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Isotonic Change Point Estimation in the AR(1) Autocorrelated Simple Linear Profiles

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ABSTRACT

Sometimes the relationship between dependent and explanatory variable(s) known as profile is monitored. Simple linear profiles among the other types of profiles have been more considered due to their applications especially in calibration. There are some studies on the monitoring them when the observations within each profile are autocorrelated. On the other hand, estimating the change point leads to meet great saving time and costs. Hence, in this paper, a maximum likelihood estimator is derived for simple linear profiles with first order autoregressive autocorrelation structure within each profile to estimate isotonic change point. The performance of the proposed estimator is appraised and compared to estimators that derived under step change and drift and a confidence set estimator presented. The results demonstrate that the proposed estimator has better performance in small and medium shifts whereas the performance of their corresponding estimators becomes better than the proposed estimator in large shifts. It is worth mentioning that knowing type of the change is not important in the proposed estimator and its only assumption is belonging of the change type to a family of isotonic shifts. Finally, the performance of the estimator is illustrated through a real case.

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1. INTRODUCTION

In some statistical process control applications, the relationship between dependent and explanatory variable (s) is monitored instead of monitoring a univariate or multivariate quality characteristics. This relationship which can be linear or nonlinear is known as a profile. According to the relationship, there are various types of profiles including simple linear profile, multiple linear profile, polynomial profile, nonlinear profile, waveform profile, spline profile and profiles based on generalized linear models. Profiles have different applications in manufacturing and service. A number of researchers such as Kang and Albin [1] have discussed practical applications of profiles. In the recent years, monitoring profiles especially simple linear profiles due to their applications especially in calibration has been considered by many researchers. Studies about monitoring profiles are done in two phases. Many researchers such as Mahmoud et al. [2] have studied Phase I monitoring of simple linear profiles. Also, there are many works on Phase II monitoring in which the parameters are assumed to be known. Researchers such as Gupta et al. [3] have Studied Phase II monitoring of simple linear profiles. Keramatpour et al. [4] proposed a remedial measure to remove the effect of autocorrelation in monitoring of autocorrelated polynomial profiles.

Many studies have been done by researchers on monitoring simple linear profiles when the sampling time between observations collapses and as a result the observations are autocorrelated. Recently, Kamranrad and Amiri [5] proposed a robust holt-winter based control chart in Phase II monitoring of a simple linear profile under within profile autocorrelation and the presence of outliers.

On the other hand, usually when the control chart declares warning about the out-of-control status, it is different with the real-time of the process change. Real-time of change in process is known as change point. A process may be in the out-of-control state due to different change types including single-step change, drift change, isotonic change, multiple-step changes, and sporadic changes. Estimating the change point leads to meet great saving on time and costs. Hence, many

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authors such as Noorossana and Shadman [6] have studied change point estimation under different situations. Also, many researchers studied estimation of the change point in the area of profile monitoring. Mahmoud et al. [2] derived an MLE for change point estimation of simple linear profile based on LRT method in Phase I. In this Phase. Sharafi et al. [7, 8] proposed an MLE to identify the real time of step and linear trend changes in monitoring of logistic regression profiles, respectively. Also, Sharafi et al. [9, 10] provided a maximum likelihood estimation approach for estimating the time of drift and step changes in Poisson profiles, respectively. Also, Kazemzadeh et al. [11] extended an MLE for linear disturbance in the parameters of multivariate linear regression profiles. Shadman et al. [12] suggested a unified framework for developing Phase I control charts in monitoring and estimating of change point in generalized linear profiles. Also, Sogandi and Amiri [13, 14] proposed step and drift estimators in Gamma regression profiles, respectively. Ayoubi et al. [15] developed the maximum likelihood approach to estimate the sporadic changes in the mean of multivariate linear profiles in Phase II. Under the sporadic change, shifts can be occurred in any directions and there is no knowledge about the change type in prior. In this method, parameters are estimated through filtering and smoothing approaches in dynamic linear model. Recently, Khedmati and Niaki [16] proposed a step change point estimator to find the real time of the change in the regression parameters of the AR(1) autocorrelated simple linear profiles in Phase II.

The isotonic changes defined as a series of shifts with the same direction in which the exact kind of shifts are undetermined. Estimation of these changes has been studied by many researchers. In this respect, for example, Sogandi and Amiri [17] derived an MLE in the Generalized Linear Model-based regression profiles under the isotonic change in Phase II.

2. PROBLEM FORMULATION

There are some studies on the monitoring of simple linear profiles with assumption that the observations within each profile are uncorrelated. However, this assumption is sometimes violated in practice when the sampling time between observations decreases. Hence, it is assumed that for the i^{th}observation in the j^{th}sample, when the process is under statistical control, the relationship between the response variable and independent variable can be written as:

$$y_{ij} = b_0 + b_1 x_i + \varepsilon_{ij}, \tag{1}$$

$$\varepsilon_{ij} = \rho\varepsilon_{(i-1)j} + l_{ij}, \tag{2}$$

in which ε_{ij}'s are the autocorrelated residuals with autoregressive (AR(1)) structure and l_{ij}'s are independently and identically distributed normal random variables. l_{ij}'s mean and variance are zero and σ^2, respectively.$l_{ij} \sim NID\ (0\ ,\sigma^2)\ , |\rho| < 1$. In Equation (1), the x-values are assumed constant in each profile. In this paper, we consider Phase II analysis. In other words, the values of parameters b_0, b_1 and σ^2 are assumed to be known.

