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RESEARCH NOTE

Damped Vibrations of Parabolic Tapered Non-homogeneous Infinite Rectangular Plate Resting on Elastic Foundation

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ABSTRACT

In the present paper, damped vibrations of non-homogeneous infinite rectangular plate of parabolically varying thickness resting on elastic foundation has been studied. Following Lévy approach, the equation of motion of plate of varying thickness in one direction is solved by quintic spline method. The effect of damping, elastic foundation and taperness is discussed with permissible range of parameters. The frequency parameter Ω decreases as damping parameter Dk increases and it decreases faster in clamped-simply supported as compared to clamped-clamped boundary conditions. It was also observed that in the presence of damping parameter Dk the frequency parameter Ω decreases continuously with increasing value of taper parameter for both the boundary conditions but variations were found in the absence of damping parameter.

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1. INTRODUCTION

In this era of science and technology, due to the significant role of plate vibration in every field of applied sciences, it is required of an accurate determination of their natural frequencies and mode shapes. Thus, the knowledge of natural frequencies of plate is of considerable importance at the design stage in order to avoid resonances. The study of vibrational behavior of plate of variable thickness has great importance due to their increasing use in aerospace industry, electronic and optical equipments and missile technology, as plates with variable thickness have significantly greater efficiency for bending, buckling and vibration as compared to plate with uniform thickness. Non–homogeneous elastic plates have acquired great importance as in many practical situations particularly in aerospace industry, naval ship design and telephone industry, etc.; require a phenomenal increase in the development of fiber reinforced material due to desirability high strength, light weight, corrosion resistance and high temperature performance. An extensive review of the work up to 1985 on linear vibration of isotropic/anisotropic plates of various geometries has been given by Liessa in his monograph [1] and in a series of review articles [2-5]. In an up to date survey of literature, the author has come across various models to account for infinite plate proposed by researchers dealing with vibration. In 1973, Jain and Soni [6, 7] investigated the free vibration of an infinite strip of variable thickness and they also analyzed the free vibrations of rectangular plate of parabolically varying thickness. On the other hand, Sobczyk [8] has considered the free vibrations of elastic plate of uniform thickness whose elastic modulus varies randomly whereas Poisson's ratio and density of the material of the plate remain constant. Plates resting on elastic foundation have applications in pressure vessels technology such as petrochemical, marine and aerospace industry, building activities in cold regions and aircraft landing in arctic operations [9, 10]. In this order, Chen [11] solved the problem of bending and vibrations of plates of variable thickness and Tomar and Gupta [12] have studied the vibrations of isotropic homogeneous infinite plate of parabolically varying thickness resting on elastic foundation. In a series

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Tomar and Gupta[13] studied the free vibration of an isotropic non-homogeneous infinite plate of parabolically varying thickness.

Thermal buckling analysis of thin functionally graded rectangular plates is presented by Kazerouni et al. [14] in 2010 andnonlinear free and forced vibration of a transversely isotropic rectangular magneto-electro-elastic thin plate with simply supported boundary conditions and closed circuit electro-magnetic boundary conditions at top and bottom surfaces of the plate is analyzed by Shooshtari and Razavi [15]. It has been observed that much less work has taken place on damped vibrations of plates. Damped vibrations of plate has highly interested because, no vibration can be thought of being in existence without damping. Recently O' Boy [16] have analyzed the damping of flexural vibration, and Robin and Rana [17, 18] discussed the damped vibration of rectangular plate variable thickness resting on elastic foundation. Recently, Shooshtari et al. [15] observed that in design and fabricate drive shafts with high value of fundamental natural frequency, using composite materials instead of typical metallic materials could provide longer length shafts with lighter weight.

Keeping this in view and practicality of problem, damped vibration of infinite plate is analyzed by employing the Lévy approach and the equation of motion of plate is solved by Quintic spline method. The plate is assumed to be of infinite extent in one of the directions (along y-axis).

The non homogeneity of the plate material is assumed to arise due to the variation in Young's modulus and density which varies exponentially. The effect of damping, non-homogeneity, elastic foundation and taperness is discussed with permissible range of parameters. Maximum deflection for the different values of the fundamental frequency of vibration is computed for clamped-clamped and clamped-simply supported boundary conditions for various values of plate parameters.

