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Nonlinear Bending Analysis of Sector Graphene Sheet Embedded in Elastic Matrix Based on Nonlocal Continuum Mechanics

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ABSTRACT

The nonlinear bending behavior of sector graphene sheets is studied subjected to uniform transverse loads resting on a Winkler-Pasternak elastic foundation using the nonlocal elasticity theory. Considering the nonlocal differential constitutive relations of Eringen theory based on first order shear deformation theory and using the von-Karman strain field, the equilibrium partial differential equations are derived. The nonlinear partial differential equations system is solved using the differential quadrature method (DQM) and a new semi analytical polynomial method (SAPM). By using the DQM or SAPM, the partial differential equations are converted to nonlinear algebraic equations, then the Newton–Raphson iterative scheme is applied to solve the resulting nonlinear algebraic equations system. The obtained results from DQM and SAPM are compared. It is observed that the SAPM results are so close to DQM. The SAPM's formulations are considerably simpler than the DQM. Different boundary conditions including clamped, simply supported and free edges are considered. The obtained results are validated with available researches, then the small scale effects is investigated on the results due to various conditions such as outer radius to thickness ratio, boundary conditions, linear to nonlinear analysis, nonlocal to local analysis ratio, angle of the sector and stiffness value of elastic foundation.

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1. INTRODUCTION

Nanostructures are attended by many researchers due to their special mechanical, thermal and electrical properties. In recent years, nano materials are widely used in nano-systems, therefore, studying their mechanical behavior is considered. A graphene sheet is the network of carbon atoms that arranged in hexagonal form. In this network, each atom has the covalent bond with three atoms around that leads to high flexibility, enormous resistance in tension and low thermal expansion.

In recent years, many researchers have studied the mechanical behavior of nanostructures. Aside from experimental works, simulating methods such as atomic simulation, combination of atomic simulation and continuum mechanics, and only continuum mechanics are used to study the mechanical behavior of nanostructures [1]. Whereas the control in experimental methods is so difficult in nano scale size and also the atomic simulation methods are expensive in calculations, consequently, the continuum mechanics method is applied considerably because of convenience in formulation and acceptable results in comparison with the other methods [2]. The continuum mechanics methods are categorized into several methods such as the Couple Stress Theory [3], Modified Strain Gradient Elasticity [4] and Eringen Nonlocal Elasticity Theory [5]. Between the mentioned methods, the Eringen Nonlocal Elasticity Theory is widely used for studying mechanical behavior of nanostructures. Based on Eringen theory, the stress at a reference point is a function of strain field at every point in the body. On the

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other hand, the covalent bond between atoms has the significant effects on mechanical properties at nano scale size and using the local elasticity theory leads to unacceptable results [6, 7]. For the first time, Hans Pieter Bohem [8] described a carbonic structure with hexagonal shape in title of the graphene. Graphite is built from several layers of graphene on each other. As well as, nanotubes discovered by Iijima [9] in Tesokoba laboratory in 1991. Thereafter, many researchers used the nonlocal elasticity theory to study the mechanical properties of nanostructures, especially vibrational and buckling issues. Investigations show that the amount of literatures in one dimensional nanostructures such as nanotubes and nanobeams is considerably more than the two dimensional ones. Lima and He [10] improved a von Karman nonlinear model for ultra-thin, elastically isotropic films with surface effect. Khanchehgardan et al. [11] are studied thermo-elastic damping in nano-beam Resonators Based on Nonlocal Theory. They concluded that with increasing the amount of nonlocal parameter and also with decreasing the length of the nano-beam, difference of the results of classical and nonlocal theory increases. Pradhan [12] investigated the buckling of rectangular graphene sheets, and proved that the ratio of critical buckling load in nonlocal to local theory is equal or less than one and found that using higher order shear deformation theories is much more accurate for thick nano plates. Reddy [13] obtained the governing equations for orthotropic beams and plates in nonlocal form considering the Von Karman nonlinear strain field that allows for large deformations. Ansari et al. [14] are studied the effects of small scale parameter for vibrations of multilayered graphene sheets in polymer environment. To this effect, they merged the equations of nonlocal elasticity and Mindlin theories and calculated the van der Waals interactions. Samayi et al. [15] studied the buckling of monolayer graphene sheets in embedded elastic foundation based on nonlocal elasticity theory. Jomezadeh and Saidi [16] investigated vibration of multilayered graphene sheets based on nonlocal elasticity theory considering nonlinear Von Karman strain field. Shen et al. [17] studied the nonlinear vibrational behavior for a simply supports single layer Graphene sheet in thermal environments in order to obtain the nonlocal parameter. Golmakani and Rezatalab [18] studied the nonlinear bending of an orthotropic rectangular graphene sheet resting on a Winkler-Pasternak elastic foundation. They found that with increase of nonlocal parameter, the maximum deflection is decreased.

In present study, the nonlinear bending of a sector graphene sheet is investigated based on the first-order shear deformation theory considering Von Karman strain field. In order to study the small scale effects on deflection, the Eringen's nonlocal theory is applied. The DQM which is a numerical method and a new semi analytical polynomial method (SAPM) are applied to solve the partial differential governing equations. First of all, the two methods are compared with each other and concluded that although the new method is extremely simple, but it is an accurate method. Since, there are not any literatures available on bending of sector graphene sheets and in order to demonstrate the accuracy of obtained results, the nonlocal parameter is considered to be zero and the results are validated with available literature. The effects of different conditions such as radius and thickness of the graphene nanoplate, outer radius to thickness ratio, nonlocal to local deflection ratio, linear to nonlinear analysis, different types of boundary conditions, and the effects of nonlocal parameter and the value of stiffness matrix on the results are investigated.