It can be shown that the autoregressive structure between the error terms leads to autocorrelation between observations in each profile. For monitoring purpose, to deal with the effects of autocorrelation we use the transformation method of Soleimani et al. [18]. The observation in each profile can be expressed as:

$$y_{ij} = b_0 + b_1 x_i + \varepsilon_{ij} \tag{3}$$

$$y_{(i-1)j} = b_0 + b_1 x_{i-1} + \varepsilon_{(i-1)j} \tag{4}$$

by replacing y_{ij} , $y_{(i-1)j}$ according to Equations (3) and (4), we have

$$\begin{aligned} y_{ij} - \rho y_{(i-1)j} &= (b_0 + b_1 x_i + \varepsilon_{ij}) - \rho(b_0 + b_1 x_{i-1} + \varepsilon_{(i-1)j}) \\ &= b_0(1-\rho) + b_1(x_i - \rho x_{i-1}) + (\varepsilon_{ij} - \rho\varepsilon_{(i-1)j}), \end{aligned} \tag{5}$$

and according to Equation (2), $\varepsilon_{ij} = \rho\varepsilon_{(i-1)j} + l_{ij}$,we have

$$y_{ij} - \rho y_{(i-1)j} = b_o(1-\rho) + b_1(x_i - \rho x_{i-1}) + l_{ij}, \tag{6}$$

where $l_{ij}{}'s$ are independent random variables with mean zero and varianceσ^2. Hence, by using the transformed variable y'_{ij}, a simple linear profile model with independent residuals is obtained .

$$y'_{ij} = y_{ij} - \rho y_{(i-1)j}\ , \tag{7}$$

$$x'_i = x_i - \rho x_{i-1}\ , \tag{8}$$

$$b'_0 = b_o(1-\rho)\ , \tag{9}$$

$$b'_1 = b_1\ . \tag{10}$$

Using Equations (7) to (10), the equation $y'_{ij} = b'_0 + b'_1 x'_i + l_{ij}$."is obtained. Therefore, by using the transformed variables x'_i and y'_{ij}a simple linear profile model with independent residuals is obtained. Then, ordinary least squares (OLS) method can be used to estimate the regression parameters.

3. T^2 CONTROL CHARTS TO MONITOR SIMPLE LINEAR PROFILES IN PHASE II

We use a T^2control to monitor the regression parameters after applying transformation method discussed in section 2. This transformation decreases the effect of autocorrelation from the regression parameters and as a result, we can easily monitor the regression parameters over time. In this section, the T^2 control chart proposed by Kang and Albin [1] is used to monitor the regression coefficients. This chart is proposed based on the least squares estimators of b_0' and b_1' that have the bivariate normal distribution. Estimator vector of $\mathbf{z}_j = (\hat{b}_{0j}', \hat{b}_{1j}')^T$ is used to construct the statistic for j^{th} sample. $\hat{b}_0'$ and $\hat{b}_1'$ have the bivariate normal distribution with mean vector $\mathbf{u}$ and variance covariance matrix $\mathbf{\Sigma}$ where $\mathbf{u}$ and $\mathbf{\Sigma}$ are computed based on the following equation equal to $\mathbf{u} = (b_0', b_1')^T$ and $\mathbf{\Sigma} = \begin{pmatrix} \sigma_0^2 & \sigma_{01} \\ \sigma_{01} & \sigma_1^2 \end{pmatrix}$, respectively.

In $\mathbf{\Sigma}$, σ_0^2, and σ_1^2 are variance of the estimator b_{0j} and b_{1j}, respectively and σ_{01} is the covariance between $\hat{b}_{0j}$ and $\hat{b}_{1j}$. Then, T^2 statistic is computed as follows:

$$T_j^2 = (\mathbf{z}_j - \mathbf{u})^T \mathbf{\Sigma}^{-1} (\mathbf{z}_j - \mathbf{u}) \tag{11}$$

when the process is in-control, the T^2 statistic follows a central chi-square distribution with two degrees of freedom. Hence, the upper control limit (UCL) for the T^2 control chart is $UCL=\chi^2_{2,\alpha}$, where $\chi^2_{2,\alpha}$ is the 100 (1-α) percentile of the chi-square distribution with two degrees of freedom.

4. PROPOSED ISOTONIC CHANGE POINT ESTIMATOR

In this section, we concentrate on change point estimation in the parameters of the simple linear profile when that error terms are autocorrelated. The change type considered in this paper is isotonic. We could use the transformed observations, write down the likelihood function based on the independent observations. However, since, the joint likelihood function of the AR(1) observations is available in the literature, surely, using the distribution of the original variable leads to more reliable results than the distribution of the transformed observations. This is the reason we used the distribution of the original variables.

The likelihood function of the AR(1) autocorrelated processes is first reported by Perry and Pignatiello [19]. Since the observations within each profile are autocorrelated based on the AR(1) model, we used this likelihood function for the problem discussed. We applied the mean of the first response variable ($b_0 + b_1 x_1$) and the other responses ($\delta_i + \rho y_{(i-1)j}$) and write down the likelihood function for observations within each profile. Through multiplying the likelihood functions of each profile until a signal is given by the T^2 control chart, the following likelihood function is obtained:

$$L(b_0,b_1,\tau|y) = \prod_{j=1}^{T} f(y_{1j},y_{2j},...,y_{nj}) = \prod_{j=1}^{T} \Big\{ (2\pi\sigma^2)^{\frac{-n}{2}} (1-\rho^2)^{\frac{1}{2}} \times exp\left[\frac{-1}{2\sigma^2} \Big\{ (1-\rho^2)\big(y_{1j} - (b_0 + b_1 x_1)\big)^2 + \sum_{i=2}^{n} \big(y_{ij} - \delta_i - \rho y_{(i-1)j} \big)^2 \Big\} \right] \Big\}, \tag{12}$$

where $\delta_i = \varepsilon_{ij} - \rho\varepsilon_{(i-1)j} - l_{ij}$ is the location parameter and T is the number of the first profile which falls out of the UCL. After some times and in unknown profile τ, the process becomes out-of-control. In fact, for profiles j=1, 2,…τ, the value of regression parameters are equal to $(b_{0(in)}, b_{1(in)})$ and for profiles $j= \tau+1, \tau+2, \ldots T$, these values change to $(b_{0j(out)}, b_{1j(out)})$, respectively. So, the logarithm of the likelihood function is equal to