2. MATHEMATICAL FORMULATION

Consider a non-homogeneous isotropic rectangular plate of length 'a', breath 'b', thickness 'h(x,y)' and density 'ρ', with resting on a winkler- type elastic foundation 'k_f' occupying the domain $0 \le x \le a, 0 \le y \le b$ in xy plane. The x-and y axes are taken along the principal directions and z–axes is perpendicular to the xy plane. The middle surface being z=0 and the origin is at one of the corners of the plate. The differential equation which governs the damped transverse vibration of such plates is given by:

$$\nabla^2(D\nabla^2 w) - (1-\upsilon)\begin{bmatrix} \dfrac{\partial^2 D}{\partial y^2}\dfrac{\partial^2 w}{\partial x^2} - 2\dfrac{\partial^2 D}{\partial x\partial y}\dfrac{\partial^2 w}{\partial x\partial y} \\ + \dfrac{\partial^2 D}{\partial x^2}\dfrac{\partial^2 w}{\partial y^2} \end{bmatrix} + \rho h\frac{\partial^2 w}{\partial t^2} + K\frac{\partial w}{\partial t} + K_f w = 0 \qquad (1)$$

where, $D = D(x,y) = \dfrac{Eh^3(x,y)}{12(1-\upsilon^3)}$,

and K is the damping constant, $w(x,y,t)$ the transverse deflection and D the flexural rigidity at any point in the middle plane of the plate. The plate is of infinite dimension in one of the directions, i.e. in the direction of y-axis. Thus, thickness of plate 'h', Young's modulus E and density ρ of the plate vary along x-axis only, the standing waves will be independent of the y-coordinate. For a harmonic solution, the deflection function w, satisfying the condition at y=0 and y=∞, is assumed

$$w(x,y,t) = W(x)e^{-\gamma t}\cos pt \qquad (2)$$

where 'p' is the circular frequency of vibration and 'm' a positive integer. Thus Equation (1) becomes Equation (3). Introducing the non-dimensional variables

$H = h/a$, $X = x/a$, $\overline{E} = E/a$, $\overline{W} = W/a$, $\overline{\rho} = \rho/a$, and $\lambda^2 = m^2\pi^2(a/b)^2$. On equating the coefficient of *sin(pt)* and *cos(pt)* independently to zero, Equation (3) reduces to Equation (4).

$$\begin{aligned} &\left[Eh^3\frac{d^4W}{dx^4} + \left\{6Eh^2\frac{dh}{dx} + 2h^3\frac{dE}{dx}\right\}\frac{d^3W}{dx^3} + \left\{6Eh\left(\frac{dh}{dx}\right)^2 + 3Eh^2\frac{d^2h}{dx^2} + 6h^2\frac{dh}{dx}\frac{dE}{dx} + h^3\frac{d^2E}{dx^2} - 2Eh^3\frac{m^2\pi^2}{b^2}\right\}\frac{d^2W}{dx^2} - \left\{6Eh^2\frac{dh}{dx} + 2h^3\frac{dE}{dx}\right\}\frac{m^2\pi^2}{b^2}\frac{dW}{dx}\right]\cos pt \\ &+ \left[\frac{m^4\pi^4}{b^4}Eh^3 + \upsilon\frac{m^2\pi^2}{b^2}\left\{3Eh^2\frac{d^2h}{dx^2} + 6Eh\left(\frac{dh}{dx}\right)^2 + 6h^2\frac{dh}{dx}\frac{dE}{dx} + h^3\frac{d^2E}{dx^2}\right\}\right]W\cos pt + 12(1-\upsilon^2)K_f W\cos pt + \\ &\left[12(1-\upsilon^2)\rho h\left\{(\gamma^2 - p^2)\cos pt + 2p\gamma\sin pt\right\} + 12(1-\upsilon^2)k\left\{-p\sin pt - \gamma\cos pt\right\}\right]W = 0 \end{aligned} \qquad (3)$$

$$\begin{aligned} &\overline{E}H^4\frac{d^4\overline{W}}{dX^4} + \left\{6\overline{E}H^3\frac{dH}{dX} + 2H^4\frac{d\overline{E}}{dX}\right\}\frac{d^3\overline{W}}{dX^3} + \left\{3\overline{E}H^3\frac{d^2H}{dX^2} + 6\overline{E}H^2\left(\frac{dH}{dX}\right)^2 + 6H^3\frac{dH}{dX}\frac{d\overline{E}}{dX} + H^4\frac{d^2\overline{E}}{dX^2} - 2\overline{E}H^4\lambda^2\right\}\frac{d^2\overline{W}}{dX^2} \\ &-\lambda^2\left\{6\overline{E}H^3\frac{dH}{dX} + 2H^4\frac{d\overline{E}}{dX}\right\}\frac{d\overline{W}}{dX} + \\ &\left[\lambda^4\overline{E}H^4 - \lambda^2\upsilon\left\{3\overline{E}H^3\frac{d^2H}{dX^2} + 6\overline{E}H^2\left(\frac{dH}{dX}\right)^2 + 6H^3\frac{dH}{dX}\frac{d\overline{E}}{dX} + H^4\frac{d^2\overline{E}}{dX^2}\right\} + 12(1-\upsilon^2)\left\{\frac{-k^2}{4a^2\overline{\rho}} - a^2p^2\overline{\rho}H^2 + HK_f\right\}\right]\overline{W} = 0 \end{aligned} \qquad (4)$$