2. GOVERNING EQUATIONS

A sector graphene sheet is considered with thickness *h*, inner radius r_i, outer radius r_o, and the total angle of τ under uniform transverse loading *q* resting on an elastic foundation. The geometry of the plate is shown in Figure 1. In this paper, all the governing equations are derived based on the first-order shear deformation theory (FSDT) that includes the neglected assumptions in classical plate theory. According to the first-order shear deformation theory of plates, the displacement field can be expressed as:

$$U\left(r,\theta,z\right)=u\left(r,\theta\right)+z\psi_1\left(r,\theta\right) \tag{1}$$

$$V\left(r,\theta,z\right)=v\left(r,\theta\right)+z\psi_2\left(r,\theta\right) \tag{2}$$

$$W\left(r,\theta,z\right)=w\left(r,\theta\right) \tag{3}$$

In Equations (1)-(3), *u*, *v* and *w* are the displacement components of the mid-plane along the *r*, θ and *z* directions, respectively. ψ_1 and ψ_2 explain the rotation functions of the transverse normal about circumferential and radial directions.

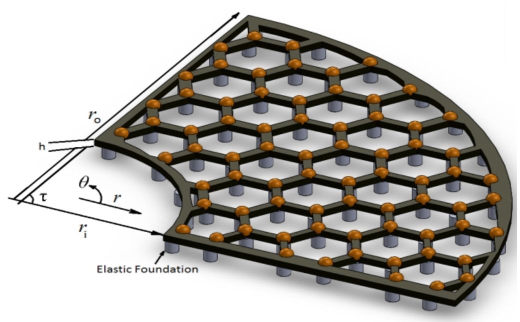

Figure 1. Geometry of sector graphene sheet

Considering von-Karman assumptions, the strain field is expressed as follows:

$$\varepsilon_r = \frac{\partial u}{\partial r} + z\frac{\partial \psi_1}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 \quad (4)$$

$$\varepsilon_\theta = \frac{1}{r}\left(\frac{\partial v}{\partial \theta} + u\right) + \frac{z}{r}\left(\frac{\partial \psi_2}{\partial \theta} + \psi_1\right) + \frac{1}{2}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}\right)^2 \quad (5)$$

$$\gamma_{rz} = \frac{\partial w}{\partial r} + \psi_1 \quad (6)$$

$$\gamma_{\theta z} = \frac{1}{r}\frac{\partial w}{\partial \theta} + \psi_2 \quad (7)$$

$$\gamma_{r\theta} = \frac{1}{r}\left(\frac{\partial u}{\partial \theta} + z\frac{\partial \psi_1}{\partial \theta} - v - z\psi_2\right) + \frac{\partial v}{\partial r} + z\frac{\partial \psi_2}{\partial r} + \frac{1}{r}\frac{\partial w}{\partial r}\frac{\partial w}{\partial \theta} \quad (8)$$

In nonlocal elasticity theory, the effects of small scale and atomic forces directly come into the constitutive equations as material parameters [5]. In this theory, the stress at reference point X is a function of strain field in every point on plate. Eringen presented a differential form of the nonlocal constitutive equation from nonlocal balance law as follow [5]:

$$\left(1 - \mu\nabla^2\right)\sigma^{NL} = \sigma^L, \mu = \left(e_0 a\right)^2 \quad (9)$$

in which, a is internal characteristic length, and e_0 is material constant which is defined by experiment. The parameter $e_0 a$ is the small-scale parameter revealing the small-scale effect on the responses of nano-size structures. The value of the small-scale parameter depends on boundary conditions, chirality, number of walls, and the nature of motions. The nonlocal stresses using the Equation (9) can be defined in polar coordinates system as follows [5]:

$$\sigma_r^{NL} - \mu\left(\nabla^2\sigma_r^{NL} - \frac{4}{r^2}\frac{\partial \sigma_{r\theta}^{NL}}{\partial \theta} - \frac{2}{r^2}\left(\sigma_r^{NL} - \sigma_\theta^{NL}\right)\right) = \sigma_r^L \quad (10)$$

$$\sigma_\theta^{NL} - \mu\left(\nabla^2\sigma_\theta^{NL} + \frac{4}{r^2}\frac{\partial \sigma_{r\theta}^{NL}}{\partial \theta} + \frac{2}{r^2}\left(\sigma_r^{NL} - \sigma_\theta^{NL}\right)\right) = \sigma_\theta^L \quad (11)$$

$$\sigma_{r\theta}^{NL} - \mu\left(\nabla^2\sigma_{r\theta}^{NL} - \frac{4}{r^2}\sigma_{r\theta}^{NL} + \frac{2}{r^2}\frac{\partial}{\partial \theta}\left(\sigma_r^{NL} - \sigma_\theta^{NL}\right)\right) = \sigma_{r\theta}^L \quad (12)$$

$$\sigma_{rz}^{NL} - \mu\left(\nabla^2\sigma_{rz}^{NL} - \frac{1}{r^2}\sigma_{rz}^{NL} - \frac{2}{r^2}\frac{\partial \sigma_{\theta z}^{NL}}{\partial \theta}\right) = \sigma_{rz}^L \quad (13)$$

$$\left(\sigma_{\theta z}^{NL} - \mu\left(\nabla^2\sigma_{\theta z}^{NL} - \frac{1}{r^2}\sigma_{\theta z}^{NL} + \frac{2}{r^2}\frac{\partial \sigma_{rz}^{NL}}{\partial \theta}\right)\right) = \sigma_{\theta z}^L \quad (14)$$