$$\begin{aligned} Log\left(L(b_0,b_1,\tau|y)\right) &= \sum_{j=1}^{\tau} log\left((2\pi\sigma^2)^{\frac{-n}{2}} (1-\rho^2)^{\frac{1}{2}} \right) + \sum_{j=1}^{\tau} \left[\frac{-1}{2\sigma^2} \Big\{ (1-\rho^2)\big(y_{1j} - (b_{0(in)} + b_{1(in)} x_1)\big)^2 + \sum_{i=2}^{n} \big(y_{ij} - \delta_{i(in)} - \rho y_{(i-1)j} \big)^2 \Big\} \right] \\ &= \sum_{j=\tau+1}^{T} log\left((2\pi\sigma^2)^{\frac{-n}{2}} (1-\rho^2)^{\frac{1}{2}} \right) + \sum_{j=\tau+1}^{T} \left[\frac{-1}{2\pi\sigma^2} \Big\{ (1-\rho^2)\big(y_{1j} - (b_{0j(out)} + b_{1j(out)} x_1)\big)^2 + \sum_{i=2}^{n} \big(y_{ij} - \delta_{ij(out)} - \rho y_{(i-1)j} \big)^2 \Big\} \right]. \end{aligned} \tag{13}$$

By replacing the estimate of vector $\mathbf{b}_j$ = $(b_{0j(out)}, b_{1j(out)})$ for $j = \tau+1, \tau+2, ..., T$ in Equation (13), the change point estimator is computed as follows:

$$\begin{aligned} \hat{\tau} = \arg\max \Bigg\{ & \sum_{j=\tau+1}^{T} \left[\frac{-1}{2\sigma^2} \Big\{ (1-\rho^2)\big(y_{1j} - (b_{0j(out)} + b_{1j(out)} x_1)\big)^2 + \sum_{i=2}^{n} \big(y_{ij} - \delta_{ij(out)} - \rho y_{(i-1)j} \big)^2 \Big\} \right] \\ & - \sum_{j=\tau+1}^{T} \left[\frac{-1}{2\sigma^2} \Big\{ (1-\rho^2)\big(y_{1j} - (b_{0(in)} + b_{1(in)} x_1)\big)^2 + \sum_{i=2}^{n} \big(y_{ij} - \delta_{i(in)} - \rho y_{(i-1)j} \big)^2 \Big\} \right] \Bigg\}. \end{aligned} \tag{14}$$

To estimate parameter τ, the matrix $\mathbf{B}_{T-\tau} = (\mathbf{b}_{\tau+1}, \mathbf{b}_{\tau+2}, ..., \mathbf{b}_T)$ should be estimated. Hence, initial estimates of each parameters of the vector $\mathbf{b}_j$ are needed. Hence, we use

$$\tilde{\mathbf{b}}_j = \begin{cases} \mathbf{b}_0 & if\ \mathbf{b}_0 \geq \mathbf{b}_j, for\ j = \tau+1, \ldots, T\ and \\ \mathbf{b}_j & if\ \mathbf{b}_0 \leq \mathbf{b}_j, for\ j = \tau+1, \ldots, T \end{cases} \tag{15}$$

We can estimate $\hat{\mathbf{b}}_j$ by solving the following convex program:

$$\begin{aligned} \hat{\tau} = \underset{\tilde{\mathbf{B}}_{T-\tau}}{\arg\max} \Bigg\{ & \sum_{j=\tau+1}^{T} \left[\frac{-1}{2\sigma^2} \Big\{ (1-\rho^2)\big(y_{1j} - (\hat{b}_{0j(out)} + \hat{b}_{1j(out)} x_1)\big)^2 + \sum_{i=2}^{n} \big(y_{ij} - \hat{\delta}_{ij(out)} - \rho y_{(i-1)j} \big)^2 \Big\} \right] \\ & - \sum_{j=\tau+1}^{T} \left[\frac{-1}{2\sigma^2} \Big\{ (1-\rho^2)\big(y_{1j} - (b_{0(in)} + b_{1(in)} x_1)\big)^2 + \sum_{i=2}^{n} \big(y_{ij} - \delta_{i(in)} - \rho y_{(i-1)j} \big)^2 \Big\} \right] \Bigg\}, \end{aligned} \tag{16}$$

subject to $\tilde{\mathbf{b}}_j \geq \tilde{\mathbf{b}}_{j-1}$ for $j = \tau+1, \ldots, T$.

We fit isotonic regression to each parameter of the vector $\tilde{\mathbf{b}}_j$ similar to Perry et al. [20] to find the estimator of vector $\mathbf{b}_j$ for $j = \tau + 1, \ldots, T$.

$$[\hat{\mathbf{b}}_j] = \mathbf{I}([\tilde{\mathbf{b}}_j]) \text{for } j = \tau + 1, \ldots, T \quad (17)$$

Among the isotonic regression algorithms, the pool adjacent violators (PAV) algorithm described by Best and Chakravarti [21] is more well-known and so it is used in this paper. Generally, the algorithm of the proposed method in detecting and change point estimation is given as follows step by step:

Step 1: Capturing signal from T^2 control chart during monitoring the process.

Step 2: Obtaining the likelihood function of joint probability distribution of AR(1) autocorrelated observations.

Step 3: Applying the PAV algorithm to estimate out-of-control parameters for τ=1, 2, …,T-1.

Step 4: Obtaining likelihood function for τ=1, 2 …, T-1.

Step 5: Estimating the change point using maximum of the log likelihood function achieved over all possible change points.