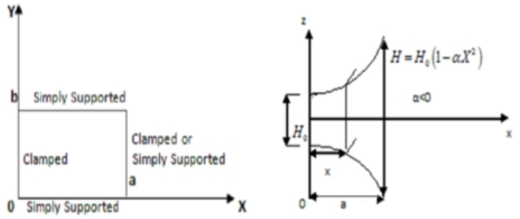


Figure 1. Boundary conditions and vertical cross-section of the parabolic tapered plate.

Substituting, $H = H_0\left(1-\alpha X^2\right)$, $\overline{E}=\overline{E_0}e^{\beta X}$, and $\overline{\rho}=\overline{\rho_0}e^{\beta X}$ where: $H_0=\left(H\right)_{X=0}, \overline{E}_0=\left(\overline{E}\right)_{X=0}, \overline{\rho}_0=\left(\overline{\rho}\right)_{X=0}$, and '$\alpha$' is the taper constant due to parabolically varying thickness of plate, and equating the coefficient of following equation is formed:

$$A_0\frac{d^4\overline{W}}{dX^4}+A_1\frac{d^3\overline{W}}{dX^3}+A_2\frac{d^2\overline{W}}{dX^2}+A_3\frac{d\overline{W}}{dX}+A_4\overline{W}=0 \tag{5}$$

where:

$$\begin{aligned}
&A_0=(1-\alpha X^2)^4, A_1=2\beta(1-\alpha X^2)^4\\
&-12\alpha X(1-\alpha X^2)^3, A_2=24\alpha^2X^2(1-\alpha X^2)^2\\
&-6\alpha(1-\alpha X^2)^3(1+2\beta X)\\
&+(\beta^2-2\lambda^2)(1-\alpha X^2)^4,\\
&A_3=2\lambda^2\left\{6\alpha X(1-\alpha X^2)^3-\beta(1-\alpha X^2)^4\right\}\\
&A_4=\lambda^4(1-\alpha X^2)^4\\
&-\upsilon\lambda^2\begin{Bmatrix}\beta(1-\alpha X^2)^4-6\alpha(1-\alpha X^2)^3(1+2\beta X)\\+24\alpha^2X^2(1-\alpha X^2)^2\end{Bmatrix}\\
&-\begin{Bmatrix}D_K{}^2I^{*2}e^{-2\beta X}+\Omega^2I^*(1-\alpha X^2)^2\\-E_f(1-\alpha X^2)e^{-\beta X}C^*\end{Bmatrix}
\end{aligned}$$

$$D_K{}^2=\frac{3(1-\upsilon^2)K^2}{a^2\overline{E_0}\overline{\rho_0}}, I^*=\frac{1}{H_0{}^2}, C^*=\frac{1}{H_0{}^3},$$

$$E_f=\frac{12(1-\upsilon^2)K_f}{\overline{E_0}}, \Omega^2=\frac{12(1-\upsilon^2)a^2\overline{\rho_0}p^2}{\overline{E_0}}$$

Ω, D_K, E_f are frequency parameter, damping parameter and elastic foundation parameter, respectively. The solution of Equation (5) together with boundary conditions at the edge X=0 and X=1 constitutes a two-point boundary value problem. As the differential equation has several plate parameters, therefore it becomes quite difficult to find its exact solution. Keeping this in mind, complex for the purpose of computation, the Quintic spline interpolation technique, is used.

3. METHOD OF SOLUTION

Let f(x) be a function with continuous derivatives in the range [0,1] and interval [0,1] be divided into 'n' subintervals by means of points X_i such that $0=X_0<X_1<X_2<...<X_n=1$.

where $\Delta X = 1/n, X_i=i\Delta X(i=0,1,2,...,n)$.

Let the approximating function $\overline{W}(X)$ for the $W(x)$ be a quintic spline with the following properties:

(i) $\overline{W}(X)$ is a quintic polynomial in each interval (X_k, X_{k+1}).