In Equations (10)-(14), ∇^2 is the Laplacian operator in polar coordinates system as follow:

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \quad (15)$$

The nonlocal force, moment and shear force components $N_i^{NL}(i = r,\theta, r\theta)$, $M_i^{NL}(i = r,\theta, r\theta)$ and $Q_i^{NL}(i = r,\theta)$, are as follows:

$$(M_r, M_\theta, M_{r\theta})^{NL} = \int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\sigma_r, \sigma_\theta, \sigma_{r\theta}\right)^{NL} z dz \quad (16)$$

$$(N_r, N_\theta, N_{r\theta}, Q_r, Q_\theta)^{NL} = \int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\sigma_r, \sigma_\theta, \sigma_{r\theta}, \kappa_s\sigma_{rz}, \kappa_s\sigma_{\theta z}\right)^{NL} dz \quad (17)$$

By using Equation (9), the relations between local and nonlocal force, moment and shear force components can be expressed as follows:

$$\left(1 - \mu\nabla^2\right)N_i^{NL} = N_i^L, i = (r,\theta, r\theta) \quad (18)$$

$$\left(1 - \mu\nabla^2\right)M_i^{NL} = M_i^L, i = (r,\theta, r\theta) \quad (19)$$

$$\left(1 - \mu\nabla^2\right)Q_i^{NL} = Q_i^L, i = (r,\theta) \quad (20)$$

$N_i^L, M_i^L(i = r,\theta, r\theta)$ and $Q_i^L(i = r,\theta)$ are the local in-plane force, moment and the shear force resultants which are defined for isotropic material as:

$$N_r^L = \frac{Eh}{1-v^2}\left(\frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2 + v\left(\frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}\right)^2\right)\right) \quad (21)$$

$$N_\theta^L = \frac{Eh}{1-v^2}\left(v\left(\frac{\partial u}{\partial r} + \frac{1}{2}\left(\frac{\partial w}{\partial r}\right)^2\right) + \frac{u}{r} + \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{1}{2}\left(\frac{1}{r}\frac{\partial w}{\partial \theta}\right)^2\right) \quad (22)$$

$$N_{r\theta}^L = \frac{Eh}{2(1+v)}\left(\frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{1}{r}\frac{\partial w}{\partial r}\frac{\partial w}{\partial \theta}\right) \quad (23)$$

$$Q_r^L = \frac{\kappa_s Eh}{2(1+v)}\left(\frac{\partial w}{\partial r} + \psi_1\right) \quad (24)$$

$$Q_\theta^L = \frac{\kappa_s Eh}{2(1+v)}\left(\frac{1}{r}\frac{\partial w}{\partial \theta} + \psi_2\right) \quad (25)$$

$$M_r^L = \frac{Eh^3}{12(1-v^2)}\left(\frac{\partial \psi_1}{\partial r} + \frac{v}{r}\left(\frac{\partial \psi_2}{\partial \theta} + \psi_1\right)\right) \quad (26)$$

$$M_\theta^L = \frac{Eh^3}{12(1-v^2)}\left(v\frac{\partial \psi_1}{\partial r} + \frac{1}{r}\left(\frac{\partial \psi_2}{\partial \theta} + \psi_1\right)\right) \quad (27)$$

$$M_{r\theta}^L = \frac{Eh^3}{24(1+v)}\left(\frac{\partial \psi_2}{\partial r} + \frac{1}{r}\frac{\partial \psi_1}{\partial \theta} - \frac{\psi_2}{r}\right) \quad (28)$$

Using the principle of stationary total potential energy, the governing equations as well as the related boundary conditions along the edges of sector plate can be derived. The equations of the total potential energy in case of nonlocal form are expressed as:

$$\delta\Pi = \delta U + \delta\Omega = 0 \tag{29}$$

$$\delta U = \iiint_V (\sigma_{rr}^{NL}\delta\varepsilon_{rr} + \sigma_{\theta\theta}^{NL}\delta\varepsilon_{\theta\theta} + \sigma_{r\theta}^{NL}\delta\gamma_{r\theta} + \kappa_s\sigma_{rz}^{NL}\delta\gamma_{rz} + \kappa_s\sigma_{\theta z}^{NL}\delta\gamma_{\theta z})dV \tag{30}$$

$$\delta\Omega = \int_{r_i}^{r_o}\int_0^{\tau}\left(q - k_w w + k_p\nabla^2 w\right)\delta w r dr d\theta \tag{31}$$

U and Ω are the strain energy and potential of applied forces, respectively. κ_s is the transverse shear correction coefficient and in this study is taken as $5/6$ for the isotropic material and k_w and k_p are the Winkler and Pasternak stiffness coefficient of elastic matrix.