5. SIMULATION STUDIES

In this section, Monte Carlo simulation is utilized to evaluate performance of the proposed estimator in terms of precision and accuracy criteria in the AR(1) autocorrelated simple linear profile. Also, the obtained results from the proposed estimator are compared to estimators derived under step change and linear drift. Also, it is assumed that the regression model of the AR(1) autocorrelated simple linear profile under in-control state is equal to $y_{ij} = 3 + 2x_i + \varepsilon_{ij}$, in which ε_{ij}'s are the autocorrelated error terms with first-order autoregressive structure according to the following equation:

$$\varepsilon_{ij} = \rho\varepsilon_{(i-1)j} + l_{ij},$$

Where l_{ij}'s are independently and identically distributed normal random variables with mean zero and variance 1. Moreover, four levels equal to 2, 4, 6, 8 are considered for explanatory variable in all of the profiles. To monitor the profiles, we use the T^2control chart with probability of Type I error equal to 0.005. Thus, the UCL is equal to $\chi^2_{2,0.005}$= 10.59. On the other hand, the explanatory variable and the obtained values for the response variables are transformed according to Soleimani et al. [18]. Hence, the T_j^2 statistics are computed for all samples according to the transformed variables. Without loss of generality, it is assumed that the type of changes is isotonic. We investigate performance of the proposed estimator under the two step changes. It is assumed that the real change point value is equal to $\tau = 50$. Hence, the first 50 profiles are simulated from AR(1) autocorrelated simple linear profile with known vector of parameters equal to (3, 2) for in-control state. If statistics fall out of the UCL during the generating of the 50 profiles, it is considered as a false alarm, so the observations are regenerated instead of them. Because, it is assumed that observations are generated under the in-control state. After 50th profile, the increasing changes are occurred in the parameters of the regression model according to $(3 + \delta_1, 2 + \delta_2)$. In addition, after 60th profile, observations are generated from the out-of-control process with $(3 + \delta_1', 2 + \delta_2')$vector such that $\boldsymbol{\delta}' > \boldsymbol{\delta}$ until the T^2control chart alarms due to occurrence of an assignable cause. At this time, the corresponding estimators are used to estimate the real time of the first change. The mentioned procedure is repeated 5000 times under different shifts in the 50th and 60th profiles to appraise performances of the proposed estimator and estimators derived under step change and linear drift. The results are summarized under the weak and strong autocorrelation coefficient equal to 0.1and 0.9 in Tables 1 and 2, respectively to compare performance of the step, drift and the proposed estimators.

TABLE 1. Accuracy and precision performances of the step, linear drift, and monotonic MLEs under a multiple step change in the parameters of simple linear profiles with 5000 simulation runs and τ =50 and $\rho = 0.1$.

Shifts (δ_1, δ_2) (δ_1, δ_2)		**(0.1,0.001) (0.2,0.005)**	**(0.2,0.005) (0.4,.01)**	**(0.4,0.01) (0.6,.05)**	**(0.6,0.05) (0.8,0.08)**	**(0.8,0.08) (1,0.1)**
Accuracy	$E(T)$	134.54	130.4	128.11	114.13	107.25
	$\hat{\tau}_{sc}$	53.12 (0.51)	52.75 (0.47)	52.16 (0.26)	51.85 (0.11)	50.65 (0.09)
	$\hat{\tau}$	50.74 (0.08)	51.54 (0.19)	52.85 (0.25)	53.21 (0.37)	53.33 (0.4)
	$\hat{\tau}_{lt}$	55.11 (0.55)	54.22 (0.48)	54.35 (0.37)	53.98 (0.3)	53.85 (0.24)
	S	0.16	0.17	0.18	0.19	0.2
$\mathbf{P_0}$	M	0.21	0.2	0.17	0.15	0.16
	L	0.11	0.11	0.12	0.12	0.13
	S	0.19	0.21	0.22	0.26	0.27
$\mathbf{P_2}$	M	0.26	0.24	0.23	0.21	0.2
	L	0.14	0.16	0.17	0.17	0.18
	S	0.22	0.24	0.25	0.28	0.31
$\mathbf{P_4}$	M	0.33	0.3	0.28	0.28	0.26
	L	0.18	0.19	0.19	0.2	0.21
	S	0.27	0.3	0.31	0.33	0.34
$\mathbf{P_6}$	M	0.4	0.38	0.36	0.36	0.35
	L	0.21	0.22	0.23	0.25	0.26
	S	0.3	0.32	0.38	0.41	0.43
$\mathbf{P_8}$	M	0.46	0.44	0.42	0.4	0.38
	L	0.28	0.29	0.3	0.3	0.32
	S	0.31	0.33	0.39	0.42	0.44
$\mathbf{P_{10}}$	M	0.5	0.48	0.43	0.41	0.4
	L	0.29	0.31	0.32	0.33	0.34

TABLE 2. Accuracy and precision performances of the step, linear drift, and monotonic MLEs under a multiple step change in the parameters of simple linear profiles with 5000 simulation runs and τ =50 and $\rho = 0.9$.

	Shifts (δ_1, δ_2) (δ_1, δ_2)	(0.1,0.001) (0.2,0.005)	(0.2,0.005) (0.4,.01)	(0.4,0.01) (0.6,.05)	(0.6,0.01) (0.8,0.05)	(0.8,0.05) (1,0.08)
Accuracy	$E(T)$	114.11	103.42	100.12	98.31	97.25
	$\hat{\tau}_{sc}$	53.2 (0.52)	52.95 (0.49)	52.15 (0.37)	51.95 (0.25)	50.7 (0.08)
	$\hat{\tau}$	50.74 (0.1)	51.54 (0.2)	52.85 (0.25)	53.21 (0.38)	53.33 (0.41)
	$\hat{\tau}_{lt}$	55.11 (0.56)	54.22 (0.49)	54.35 (0.38)	53.98 (0.31)	53.45 (0.25)
	S	0.1	0.11	0.12	0.14	0.14
P_0	M	0.15	0.13	0.12	0.12	0.1
	L	0.06	0.07	0.08	0.08	0.09
	S	0.11	0.12	0.13	0.14	0.16
P_2	M	0.16	0.15	0.14	0.12	0.1
	L	0.08	0.09	0.09	0.1	0.13
	S	0.18	0.19	0.21	0.22	0.23
P_4	M	0.3	0.27	0.2	0.2	0.19
	L	0.15	0.16	0.15	0.17	0.18
	S	0.2	0.21	0.23	0.24	0.29
P_6	M	0.36	0.34	0.21	0.22	0.19
	L	0.16	0.15	0.13	0.1	0.1
	S	0.25	0.26	0.31	0.3	0.33
P_8	M	0.37	0.36	0.29	0.28	0.26
	L	0.2	0.21	0.23	0.25	0.27
	S	0.27	0.29	0.33	0.35	0.4
P_{10}	M	0.41	0.42	0.38	0.36	0.34
	L	0.21	0.23	0.25	0.29	0.32

TABLE 3. Accuracy and precision performances of the step, linear drift, and isotonic MLEs under a step change in the parameters of simple linear profiles with 5000 simulation runs and τ =50 and $\rho = 0.1$.

