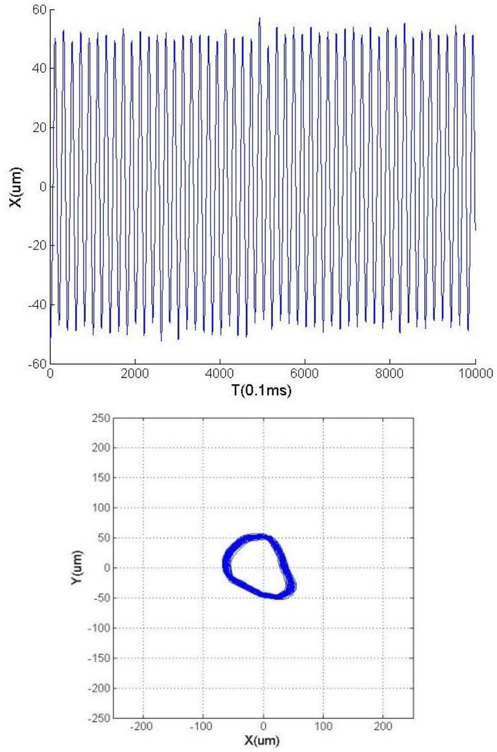

Figure 16. H_∞ control AMB, vibration of horizontal direction and orbit of the rotor at speed n=3000rpm.

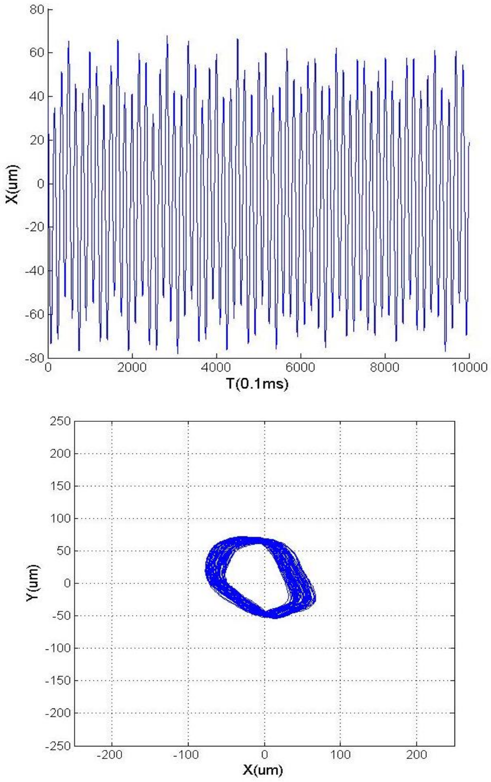

Figure 17. H_∞ control AMB, vibration of horizontal direction and orbit of the rotor at speed n=4200rpm.

6. CONCLUSIONS

H_∞ control for AMBs is discussed in this paper. Firstly, the standard design of H_∞ control is described. Several important parameters of H_∞ control are given by analyzing the impact of interference signals and Model uncertainty in feedback system, then standard framework of H_∞ mixed sensitivity is established. The mixed sensitivity and weighting functions of H_∞ control are discussed in detail, and the AMBs´ controller is designed by H_∞ standard design.

The test rig is designed for AMBs-flexible rotor system. The experimental results clearly show that the H_∞ control model designed in the paper improves the performance and robustness for the AMBs system, and ensure the flexible rotor system smoothly through first-order bending critical speed of the flexible rotor system. This article is intended to be a reference for AMBs´ control method. The next work will focus on controller design using DSPs.

7. ACKNOWLEDGEMENT

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چکیده

کنترل $H\infty$ برای یاتاقانهای مغناطیسی فعال (AMBS) با سیستم روتور انعطاف پذیر در این مقاله طراحی شده است. معادلات حرکت AMBS و سیستم روتور انعطاف پذیر بر اساس روش المان محدود (FEM) ساخته شده است. ماتریس تابع وزن از کنترل $H\infty$ برای AMBS برای هر دو حساسیت و حساسیت مکمل تئوری کنترل$H\infty$ مورد مطالعه قرار گرفت. آزمایش بر روی یک دکل آزمون چرخشی با یاطاقان های انعطاف پذیر مغناطیسی با درجه آزادی چهار تکمیل شد. نتایج تجربی نشان می دهد که روش کنترل $H\infty$ نسبت به کنترل سنتی PID دارای توانایی بهتری برای کاهش لرزش می باشد. کنترل $H\infty$ توسط اثربخشی ایمنی تداخل و ثبات قدرتمند خصوصیت دهی می شود. از یک قله تا قله دیگر دامنه ارتعاش روتور انعطاف پذیر در اولین سرعت بحرانی روتورهای انعطاف پذیر کمتر از 60 µm است. نتایج نشان می دهد که کنترل کننده $H\infty$ برای یاتاقانهای مغناطیسی فعال با سیستم روتور انعطاف پذیر از طریق اولین سرعت بحرانی سیستم روتور انعطاف پذیر پایدار است.

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