(ii) $\overline{W}(X)=W(X_k), k=0,1,2,...,n.$

(iii) $\frac{d\overline{W}}{dX}, \frac{d^2\overline{W}}{dX^2}, \frac{d^3\overline{W}}{dX^3}$ and $\frac{d^4\overline{W}}{dX^4}$ are continuous.

In view of above axioms, the quintic spline takes the form:

$$\overline{W}(X)=a_0+\sum_{i=1}^{4}a_i(X-X_0)^i+\sum_{j=0}^{n-1}b_j(X-X_j)_*^5 \tag{6}$$

where: $(X-X_j)_*=\begin{cases}0, & if\ X\le X_j\\(X-X_j), & if\ X>X_j\end{cases}$ and $a_0, a_1, a_2, a_3, a_4, b_{n-1}$

are (n+5) unknown constants. Thus for the satisfaction at the n[th] knot, Equation (5) reduced to:

$$\begin{aligned}
&A_4a_0+[A_4(X_m-X_0)+A_3]a_1\\
&+[A_4(X_m-X_0)^2+2A_3(X_m-X_0)+2A_2]a_2\\
&+[A_4(X_m-X_0)^3+3A_3(X_m-X_0)^2+6A_2(X_r-X_0)+6A_1]a_3\\
&+[A_4(X_m-X_0)^4+4A_3(X_m-X_0)^3+12A_2(X_m-X_0)^2\\
&+24A_1(X_m-X_0)+24A_0]a_4\\
&+\sum_{j=0}^{n-1}b_j[A_4(X_m-X_j)_*^5+5A_3(X_m-X_j)_*^4+20A_2(X_m-X_j)_*^3\\
&+60A_1(X_m-X_j)_*^2+120A_0(X_m-X_j)_*]=0
\end{aligned} \tag{7}$$

For m=0(1)n, above system contains (n+1) homogeneous equation with(n+5) unknowns, a_i, i=0(1)4, b_j, j=0, 1, 2, …, (n-1), and can be represented in matrix form as:

$$[A][B]=[0] \tag{8}$$

where [A] is a matrix of order (n+1) × (n+5) while [B] and [0] are column matrices of order (nx5).

4. BOUNDARY CONDITIONS AND FREQUENCY EQUATIONS

The following two cases of boundary conditions have been considered:

(i) (C-C): clamped at both the edge X=0 and X=1.

(ii) (C-SS): clamped at X=0 and simply supported at X=1.

The relations that should be satisfied at clamped and simply supported are:

$$W = \frac{dW}{dX} = 0\,;\; W = \frac{d^2W}{dX^2} = 0 \tag{9}$$

respectively. Applying the boundary conditions C-C to the displacement function by Equation (6), one obtains a set of four homogeneous equations in terms of (n+5) unknown constants which can be written as:

$$[B^{cc}][B]=[0] \tag{10}$$

where B^{cc} is a matrix of order 4x(n+5). Therefore, the Equation (8) together with Equation (10) gives a complete set of (n+5) homogeneous equations having (n+5)unknowns which can be written as:

$$\left[\frac{A}{B^{cc}}\right][B]=[0] \tag{11}$$

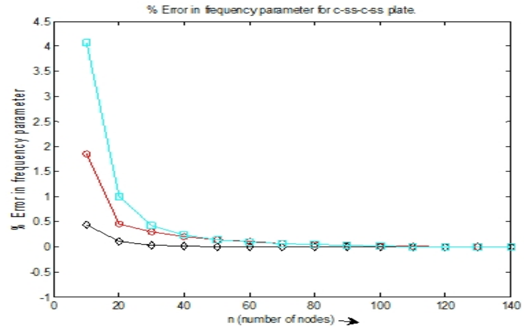
For a non-trivial solution of Equation (11), the characteristic determinant must vanish, i.e.

$$\left|\frac{A}{B^{cc}}\right| = 0 \tag{12}$$

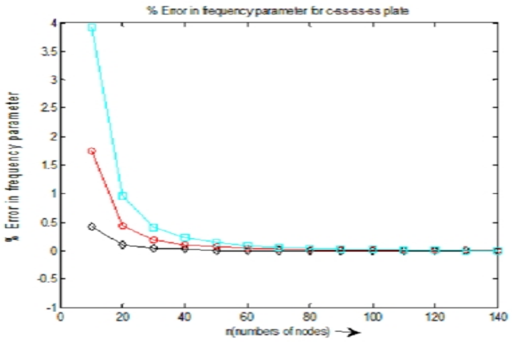
Similarly, for (C-SS) plate the frequency determinant can be written as:

$$\left|\frac{A}{B^{ss}}\right| = 0 \tag{13}$$

where B^{ss} is a matrix of order 4 × (n+5).