	Shifts (δ_1, δ_2) (δ_1, δ_2)	(0.1,0.001)	(0.2,0.005)	(0.4,0.01)	(0.6,0.05)	(0.8,0.08)
Accuracy	$E(T)$	135.64	131.24	127.7	145.31	117.22
	$\hat{\tau}_{sc}$	54.12 (0.5)	53.75 (0.46)	53.16 (0.35)	52.85 (0.24)	51.65 (0.2)
	$\hat{\tau}$	50.74 (0.09)	51.54 (0.15)	52.85 (0.25)	53.21 (0.37)	53.33 (0.37)
	$\hat{\tau}_{lt}$	56.21 (0.5)	54.15 (0.42)	54.55 (0.35)	53.78 (0.32)	53.25 (0.39)
	S	0.18	0.19	0.2	0.22	0.24
P_0	M	0.23	0.25	0.19	0.17	0.15
	L	0.11	0.12	0.12	0.13	0.13
	S	0.2	0.22	0.23	0.27	0.29
P_2	M	0.27	0.25	0.24	0.22	0.21
	L	0.14	0.16	0.17	0.19	0.19
	S	0.23	0.25	0.27	0.29	0.3
P_4	M	0.33	0.3	0.29	0.28	0.26
	L	0.19	0.2	0.21	0.21	0.23
	S	0.28	0.31	0.32	0.34	0.34
P_6	M	0.4	0.38	0.36	0.35	0.35
	L	0.2	0.21	0.22	0.23	0.26
	S	0.31	0.33	0.38	0.41	0.43
P_8	M	0.47	0.45	0.43	0.42	0.38
	L	0.28	0.29	0.3	0.31	0.32
	S	0.33	0.35	0.39	0.43	0.44
P_{10}	M	0.48	0.46	0.44	0.4	0.4
	L	0.3	0.31	0.32	0.33	0.34

TABLE 4. Accuracy and precision performances of the step, linear drift, and isotonic MLEs under a step change in the parameters of simple linear profiles with 5000 simulation runs and τ =50 and $\rho = 0.9$.

	Shifts (δ_1, δ_2)	(0.1,0.001)	(0.2,0.005)	(0.4,0.01)	(0.6,0.05)	(0.8,0.08)
Accuracy	$E(T)$	124.64	130.34	117.72	115.4	107.34
	$\hat{\tau}_{sc}$	55.21 (0.5)	54.55 (0.46)	54.26 (0.36)	52.98 (0.23)	51.85 (0.21)
	$\hat{\tau}$	50.94 (0.1)	51.64 (0.15)	52.99 (0.24)	53.43 (0.36)	53.75 (0.37)
	$\hat{\tau}_{lt}$	56.41 (0.51)	54.35 (0.4)	54.85 (0.36)	53.98 (0.32)	53.88 (0.37)
P_0	S	0.17	0.18	0.19	0.2	0.23
	M	0.24	0.23	0.19	0.17	0.16
	L	0.11	0.12	0.13	0.14	0.14
P_2	S	0.21	0.23	0.24	0.28	0.3
	M	0.28	0.26	0.25	0.23	0.22
	L	0.16	0.18	0.19	0.21	0.23
P_4	S	0.24	0.26	0.28	0.3	0.33
	M	0.35	0.32	0.3	0.3	0.28
	L	0.2	0.21	0.22	0.22	0.23
P_6	S	0.32	0.32	0.33	0.35	0.36
	M	0.4	0.4	0.38	0.37	0.35
	L	0.24	0.24	0.25	0.27	0.28
P_8	S	0.32	0.34	0.39	0.43	0.45
	M	0.48	0.46	0.44	0.42	0.39
	L	0.3	0.29	0.31	0.32	0.34
P_{10}	S	0.33	0.35	0.4	0.42	0.45
	M	0.5	0.48	0.44	0.4	0.38
	L	0.31	0.33	0.35	0.36	0.37

TABLE 5. Accuracy and precision performances of the step, linear drift, and isotonic MLEs under a disturbance linear trend in the parameters of simple linear profiles with 5000 simulation runs and τ =50 and $\rho = 0.1$.

Shift		0.001	0.002	0.006	0.009	0.01
Accuracy	$E(T)$	157.61	154.4	150.34	144.25	132.3
		62.81 (0.23)	56.75 (0.25)	54.17 (0.3)	53.41 (0.24)	54.98 (0.2)
		51.25 (0.1)	51.38 (0.19)	52.2 (0.27)	53.26 (0.3)	53.89 (0.32)
	$\hat{\tau}_{lt}$	61.06 (0.36)	54.45 (0.29)	52.34 (0.25)	51.25 (0.14)	50.89 (0.08)
P_0	S	0.02	0.02	0.03	0.04	0.05
	M	0.06	0.05	0.05	0.03	0.02
	L	0.03	0.03	0.04	0.06	0.07
P_2	S	0.04	0.06	0.07	0.06	0.09
	M	0.1	0.09	0.08	0.07	0.06
	L	0.05	0.07	0.11	0.12	0.13
P_4	S	0.05	0.08	0.09	0.09	0.1
	M	0.14	0.1	0.09	0.1	0.12
	L	0.07	0.09	0.14	0.18	0.27
P_6	S	0.07	0.1	0.11	0.12	0.13
	M	0.19	0.18	0.17	0.15	0.14
	L	0.09	0.1	0.16	0.17	0.21
P_8	S	0.1	0.1	0.12	0.13	0.12
	M	0.2	0.19	0.18	0.16	0.15
	L	0.11	0.12	0.18	0.19	0.2
P_{10}	S	0.12	0.13	0.15	0.16	0.17
	M	0.29	0.26	0.24	0.23	0.21
	L	0.15	0.17	0.24	0.25	0.26

TABLE 6. Accuracy and precision performances of the step, linear drift, and isotonic MLEs under a single step change in the parameters of simple linear profiles with 5000 simulation runs and τ =50 and $\rho = 0.9$.