Using the variation principals, the nonlocal equilibrium equations of sector graphene sheets are obtained in cylindrical coordinates system in terms of nonlocal force, moment and the shear force resultants as:

$$\delta u : N_{r,r}^{NL} + \frac{1}{r}\left(N_{r\theta,\theta}^{NL} + N_r^{NL} - N_\theta^{NL}\right) = 0 \tag{32}$$

$$\delta v : N_{r\theta,r}^{NL} + \frac{1}{r}\left(N_{\theta,\theta}^{NL} + 2N_{r\theta}^{NL}\right) = 0 \tag{33}$$

$$\delta w : Q_{r,r}^{NL} + \frac{1}{r}\left(Q_{\theta,\theta}^{NL} + Q_r^{NL}\right) + \left(q - k_w w + k_p\nabla^2 w + N_r^{NL}\frac{\partial^2 w}{\partial r^2} + N_\theta^{NL}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right) + 2N_{r\theta}^{NL}\left(\frac{1}{r}\frac{\partial^2 w}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial w}{\partial\theta}\right)\right) = 0 \tag{34}$$

$$\delta\psi_1 : M_{r,r}^{NL} + \frac{1}{r}\left(M_{r\theta,\theta}^{NL} + M_r^{NL} - M_\theta^{NL}\right) - Q_r^{NL} = 0 \tag{35}$$

$$\delta\psi_2 : M_{r\theta,r}^{NL} + \frac{1}{r}\left(M_{\theta,\theta}^{NL} + 2M_{r\theta}^{NL}\right) - Q_\theta^{NL} = 0 \tag{36}$$

Substituting Equations (10)-(14) in Equations (16)-(17) and then inserting the resulted nonlocal form of force, moment and shear force resultants into Equations (32)-(36) leads to five equilibrium Equations (37)-(41) which can be defined in local form as follows:

$$\delta u : N_{r,r}^{L} + \frac{1}{r}\left(N_{r\theta,\theta}^{L} + N_r^{L} - N_\theta^{L}\right) = 0 \tag{37}$$

$$\delta v : N_{r\theta,r}^{L} + \frac{1}{r}\left(N_{\theta,\theta}^{L} + 2N_{r\theta}^{L}\right) = 0 \tag{38}$$

$$\delta w : Q_{r,r}^{L} + \frac{1}{r}\left(Q_{\theta,\theta}^{L} + Q_r^{L}\right) + \left(1 - \mu\nabla^2\right)\left(q - k_w w + k_p\nabla^2 w + N_r^{L}\frac{\partial^2 w}{\partial r^2} + N_\theta^{L}\left(\frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial\theta^2}\right) + 2N_{r\theta}^{L}\left(\frac{1}{r}\frac{\partial^2 w}{\partial r\partial\theta} - \frac{1}{r^2}\frac{\partial w}{\partial\theta}\right)\right) = 0 \tag{39}$$

$$\delta\psi_1 : M_{r,r}^{L} + \frac{1}{r}\left(M_{r\theta,\theta}^{L} + M_r^{L} - M_\theta^{L}\right) - Q_r^{L} = 0 \tag{40}$$

$$\delta\psi_2 : M_{r\theta,r}^{L} + \frac{1}{r}\left(M_{\theta,\theta}^{L} + 2M_{r\theta}^{L}\right) - Q_\theta^{L} = 0 \tag{41}$$

Because of small numbers, for sake of generality and convenience, the following non-dimensional terms are introduced as follow:

$$r^* = \frac{r}{r_o}; u^* = \frac{u}{h}; v^* = \frac{v}{h}; w^* = \frac{w}{r_o}; z^* = \frac{z}{h}; \psi_1^* = \psi_1; \psi_2^* = \psi_2$$

$$\eta = \frac{h}{r_o};\ \mu^* = \frac{\mu}{r_o^2};\ q^* = \frac{q}{E};\ k_w^* = \frac{k_w r_o}{E}; Q^* = \frac{q^*}{\eta}; k_p^* = \frac{k_p}{Eh}$$

$$\nabla^{*2} = \frac{\partial^2}{\partial r^{*2}} + \frac{1}{r^*}\frac{\partial}{\partial r^*} + \frac{1}{r^{*2}}\frac{\partial^2}{\partial\theta^2}; \nabla^2 = \frac{\nabla^{*2}}{r_o^2}$$

3. NUMERICAL SOLUTION

In this paper, two different methods are used to obtain the results. First, the DQM method is applied which is a popular method for solving differential equations. Then, a new semi analytical polynomial method is introduced.

3. 1. Solution using Differential Quadrature Method (DQM)

The DQ method based on the approximation of partial derivative of a function at a point is affected by the values of the function in the whole domain [19].

According to DQ Method, the derivatives of a two dimensional function $f(r,\theta)$ at points r, θ can be defined as a linear sum of the function values [18, 20]. The number of discrete grid points and point distribution in DQM is arbitrary. However, it has been shown that the grid point distribution which is based on Gauss-Chebyshev-Lobatto points, gives more accurate results [21]. This method distributes points in unequal distances in a way that the points set mainly near the boundaries.

By use of DQM, partial differential equations turn to nonlinear algebraic equations system. Now, this system can be solved using several numerical methods. Because of high rate of convergence, in this paper, the Newton-Raphson iterative method is used to solve the nonlinear algebraic equations system.

3. 2. Semi Analytical Polynomial Method (SAPM)

In this method, every function of differential equations is approximated by a polynomial in general form depends on the grid point distribution. Contrary to collocation method [22], there is no need for the introduced polynomial functions to satisfy the boundary conditions. Every PDE or set of PDE's system will be solved conveniently and quickly with different types of boundary conditions. Contrary to DQM, in this method it is not needed to calculate weighted residuals coefficients for each derivative. Considering a partial differential equation as follow:

$$\frac{\partial^n f(r,\theta)}{\partial^n r}+\frac{\partial^{(n-1)} f(r,\theta)}{\partial^{(n-1)} r}+\ldots+\frac{\partial f(r,\theta)}{\partial r}+\frac{\partial^n f(r,\theta)}{\partial^n \theta}+$$
$$\frac{\partial^{(n-1)} f(r,\theta)}{\partial^{(n-1)} \theta}+\ldots+\frac{\partial f(r,\theta)}{\partial \theta}+\frac{\partial^n f(r,\theta)}{\partial r \partial^{(n-1)} \theta}+\frac{\partial^n f(r,\theta)}{\partial r^2 \partial^{(n-2)} \theta} \tag{42}$$
$$+\ldots+\frac{\partial^n f(r,\theta)}{\partial^{(n-1)} \partial \theta}=0$$