(a)



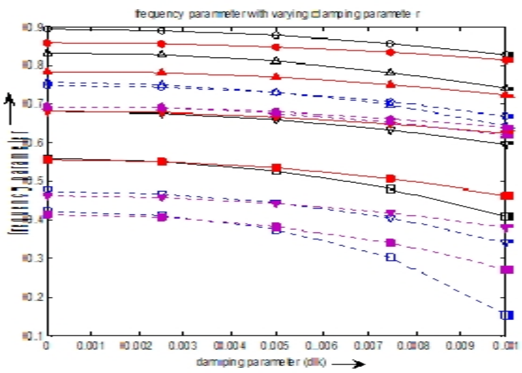
(b)

Figure 2. Percentage error in frequency parameter Ω: (a) C-C plate (b) C-SS plate, for a/b=0.25, α=0.0, Dk=0.0, Percentage error=$[(\Omega_n - \Omega_{140})/\Omega_{140}]$x100; n=10(10)140.

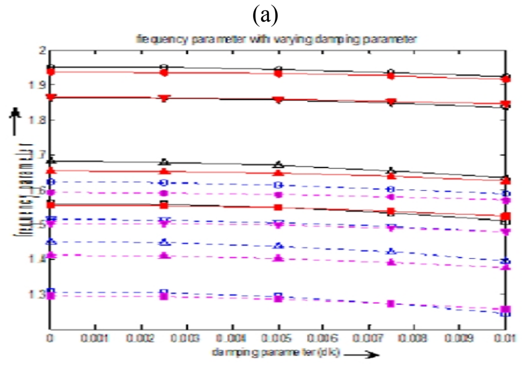
5. NUMERICAL RESULTS AND DISCUSSION

The frequency Equations (12) and (13) provide the values of frequency parameter Ω for various values of plate parameters. In the present paper, first three frequency modes of vibration have been computed for the above mentioned two boundary conditions for different values of foundation parameter Ef=0.0(0.005)0.02, damping parameter Dk=0.0(0.025)0.01 and taper parameter α=0.0(0.1)0.4 for non-homogeneity parameter β=0.0, 0.4, Poisson ratio's ν=0.3, thickness of plate h=0.03 and aspect ratio a/b=0.25.

The numerical method provides approximate values.Therefore, in order to minimize the error, there is an urgent need to determine the optimum size of interval length ΔX. In the present problem, a computer program was developed and executed for n=10(10)150 and observed that no consistent improvement in results while n≥140 for clarity (Figure 2a -2b). Therefore, the results are obtained for n=140 and depicted through graphs (3-7). It is found that frequency parameter for clamped plate is greater than that of simply supported plate whatever the values of other parameters are.



(a)



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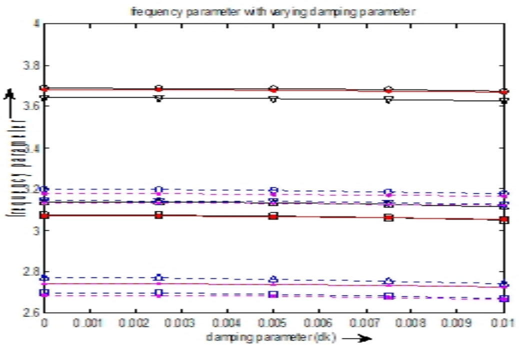
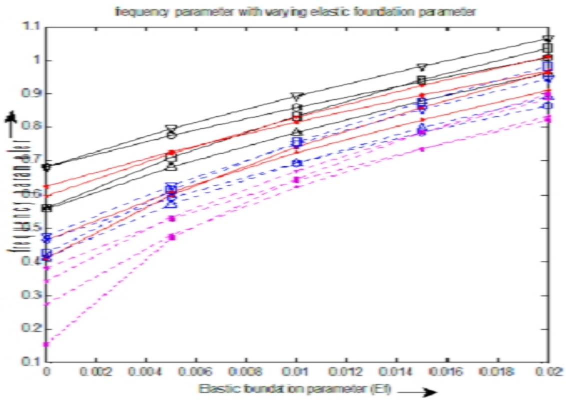