Shift		0.001	0.002	0.006	0.009	0.01
Accuracy	$E(T)$	157.61	154.44	150.34	144.25	132.3
		63.66 (0.23)	56.9 (0.25)	54.19 (0.3)	53.77 (0.24)	55.05 (0.2)
		51.6 (0.08)	51.44 (0.11)	52.31 (0.15)	53.36 (0.28)	53.45 (0.33)
	$\hat{\tau}_{lt}$	61.95 (0.33)	56.08 (0.27)	51.18 (0.19)	50.73 (0.15)	50.24 (0.09)
P_0	S	0.01	0.01	0.03	0.03	0.04
	M	0.05	0.04	0.04	0.02	0.01
	L	0.01	0.02	0.03	0.05	0.06
P_2	S	0.03	0.05	0.06	0.05	0.06
	M	0.09	0.08	0.07	0.06	0.05
	L	0.04	0.07	0.1	0.11	0.12
P_4	S	0.04	0.07	0.07	0.08	0.09
	M	0.12	0.1	0.08	0.09	0.1
	L	0.06	0.08	0.11	0.16	0.15
P_6	S	0.06	0.08	0.1	0.12	0.13
	M	0.16	0.13	0.15	0.13	0.1
	L	0.09	0.1	0.15	0.17	0.21
P_8	S	0.08	0.09	0.1	0.12	0.11
	M	0.19	0.18	0.17	0.16	0.14
	L	0.1	0.11	0.16	0.18	0.19
P_{10}	S	0.11	0.13	0.14	0.15	0.17
	M	0.28	0.25	0.23	0.22	0.19
	L	0.14	0.15	0.23	0.24	0.25

Also, a step change is imposed at 50th sample in Tables 3 and 4 under autocorrelation coefficients of 0.1 and 0.9, respectively. Afterward, a linear disturbance is considered in parameters of the simple linear profile at 50th observation. These simulation runs are provided in Tables 5 and 6 under the same autocorrelation coefficients.

In these tables, $\hat{\tau}$,$\hat{\tau}_{sc}$, and $\hat{\tau}_{lt}$denote the mean of the proposed estimator, step estimator, and linear drift estimator, respectively and the corresponding standard errors of the estimators are shown in parentheses. Also, in each simulation run, E(T) is the expected value of the number of samples taken until the first alarm happens, so E(T) = ARL+50 for given shifts. Also, results contain precision (P_i) under the given shifts. They are the percent of results which distance of the estimated change point from the exact change point is $i$or less than $i$and here i is equal to 0, 2,…, 10. As seen in results, the obtained precision and accuracy by using three estimators approve satisfactory performance of the proposed estimator. In Tables 1 and 2 which are under multiple changes, the estimators can estimate only the first change point. Results of Tables 1 to 4 show that the proposed isotonic estimator has superior performance in small to moderate shift sizes compared to the other estimators, when single or two step shift is exposed to parameters of simple linear profiles. However, in moderate to large shifts, step estimator has better performance because it is designed under the assumption of step change. Note that, drift estimator has worse performance than the other estimators under different shifts. Similarly, results of Tables 5 and 6 demonstrate that proposed change point estimator has better performance in the small to moderate shifts and drift estimator works better than the other estimator under moderate to large shifts. Meanwhile, step change point estimator has worse performance than the other estimators under these shifts. It is proved that although $\hat{\tau}_{sc}$ and $\hat{\tau}_{lt}$ are more accurate than the proposed estimator under the large shifts with assumptions of a step and drift change points respectively, isotonic change point estimator does not need any known assumption about the type of change. In addition, as shown in all of the tables, decreasing autocorrelation coefficient causes that the proposed estimator performs better than the other two estimators in terms of both accuracy and precision criteria.

6. CARDINALITY AND COVERAGE PERCENTAGE OF CONFIDENCE SET ESTMATOR

In this section, a confidence set for assumed D and $\boldsymbol{\delta}$ values is a window including possible change points whose log likelihoods fall within the log-likelihood of the estimated change point (τ) minus determined reference value i.e. D. The number of points in the confidence set is referred to cardinality, and the probability of the coverage is estimated as a fraction of the cardinality of the confidence set to the number of samples taken until an alarm given by control chart (T). In this regard, Box and Cox [22] proposed constructing confidence region using the likelihood function by a confidence set according to following form:

$$CS = \{t : \log_e L(t) > \log_e L(\hat{\tau}) - D\}, \quad (18)$$

in which $\log_e L(\hat{\tau})$ is the maximum of the log likelihood function achieved over all possible change points t. As seen in Equation (18), if the value of the log likelihood function at time t i.e. $\log_e L(t)$ exceeds the maximum of the log likelihood function minus D, the t is included in the confidence set. We use critical values of D between 1 and 6 and different $\boldsymbol{\delta}$ and $\boldsymbol{\delta}'$ vectors to compute the cardinality and coverage percentage of confidence set estimator. Figure 1 provides a surface plot demonstrating the relationship among cardinality, coverage,$\boldsymbol{\delta}$and D for the confidence set estimator. For instance, if the $\boldsymbol{\delta}$, $\boldsymbol{\delta}'$, and D equal to (0.1,0.001), (0.2, 0.005), and 6 respectively, the obtained confidence set will produce an expected cardinality close to 20, 21, and 22 for the proposed, step, and drift estimators, respectively. Also, percent of possible change points involve the estimated change point for the proposed, step and drift estimators are %95, %89, and %83, respectively. This figure demonstrates that the surface achieved from the confidence set of the proposed estimator superimposed on the surface obtained from confidence set of step and drift estimators shown with dashed surface under increasing two step changes. In other words, for any determined values of D, the confidence set estimator for drift and step changes will yield less coverage percentage and cardinality than the ones by the proposed estimator.