The two variables function *f(r,θ)* is considered as follow:

$$f(r,\theta)=\sum_{i=1}^{N}\sum_{j=1}^{M} a_{(i+j-(1-(i-1)(M-1)))} r^{(i-1)} \theta^{(j-1)} \tag{43}$$

In Equation (43), N is the number of grid points in r direction and M for θ direction. The grid points are shown in Figure 2. By substituting Equation (43) in (42), the partial differential equation converts to algebraic equations. According to the Figure 2, the numbers of $2N+2M-4$ equations are derived from boundary conditions (dark points) and $(N-2)\cdot(M-2)=N\cdot M-2N-2M+4$ equations from Equation (43) (bright points). So, there are $M\cdot N$ algebraic equations and $M\cdot N$ unknown a_i. By substituting $a_i, i=1..M\cdot N$ in Equation (43), the function *f(x,y)* will be determined. For the set of partial differential equations, the similar method will apply.

If the differential equations are nonlinear, by substituting Equation (43) in governing equations, the resulted algebraic equations will be nonlinear. So, a numerical method must be applied to solve the set of nonlinear equations. In this paper, as the procedure is explained for DQM, the Newton-Raphson numerical method is investigated to solve the nonlinear algebraic equations system. Now, the dimensionless displacements and rotations are introduced in dimensionless form as follows:

$$u^*=\sum_{i=1}^{N}\sum_{j=1}^{M} a_{(i+j-(1-(i-1)(M-1)))} r^{*(i-1)} \theta^{(j-1)} \tag{44}$$

$$v^*=\sum_{i=1}^{N}\sum_{j=1}^{M} a_{(i+j-(1-(i-1)(M-1))+M\cdot N)} r^{*(i-1)} \theta^{(j-1)} \tag{45}$$

$$w^*=\sum_{i=1}^{N}\sum_{j=1}^{M} a_{(i+j-(1-(i-1)(M-1))+2M\cdot N)} r^{*(i-1)} \theta^{(j-1)} \tag{46}$$

$$\psi_1^*=\sum_{i=1}^{N}\sum_{j=1}^{M} a_{(i+j-(1-(i-1)(M-1))+3M\cdot N)} r^{*(i-1)} \theta^{(j-1)} \tag{47}$$

$$\psi_2^*=\sum_{i=1}^{N}\sum_{j=1}^{M} a_{(i+j-(1-(i-1)(M-1))+4M\cdot N)} r^{*(i-1)} \theta^{(j-1)} \tag{48}$$

Substituting the polynomial functions (44-48) into the governing equations, the PDE's was inverted to algebraic equations. Comparison between Equations (44-48) with the relations for DQM, it is obvious that the formulations and coding by computer programs in SAPM method are significantly simpler than DQM.

4. BOUNDARY CONDITIONS

In this paper, the boundaries are considered in the category of the simply supported (S), clamped (C) and free edges (F) which the conditions in each one are defined as Figure 3.

S:	$u=v=w=\psi_2=M_r=0$	$r=r_i,r_o$
	$u=v=w=\psi_1=M_\theta=0$	$\theta=0,\tau$
C:	$u=v=w=\psi_2=\psi_1=0$	$r=r_i,r_o$
	$u=v=w=\psi_2=\psi_1=0$	$\theta=0,\tau$
F:	$N_r=N_{r\theta}=Q_r=M_r=M_{r\theta}=0$	$r=r_i,r_o$
	$N_r=N_{r\theta}=Q_\theta=M_\theta=M_{r\theta}=0$	$\theta=0,\tau$

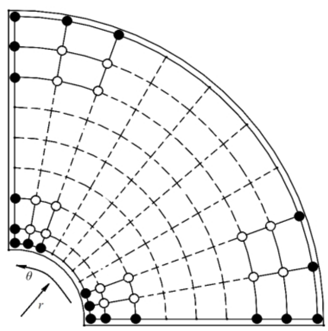

Figure 2. The division of two directional domain for sector plate

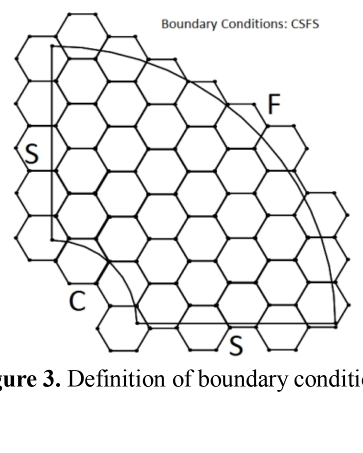

Figure 3. Definition of boundary conditions

TABLE 2. Comparison dimensionless deflection with the references

$\overline{w} = \left(\frac{D}{q r_o^4}\right) w \times 10^4$

[26]	[25]	[24]	[23]	Preset study SAPM 9×9	SAPM 7×7	DQM 9×9	DQM 7×7	$\frac{r_i}{r_o}$
2.840	2.760	2.841	2.860	2.848	2.801	2.835	2.788	0.25
1.410	1.420	1.426	1.430	1.438	1.415	1.433	1.414	0.5
0.100	0.090	1.043	0.099	0.093	0.090	0.09	0.088	0.75

5. NUMERICAL RESULTS

First, it is considered a sector nano-plate to check the convergence of DQM and SAPM. Figure 4, shows that the desirable converged results are obtained for only seven nodes in each direction. As well as, it is concluded that with the same nodes in each direction, the results are so close to each other using DQM and SAPM. Because of convenience in formulations and coding by computer programs, using of SAPM is highly recommended.