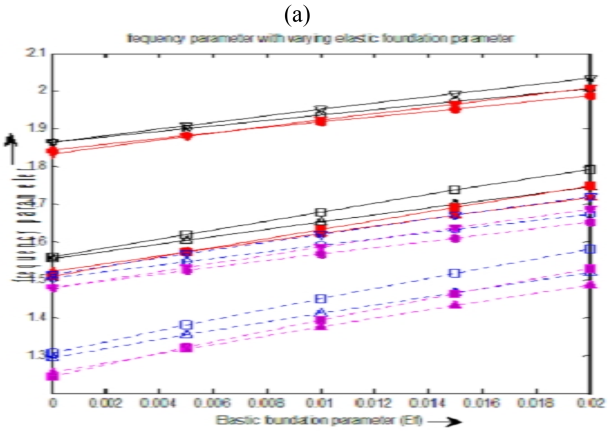
(c)

Figure 3. Natural frequencies for C-C and C-SS plates: (a) First mode (b) Second mode (c) Third mode ,for a/b=0.25 . —,C-C; --,C-SS;▽, α=0.0 , E_f=0.0, β=0.0;▼, α=0.0 , E_f=0.0, β=0.04;o, α=0.0 , E_f=0.01,β=0.0; ●, α=0.0 , E_f=0.01, β=0.01; □, α=0.4 , E_f=0.0, β=0.0;■, α=0.4 , E_f=0.0, β=0.01; Δ, α=0.4 , E_f=0.01, β=0.0; ▲ , α=0.4 , E_f=0.01, β=0.01.

Figure 3 shows the values of frequency parameter Ω with the increasing value of damping parameter (Dk) for the fixed value of non homogeneity parameter β, taper constant α and foundation parameter Ef for first three modes of vibration of C-C and C-SS plates. Figure 3a shows the behavior of frequency parameter Ω which decreases with the increasing values of damping parameter Dk for two different values of taper parameter α=0.0, 0.4, foundation parameter Ef =0.0, 0.01 and non homogeneity parameter β=0.0, 0.4 for both plates. The rate of decrease of Ω with damping parameter Dk for C-SS is higher than that for C-C plate keeping all other plate parameters fixed. This rate decreases with the increase in the value of non homogeneity parameter β. A similar inference can be drown from Figure 3b and 3c, when the plate is vibrating in the second mode as well as in the third mode of vibration except that the rate of decrease of Ω with Dk is lesser as compared to the first mode. Figure 4 provides the inference of foundation parameter Ef on frequency parameter Ω for two values of damping parameter Dk=0.0, and 0.01, for the fixed value of taper parameter α=0.0, 0.4 and non homogeneity parameter β=0.0, 0.4. It is noticed that the frequency parameter Ω increases continuously with the increasing value of foundation parameter Ef for C-C and C-SS plates, whatever be the value of other plate parameters. It is found that the rate of increases of frequency parameter Ω for C-SS plate is higher than C-C plate for three modes. Figure 4a gives the inference of foundation parameter Ef on frequency parameter Ω for the first mode of vibration. This rate increases with the increase in the value of foundation parameter Ef, it decreases with the increases in the number of modes, as clear from 3b and 3c when the plate is vibrating in the second and third mode of vibration. From Figure 4b, the effect of foundation parameter is found to increase the frequency parameter Ω, however the rate of increase reduces to more than half of the first mode for both boundary conditions. In case of the third mode, this rate of increase further decreases and becomes nearly half of the second mode as is evident from Figure 4c.



(a)



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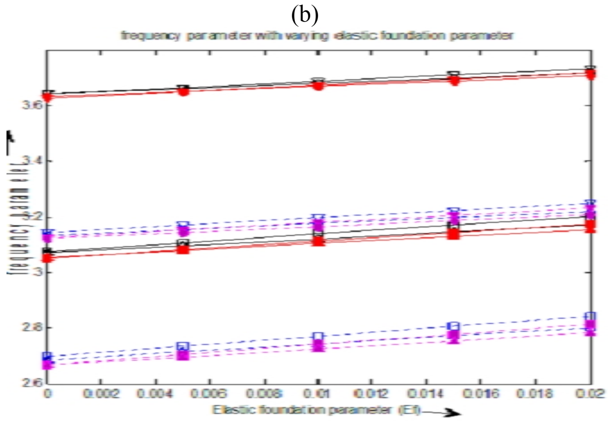
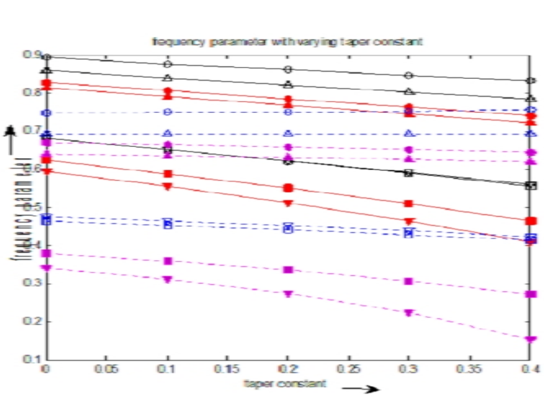