7. A REAL CASE

In this section, a case study is provided to show the performance of the proposed change point estimator in practice. In this regard, we apply a real data set in Soleimani et al.[18] taken from 10 apple trees which 25 apples are fortuitously gathered on each tree.

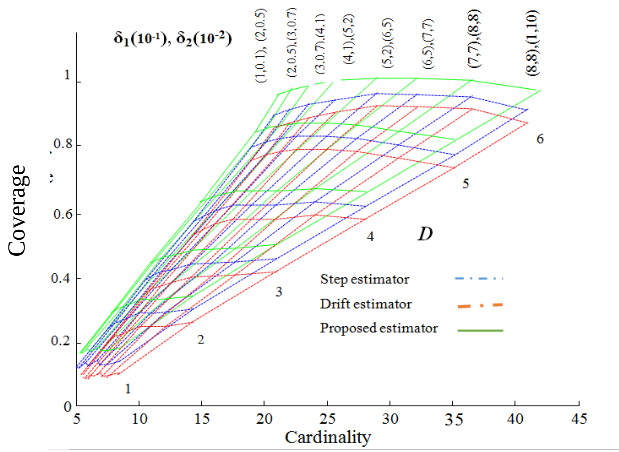

Figure 1. Estimated cardinality of confidence set and the corresponding coverage probability for drift, step, and proposed change point estimators under increasing two step changes.

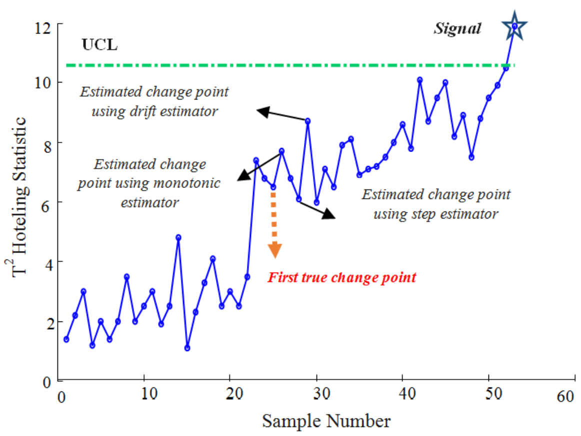

Figure 2. Comparison of the change point estimators for data of the apple trees under given isotonic shift in the slope parameter using the estimators.

Afterwards, these data are broken down to contain only the apples with the largest sizes to meet desirable values. Hence, the 80 apples with the largest size are remained in the data set from the existing 250 apples. Every 2 weeks for 12 weeks the diameter of each apple is evaluated. Consequently, there are 6 samples on each apple that constitutes a simple linear profile according to Schabenberger and Pierce [23].

On another side, Durbin–Watson test results ratify which there are the AR (1) structure between observations in each profile. In this case, response and explanatory variables are apple diameter and time, respectively and the relationship between them is a critical quality characteristic as a profile that should be monitored over time. So, the AR (1) autocorrelated simple linear profile is simulated for each apple and is monitored by using Hotelling T^2 control chart. The data analysis provides 0.7 and 0.0004 as the correlation coefficient and variance of the data, respectively. Also, the regression parameters are as follows from Soleimani et al. [18]:

$$y'_{ij} = 0.87 + 0.02x' + \varepsilon_{ij},$$

in which ρ is considered as 0.7. After applying the transformation method on the observations and monitoring the profiles, a step change with magnitude of the 0.005 is imposed in slope parameter of the regression model from 25th profile. Then, the second shift is given in 30th profile with slope of the linear trend disturbance 0.002. After that, the observations are simulated until the T^2 control chart alarms. At the same time, the change point is estimated by the proposed method as well as the drift and step change point estimators. As shown, the proposed estimator outperforms an MLE designed for step and drift changes.

8. CONCLUSION AND FUTURE RESEARCHES

In this paper, an MLE was derived for simple linear profiles with AR(1) autocorrelation structure within each profile to estimate isotonic change point. The performance of the proposed estimator was appraised in terms of accuracy and precision. Also, the performance of the proposed estimator was compared to change point estimators derived for step change and linear drift under different shifts and autocorrelation coefficients. Furthermore, the performance of the proposed change point estimator was compared under increasing shifts in regression parameters in simple linear profiles. Based on the results, if regression parameters placed under a single step change, linear disturbance, and multiple changes, performance of the corresponding estimators are better than the others under large shifts. However, the proposed estimator indicated better results than the others for small and medium shifts. It is worth mentioning that knowing the type of change is a main assumption in deriving an MLE of step and linear trend shifts whereas knowing the change type is not essential in the proposed change point estimator. In addition, cardinality and coverage percent of a confidence set estimator was analyzed under a type of isotonic shift. The results showed that the performance of the proposed confidence set estimator was better than the confidence set estimator derived for step and drift changes. At last, the application of the proposed estimator was shown through a real case. Investigating the other methods such as clustering for isotonic change point estimation can be considered as future researches.