$E = 1.06\times10^{12} Pa,\ q^* = 1\times10^7 / E, \mu = 1\ \text{nm}^2$

$r_i / r_o = 0.2, h = 0.34\ \text{nm}, \tau = \pi / 2$

One of the benefits of sectorial shape is capability for obtaining the rectangular, annular and circular geometry. In this paper, a rectangular graphene sheet is estimated by a sector and the results are compared with available paper. The dimensionless deflection with $\mu = 4\text{nm}^2$ and the transverse load $q = 1\text{GPa}$ is compared with rectangular graphene sheet in Table 1 for SSSS boundary conditions. Since no research exist in nonlocal bending of sector plates; therefore, in order to validate with the other researches, the nonlocal parameter is considered to be zero. The central deflection reported in references [23-26] is compared with the ones obtained by the present solution with two methods DQM and SAPM with 7×7 and 9×9 grid points in Table 2. The properties of the plate are given as follow: $(r_0 - r_i)/h = 100,\ q = 1\,\text{Pa}, E = 200\times10^9\,\text{Pa},\ \tau = \pi/3$

Now, a sector graphene plate is considered with the properties below ($\theta = \pi, k_p^* = 0$):

$E = 1.06\times10^{12}\text{Pa},\ Q^* = 50\times10^7 / E, k_w^* = 0.005331, r_i / r_o = 0.2$

The central deflection is given using two methods DQM and SAPM for $e_0 a = 1\text{nm}$ and 7×7 grid point distribution in Table 3. It is observed that the obtained results from two different methods are so close. Consequently, by using the SAPM method, the expected accuracy of the result is obtained with the same grid point distribution toward DQM, but shorter in processing time by computer.

Figure 5 shows the dimensionless deflection versus nonlocal parameter for two types of boundary conditions SSSS and CCCC both with and without an elastic foundation ($q^* = 4\times10^6 / E$).

TABLE 1. Comparison dimensionless deflection with Ref [18]

q (GPa)	Dimensionless deflection ×1000 [18] h= 0.34nm	[18] h= 0.68nm	Present h= 0.34nm	Present h= 0.68nm
0.01	0.6	0.4	0.591	0.392
0.05	2.8	1.9	2.76	1.87
0.1	5.6	3.7	5.56	3.67
0.5	28	18.6	27.83	18.45
1	56	37.3	55.65	37

TABLE 3. Comparison of dimensionless deflection for two DQM and SAPM methods

	q (GPa)	Central dimensionless Deflection DQM	SAPM
	0.15	0.0030321056189380	0.0030321056187031
CCCC	0.2	0.0040428074919197	0.004042807491599
	0.5	0.0101070187298200	0.01010701872884

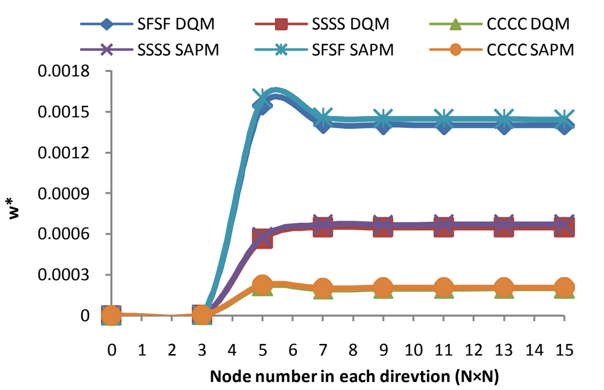

Figure 4. Dimensionless deflection versus the number of grid points for DQM and SAPM domain

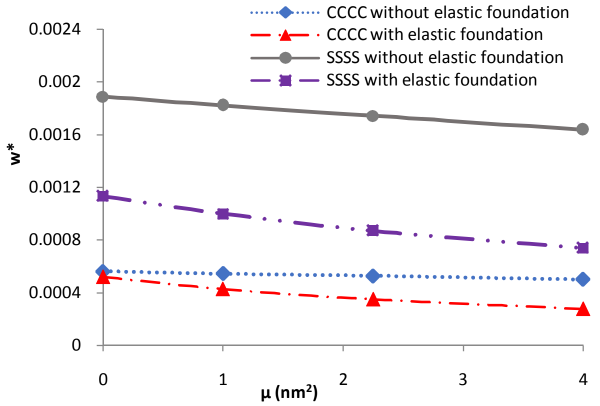

Figure 5. Variation of dimensionless deflection versus the nonlocal parameter in presence and absence of elastic foundation ($k_p = 1.13$ Pa.m)