(c)

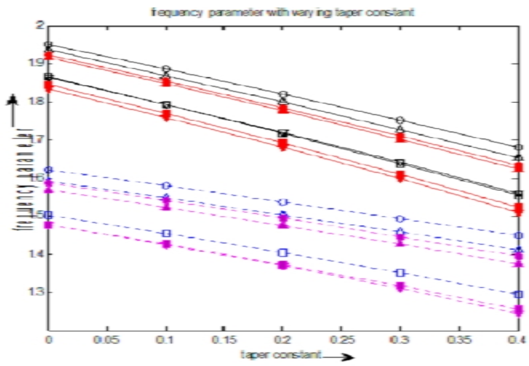
Figure 4. Natural frequencies for C-C and C-SS plates: (a) First mode (b) Second mode (c) Third mode ,for a/b=0.25 . —,C-C; ----,C-SS; ▽, α=0.0 , *Dk*=0.0, β=0.0; ▼, α=0.0 , *Dk* =0.01, β=0.0; ○, α=0.0 , *Dk* =0.0,β=0.4; ●, α=0.0 , *Dk* =0.01, β=0.4; □, α=0.4 , *Dk* =0.0, β=0.0; ■ , α=0.4 , *Dk* =0.1, β=0.0; Δ, α=0.4 , *Dk* =0.0, β=0.4; ▲ , α=0.4 , *Dk* =0.01, β=0.4.

Figure 5 shows the effect of taper parameter α on frequency parameter Ω for two different values of damping parameter Dk =0.0 and 0.01respectively, for the fixed value of taper parameter α=0.0, 0.4, foundation parameter Ef =0.0, 0.01and β=0.0, 0.4. Figure 5a provides the graphs of frequency parameter Ω verses taper parameter α for the first mode of vibration. It is observed in the presence of damping parameter Dk, i.e. for Dk =0.1,that the frequency parameter Ω decreases continuously with increasing values of taper parameter α for both the boundary conditions, whatever be the value of other plate parameters. In the absence of damping parameter Dk, .i.e for Dk =0.0, it has been observed that the frequency parameter Ω decreases with increasing values of taper parameter α for C-C plates and it has also been observed that for C-SS plate, there is a continuously decrement in the value of frequency parameter Ω for fixed values of Ef =0.0, and β=0.0, 0.4. But, fluctuations in results we have found with Ef as constant and with β=0.0, 0.4. The values were found to be increasing in Ef=0.01 and β=0.0. In case where Ef=0.01 and β= 0.4, the results had both increasing and decreasing values. The local minima was observed as α=0.3. Figures 5b and 5c show the continuous decrement in frequency parameter Ω with the increased value in α, for second and third mode, respectively.

The normalized displacements for the two boundary conditions C-C and C-SS, considered in this paper are shown in Figure 6 and 7, respectively. The plate thickness varies parabolically in X-direction and the plate is considered resting on elastic foundations Ef=0.02 with damping parameter Dk=0.01. Mode shapes for a rectangular plate i.e, a/b=0.25 have been computed and observed that the nodal lines are seen to shift towards the edge, i.e. X=1 as the edge X=0 increases in thickness for both the plates. No special change was seen in the pattern of nodal lines by taking different values of β and Ef. As normalized displacements were differing only at the third or fourth place after decimal for both boundary conditions.



(a)



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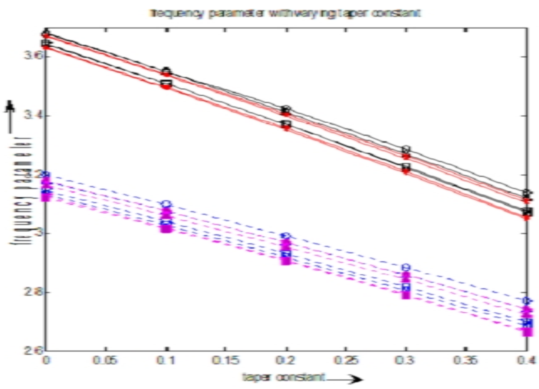

(c)