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10. REFERENCES

1. Kang, L. and Albin, S., "On-line monitoring when the process yields a linear", ***Journal of Quality Technology***, Vol. 32, No. 4, (2000), 418-426.
2. Mahmoud, M.A., Parker, P.A., Woodall, W.H. and Hawkins, D.M., "A change point method for linear profile data", ***Quality and Reliability Engineering International***, Vol. 23, No. 2, (2007), 247-268.
3. Gupta, S., Montgomery, D. and Woodall, W., "Performance evaluation of two methods for online monitoring of linear calibration profiles", ***International Journal of Production Research***, Vol. 44, No. 10, (2006), 1927-1942.
4. Keramatpour, M., Niaki, S., Khedmati, M. and Soleymanian, M., "Monitoring and change point estimation of ar (1) autocorrelated polynomial profiles", ***International Journal of Engineering-Transactions C: Aspects***, Vol. 26, No. 9, (2013), 933-942.
5. Kamranrad, R. and Amiri, A., "Robust holt-winter based control chart for monitoring autocorrelated smple linear profiles with contaminated data", ***To Appear in Scientia Iranica***, , (2015).
6. Noorossana, R. and Shadman, A., "Estimating the change point of a normal process mean with a monotonic change", ***Quality and Reliability Engineering International***, Vol. 25, No. 1, (2009), 79-90.
7. Sharafi, A., Aminnayeri, M., Amiri, A. and Rasouli, M., "Estimating the change point of binary profiles with a linear trend disturbance", ***International Journal of Industrial Engineering***, Vol. 24, No. 2, (2013), 123-129.
8. Sharafi, A., Aminnayeri, M. and Amiri, A., "Identifying the time of step change in binary profiles", ***The International Journal of Advanced Manufacturing Technology***, Vol. 63, No. 1-4, (2012), 209-214.
9. Sharafi, A., Aminnayeri, M. and Amiri, A., "Identifying the change point in monitoring poisson regression profiles with linear trend disturbance", ***Journal of Quality Engineering and Management***, Vol. 1, No. 1, (2012), 39-44.
10. Sharafi, A., Aminnayeri, M. and Amiri, A., "An mle approach for estimating the time of step changes in poisson regression profiles", ***Scientia Iranica***, Vol. 20, No. 3, (2013), 855-860.

11. Kazemzadeh, R.B., Noorossana, R. and Ayoubi, M., "Change point estimation of multivariate linear profiles under linear drift", ***Communications in Statistics-Simulation and Computation***, Vol. 44, No. 6, (2015), 1570-1599.
12. Shadman, A., Mahlooji, H., Yeh, A.B. and Zou, C., "A change point method for monitoring generalized linear profiles in phase i", ***Quality and Reliability Engineering International***, (2014).
13. Sogandi, F. and Amiri, A., "Estimating the time of a step change in gamma regression profiles using mle approach", ***International Journal of Engineering-Transactions B: Applications***, Vol. 28, No. 2, (2014), 224-231.
14. Sogandi, F. and Amiri, A., "Change point estimation of gamma regression profiles with a linear trend disturbance", ***International Journal of Quality Engineering and Technology***, Vol. 4, No. 4, (2014), 352-368.
15. Ayoubi, M., Kazemzadeh, R.B. and Noorossana, R., "Change point estimation in the mean of multivariate linear profiles with no change type assumption via dynamic linear model", ***Quality and Reliability Engineering International***, (2015).
16. Khedmati, M. and Niaki, S.T.A., "Identifying the time of a step change in ar (1) auto-correlated simple linear profiles". ***Journal of Industrial Engineering International***, (2015)
17. Sogandi, F. and Amiri, A., "Monotonic change point estimation of generalized linear model-based regression profiles", ***Communications in Statistics-Simulation and Computation***, just-accepted, (2015).
18. Soleimani, P., Noorossana, R. and Amiri, A., "Simple linear profiles monitoring in the presence of within profile autocorrelation", ***Computers & Industrial Engineering***, Vol. 57, No. 3, (2009), 1015-1021.
19. Perry, M.B. and Pignatiello Jr, J.J., "Identifying the time of step change in the mean of autocorrelated processes", ***Journal of Applied Statistics***, Vol. 37, No. 1, (2010), 119-136.
20. Perry, M.B., Pignatiello, J.J. and Simpson, J.R., "Estimating the change point of the process fraction non-conforming with a monotonic change disturbance in spc", ***Quality and Reliability Engineering International***, Vol. 23, No. 3, (2007), 327-339.
21. Best, M.J. and Chakravarti, N., "Active set algorithms for isotonic regression; a unifying framework", ***Mathematical Programming***, Vol. 47, No. 1-3, (1990), 425-439.
22. Box, G.E. and Cox, D.R., "An analysis of transformations", ***Journal of the Royal Statistical Society. Series B (Methodological)***, (1964), 211-252.
23. Schabenberger, O. and Pierce, F.J., "Contemporary statistical models for the plant and soil sciences, CRC press, (2001).

Isotonic Change Point Estimation in the AR(1) Autocorrelated Simple Linear Profiles

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چکیده

گاهی اوقات رابطه بین متغیرهای مستقل و وابسته تحت عنوان پروفایل پایش می‌شود. پروفایل‌های خطی ساده در بین انواع پروفایل‌ها به دلیل کاربردهای آنها خصوصاً در کالیبراسیون بیشتر مورد توجه قرارگرفته‌اند. تحقیقاتی نیز در زمینه پایش پروفایل‌ها زمانی که مشاهدات درون هرپروفایل خودهمبسته هستند وجود دارند. از طرف دیگر تخمین نقطه تغییر منجر به صرفه جویی در زمان و هزینه می‌شود. بنابراین در این مقاله یک برآوردکننده ماکزیمم درست‌نمایی برای پروفایل-های خطی ساده خود همبسته اتورگرسیون مرتبه اول در مشاهدات داخل هر پروفایل به منظور تخمین نقطه تغییر یکنوای افزایشی پیشنهاد شده است. عملکرد برآوردکننده پیشنهادی ارزیابی شده و با برآوردکننده‌های تغییرات پله ای و تدریجی نیز مقایسه می‌شود. همچنین در ادامه یک مجموعه اطمینان برای برآورد کننده ارائه می‌شود. نتایج شبیه سازی‌ها نشان می دهد که برآوردکننده پیشنهادی در تغییرات کوچک و متوسط عملکرد بهتری دارد درحالیکه در تغییرات بزرگ عملکرد برآوردکننده های پله ای و تدریجی بهتر است. اما این نکته حائز اهمیت است که در برآوردکننده پیشنهادی دانستن نوع تغییر از قبل لزومی ندارد و تنها کافی است که نوع تغییر از خانواده تغییرات یکنوای افزایشی باشد. نهایتاً عملکرد برآورد کننده با یک مثال واقعی نشان داده شده است.

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