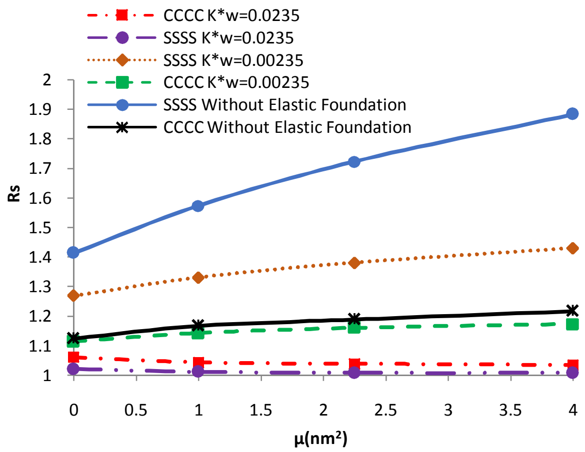

Figure 6. Linear to nonlinear deflection ratio R_s versus nonlocal parameter

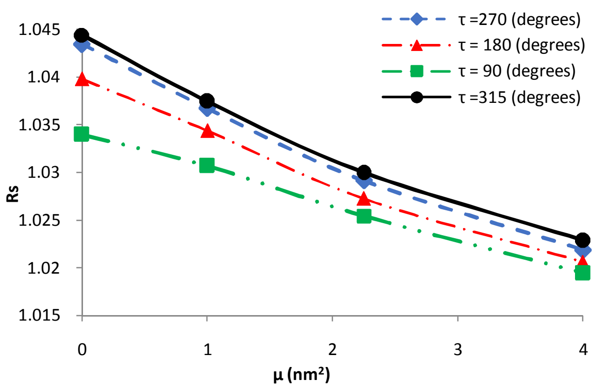

Figure 7. Diagram of R_s versus the nonlocal parameter for different sector angles (SSSS)

It is observed that the maximum deflection in CCCC boundary conditions is significantly less than SSSS. When there is not any elastic foundation, the increase of nonlocal parameter leads to decrease of the deflection gently. The maximum deflections are closer to each other in CCCC and SSSS boundary conditions in presence of Pasternak elastic foundation and the increase of nonlocal parameter has the more effects on the results. To study the differences between linear to nonlinear analyses, the parameter R_s is defined as follow [18]:

$$R_s = \frac{W^*_{Linear}}{W^*_{Nonlinear}} \tag{49}$$

Figure 6 is pictured variation of ratio R_s versus nonlocal parameter for two types of boundary conditions in presence and absence of Winkler elastic foundation with the properties below:

$$E = 1.06\times10^{12}\,\text{Pa},\; Q^* = 30\times10^7 / E, r_i / r_o = 0.2,\; \theta = \pi$$

Existence of elastic foundation causes decrease of flexibility, so the effect of boundary conditions decreases. Along the increase of the Winkler stiffness value, the results of two different boundary conditions and linear and nonlinear analysis approach to each other. Also, with increase of small scale effects, the R_s slope decreases gently. With increase of the Winkler stiffness value, the small scale effects decreases.

The differences between linear and nonlinear analysis goes up in absence of elastic foundation. This raising of R_s ratio is more significant for SSSS than CCCC boundary condition. The increase of nonlocal parameter has the more effects on the results in absence of elastic foundation especially for SSSS boundary conditions. However, these effects are decreased along the increase of nonlocal parameter.

In Figure 7, it can be concluded that increase of sector angle causes increase of dimensionless deflection and R_s parameter. With increase of μ, two linear and nonlinear FSDT analysis approach to each other and the effects of nonlocal parameter on the results are raised with increase of sector angle. As well as, the differences between R_s parameter along the raise of sector angle reduced.

The parameter R_m introduces to compare the results of nonlocal elasticity theory with local, as follow [18]:

$$R_m = \frac{W^*_{Nonlocal}}{W^*_{Local}} \tag{50}$$

$w^*_{Nonlocal}$ and w^*_{Local} are the dimensionless deflections in nonlocal and local elasticity theory, respectively. Figure 8, shows the variation of R_m versus dimensionless transverse load in presence and absence of elastic foundation.

According to Figure 8, it is concluded that two local and nonlocal elasticity theories are receded from each other in presence of elastic foundation, but along the raise of transverse loading the local and nonlocal theories approach. The results of nonlocal and local elasticity theories have the less difference in absence of elastic foundation. Contrary to the statement that the elastic foundation exists, the differences between two theories are increased along the growing of transverse loading. Also, the presence or absence of elastic foundation do not have any effects on the differences between two nonlocal and local elasticity theories in large amount of transverse loads and the results tend to a constant value.

The effect of sector's angle on R_m is shown in Figure 9 considering the nonlocal parameter $e_0 a = 1.5$nm for SSSS boundary conditions in absence of elastic foundation. The transverse load and the deformation of the plate increase with increasing of sector angle. So, the differences between the two theories increase with the raise of sector angle. Consequently, for greater angles use of nonlocal theory is more recommended. Also, with increase of sector angle, the two theories are separated from each other with higher rate.

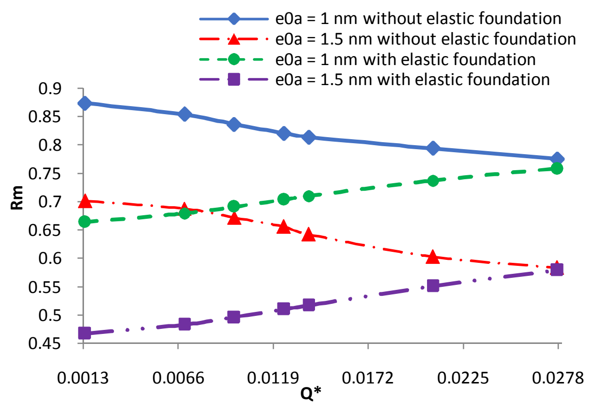

Figure 8. R_m ratio versus the non-dimensional transverse loading in presence and absence of elastic foundation (CCCC)