Figure 5. Natural frequencies for C-C and C-SS plates: (a) First mode (b) Second mode (c) Third mode ,for a/b=0.25 . —,C-C; ----,C-SS;▽, *Dk*=0.0 , E_f=0.0, β=0.0; ▼,*Dk*=0.01 , E_f=0.0, β=0.0; ○,*Dk* =0.0 , E_f=0.01,β=0.0; ●,*Dk* =0.01 , E_f=0.01,β=0.0; □,*Dk* =0.0 , E_f=0.0, β=0.4; ■,*Dk* =0.01 , E_f=0.0, β=0.4; Δ,*Dk* =0.0 , E_f=0.01, β=0.4; ▲ ,*Dk* =0.01,E_f=0.01, β=0.4

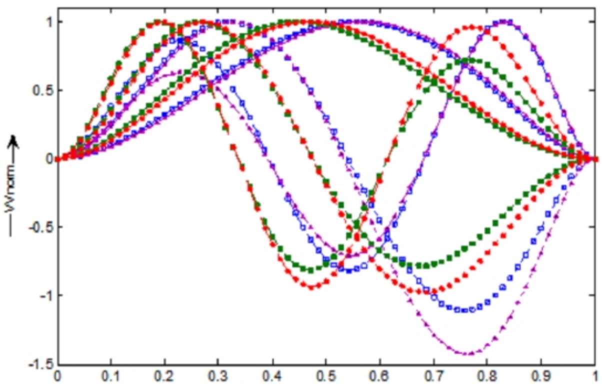

Figure 6. Normalized displacements for C-SS-C-SS plate, for a/b=0.25; h=0.03,Dk=0.01, E_f=0.02 ;—,First mode; ——,second mode ; ……,third mode; ●,α=-0.5,β=-0.5; □, α=0.5, β=0.5;▲, α=0.5,β=-0.5;■, α=-0.5,β=0.5;

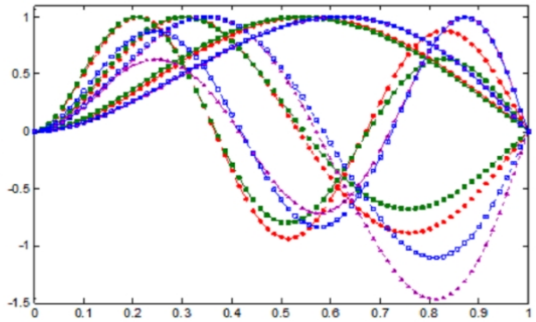

Figure7. Normalized displacements for C-SS-SS-SS plate, for a/b=0.25; h=0.03,Dk=0.01, E_f=0.02;—,First mode;——, second mode; ……,third mode;●,α=-0.5,β=-0.5; □, α=0.5, β=0.5;▲, α=0.5,β=-0.5;■, α=-0.5,β=0.5;

6. CONCLUSION

In the present study, results are computed using MATLAB within the permissible range of parameters up to the desired accuracy (10^{-8}), which validates the actual phenomenon of vibrational problem. Variation in thickness, elastic foundation and damping parameter are of great interest since it provides reasonable approximation to linear vibrations. One of the major causes of plate failures in industrial machines is from undamped/damped vibration, which results in high cyclic fatigue. Determination of vibration frequencies is of utmost importance for assessment of failure life. The present paper finds the required damping to reduce the plate vibrations. After assessment of inherent material damping, the balance damping may be provided by external means e.g. friction damping or lacquer damping. Thus, the present study may be helpful in designing plates which requires an accurate determination of their natural frequencies and mode shapes also by proper choice of these parameters required natural frequency can be achieved for vibrational model.Thus, the present study may be useful for design engineers especially in rigid roadway pavement under moving traffic loads.

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Damped Vibrations of Parabolic Tapered Non-homogeneous Infinite Rectangular Plate Resting on Elastic Foundation

RESEARCH NOTE

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چکیده

در مقاله حاضر، ارتعاشات میرا شده ناهمگن صفحه مستطیلی بی نهایت با ضخامت متغیر سهموی متکی بر پایه کش‌سان مطالعه شده است. پس از روی کرد لوی، معادله حرکت صفحه با ضخامت متغیر در یک جهت با استفاده از روش نوار باریک quintic حل شده است. اثر نوسانات کم، پایه کش‌سانو شیب با محدوده مجاز پارامترها بحث شده است. متغیر فرکانس Ω به عنوان پارامتر میرایی با افزایش DK، کاهش می یابد و در گیره ساده، در مقایسه با شرایط مرزی گیره–فشرده سریعتر کاهش می یابد. همچنین، مشاهده شد که در حضور نوسانات پارامترDK ،پارامتر فرکانس Ω به طور پیوسته با افزایش مقدار پارامتر شیب‌دار برای هر دو شرایط مرزی کاهش می یابد، اما تغییرات در غیاب پارامتر میرایی مشاهده شد.

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