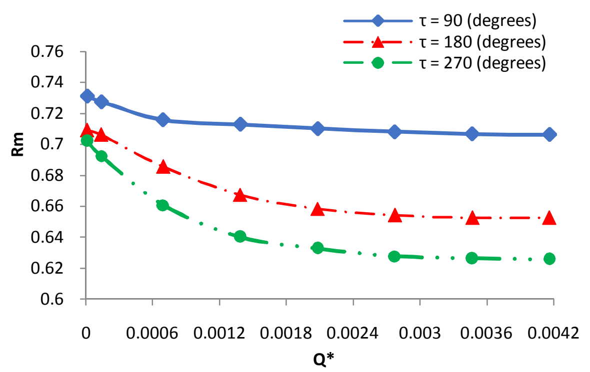

Figure 9. Diagram of R_m parameter versus non-dimensional transverse loading and different sector angle

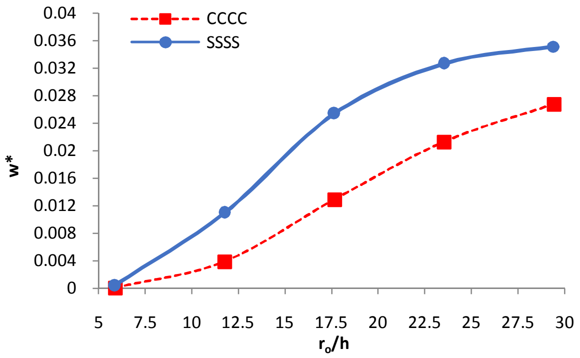

Figure 10. Dimensionless deflection versus the variation of r_o / h

The effect of increase of radius on deflection is studied in Figure 10 for two different types of boundary conditions SSSS and CCCC, with the plate conditions below:

$E = 1.06 \times 10^{12}$ Pa, $q^* = 50 \times 10^7 / E$, k_w=1Gpa/nm

$r_i = 1$nm, $\tau = \pi / 2$, $h = 0.34$ nm, k_p=0.01Pa.m

It is observed that with increase of outer radius (r_o / h, h is constant), the maximum deflection is increased. In the beginning, the maximum deflection goes up with a rising slope during the increase of outer radius. However, in continue of increase of r_o / h, the variation slope tends to go down to a constant value. Also, the variations for SSSS boundary conditions are more than CCCC.

6. CONCLUSION

In this paper, the nonlinear bending analysis for sector nano graphene plate resting on an elastic matrix with two parameters. Winkler and Pasternak is investigated using nonlocal elasticity theory. The first order shear deformation theory (FSDT) is applied considering the nonlinear Von Karman strain field. The equilibrium equations are derived using the principals of minimum potential energy method. The most important conclusions can be classified as follows:

- The presented semi analytical polynomial method (SAPM) is highly accurate, considerably simpler in formulations and coding by computer programs and its rate of processing time is about fifty percent more than DQM.
- Increase in nonlocal parameter has much more effects on nonlinear analysis in comparison with the linear analysis.
- The small scale effects on deflection reduce when the plate is rested on an elastic foundation.

- The differences between two nonlocal and local elasticity theories are increased in presence of elastic foundation. However, the effects of elastic foundation reduce due to the increase of the transverse loading.
- The differences between nonlocal and local elasticity theories are raised along the increase of sector angle and the two theories go on each other with growing of thickness.

Some of possible directions for future research can be summarized as follows:

- Nonlinear bending analysis of multilayers sector graphene sheets.
- Using higher order shear deformation theory (HSDT) instead of FSDT in order to obtain the more precise results.
- Buckling of sector graphene sheet and deriving the buckling of circular/annular and rectangular graphene plates from sectorial shape.

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Nonlinear Bending Analysis of Sector Graphene Sheet Embedded in Elastic Matrix Based on Nonlocal Continuum Mechanics

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چکیده

در این تحقیق خمش غیرخطی صفحات قطاعی گرافن بر پایه الاستیک توسط تئوری الاستیسیته غیرموضعی ارینگن مورد بررسی قرار می‌گیرد. برای این منظور معادلات تعادل حاکم بر ورق قطاعی گرافن با در نظر گرفتن روابط غیر موضعی تنش و تئوری مرتبه اول برشی و کرنش های غیر خطی فون-کارمن بدست آمده است. برای حل دستگاه معادلات دیفرانسیل غیر خطی، از روش عددی مربعات دیفرانسیلی(DQM) و یک روش نیمه تحلیلی بر پایه چندجمله‌ای (SAPM) استفاده شده است. روش حل نیوتن-رافسون برای حل دستگاه جبری غیرخطی به دست آمده بکار گرفته شده و نتایج دو روش با یکدیگر مقایسه و ملاحظه شد نتایج بدست آمده از روش SAPM با فرمول‌بندی ساده‌تر دقت بسیار خوبی در مقایسه با روش DQM دارد. شرایط تکیه‌گاهی مختلف از جمله گیردار، مفصلی و آزاد در نظر گرفته شده است. جهت اعتبار سنجی نتایج بدست آمده با داده‌های تحقیقات دیگر مقایسه شده است. در بررسی نتایج، اثر تغییرات ضریب غیرموضعی بر نتایج بر حسب تغییرات ضخامت، شرایط مرزی، سختی پایه الاستیک و بار اعمالی، تاثیر تحلیل خطی و غیرخطی تئوری مرتبه اول برشی، تعداد گره‌ها در دامنه هندسه قطاع بر دقت جواب‌ها و نیز تاثیر زاویه قطاع و ضریب غیرموضعی بر خیز و نسبت خیز غیرموضعی به موضعی در شرایط مختلف مورد بررسی قرار گرفته است

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