



## A Reliable Multi-objective $p$ -hub Covering Location Problem Considering of Hubs Capabilities

M. Bashiri, M. Rezanezhad\*

Department of Industrial Engineering, Shahed University, Tehran, Iran

### PAPER INFO

#### Paper history:

Received 01 November 2014

Received in revised form 18 December 2014

Accepted 29 January 2015

#### Keywords:

$P$ -hub Covering

Hub Capability

Reliability

Multi Objective

### ABSTRACT

In the facility location problem usually reducing total transferring cost and time are common objectives. In the  $p$ -hub covering problem it is attempted to locate hubs and allocate customers to established hubs while allocated nodes to hubs are inside of related hubs covering radius. In this paper, we attempt to consider capability of established hubs to achieve a more reliable network. Also, the proposed model try to construct a network with more covering reliability by determining operating covering radius inside of nominal radius. Then, a sensitivity analysis is performed to analyze effect of parameters in the model. The proposed multi objective model is solved by  $\epsilon$ -constraint algorithm for small size instances. For large scale instances a non-dominated sorting genetic algorithm (NSGA-II) is presented to obtain Pareto solutions and its performance is compared with results of  $\epsilon$ -constraint algorithm. The model and solution algorithm were analyzed by more numerical examples such as Turkish network dataset. The sensitivity analysis confirms that the network extracted by the proposed model is more efficient than classic networks.

doi: 10.5829/idosi.ije.2015.28.05b.10

## 1. INTRODUCTION

In the hub covering problems, it is tried to satisfy demand points with less cost. So some points should be located as hub facilities to transfer goods with a discount rate. Each located hub can cover demands inside its covering radius. Between established hubs, the amount of cargo is greater than usual transportation, so suitable vehicles for carrying the cargos is essential to decrease transportation time between hubs. Utilizing mentioned vehicles can lead to lower transportation cost but will increase fixed costs. Some services in the hub nodes such as switching, transshipment and sorting needs special technology and facility [1]. So, in the real world, during locating hubs, other characteristics of hub facilities should be determined according the hubs operations, number of nodes, zone condition and etc.

In this study, the model of mohammadi et al. [2] is extended and three characteristics of cargos preparation in hubs are considered which are named sorting time, sorting cost and reliability of hubs. In the real world, mentioned characteristics are usually desirable.

As one of applications of the proposed model, the networks can be mentioned that some finishing operations are implemented inside of hub nodes. For example in international trading, customs of each country can be considered as a hub node while some operations such as quality, document and legal controls should be done in such nodes. In mentioned networks, reliability is a fundamental aspect and should be considered. Kim et al. [3] considered the reliability computation formula in the hub location problem. In this paper reliability is considered during network design while the reliability has two dimensions in the proposed model:

1. Reliability of established hub nodes and their facility.
2. Covering reliability.

\*Corresponding Author's Email: [Rezanezhad\\_mr@gmail.com](mailto:Rezanezhad_mr@gmail.com) (M. rezanezhad)

Therefore novelty of this paper can be summarized as follows:

- Four objective functions are considered to cover mentioned characteristics, so a reliable multi objective model for hub covering location problem is presented.
- Technological hub facilities are considered while in previous studies all hub facilities are assumed to be the same.
- Optimum type of facilities are determined according to their capabilities.
- The proposed model is solved by NSGA-II algorithm and Pareto solutions are identified.

The remainder of this paper is organized as follows. Section 2 contains a review about the hub covering problem. In Section 3 the extended mathematical model is presented. In the section 4, NSGAI algorithm is defined to solve the problem. In the section 5 numerical examples and sensitivity analysis are presented. Finally, in the last section concluding remarks are defined.

## 2. LITERATURE REVIEW

O'Kelly [4] proposed the first mathematical formulation and solution method for the hub location problem. Campbell [5] compared rectilinear distance transportation costs for hub networks with two types of demands to investigate how well an idealized model predicts the cost for the real-world demand. Campbell [6] presented an integer programming formulation for four types of discrete hub location problems: the  $p$ -hub median problem, the uncapacitated hub location problem,  $p$ -center and covering problems.

Many papers have been published about application of hub location problems. Some applications in the hub location problem are airlines and airports industries, emergency services, supply chain management–logistic, transportation systems and so on. Toh [7], Shaw [8] and Alder et al. [9] discussed application of hub location network for airlines and aviation industry. Hakimi [10] and Berman et al. [11] studied application of hub location problem in emergency services and similarly Wang et al. [12] and Ishfaq et al. [13] studied it in the supply chain management–logistic. Don et al. [14], Cunha et al. [15], Takno et al. [16], Gelareh et al. [17] studied the hub problem in the transportation system.

Many researches tried to extend hub location problems. Campbell [5] and Campbell [6] studied the hub arc location problem, Baharet al. [18] studied the latest arrival hub location problem and Kim et al. [3], An et al. [19] and Azizi et al. [20] studied reliable hub location problem. They are some of recently extensions to the traditional hub location problem which are described more in the following. Bahar et al. [18] considered unloading, loading and sorting operators for

latest arrival hub location problem with stopovers and proposed nonlinear and linear integer programming. Kim et al. [3] presented a reliable  $p$ -hub location problem, which focuses on maximizing network performance in terms of reliability by locating hubs for delivering flows among city nodes. Karimi et al. [21] proposed a capacitated hub location problem by hierarchical approach. Yahyaei et al. [22] presented a multi-criteria logistic hub location problem and utilized a Meta-model to reduce uncertainty effect. Mohammadi et al. [2] present a stochastic multi-objective multi-mode transportation model for hub covering location problem under uncertainty.

Also the review of Zanjirani Farahani et al. [23] can be proposed as a comprehensive review of the hub location problems. To the best of our knowledge, there is no previous study to determine hub capability in the hub covering location problem while its consideration with other decisions simultaneously can improve network efficiency.

Also, about considering reliability in the hub network, An et al. [19] proposed a set of two-stage robust optimization models to design reliable  $p$ -median facility location networks subjected to disruptions. Azizi et al. [20] proposed a model for hub location problem that build reliable network by explicitly considering possible disruption at hub facilities in the design stage. They used a backup facility in hubs when they stops normal operating. The entire demand in that hub is initially served by the backup facility. Eghbalizarch et al. [24] studied carrying different cargo type in the hub network under various levels of services such as price, reliability and transit time and solve their model by different evolution algorithms. Hamidi et al. [25] proposed a new concept of hub location problem that called preventive reliable hub location problem. The another added three objective functions in the usual model to make model more reliable and save all common properties, called fake hub, fake allocation and fake follow. Kim et al. [3], Davari et al. [26], Zarandi et al. [27] are some examples of studies that considered path reliability in the hub location problem. Another studies about reliability prepared in Table 1. In the previous studies, covering reliability was never considered. Because of importance of covering reliability and path reliability we aim to consider mentioned dimensions of reliability in the proposed model and propose a more reliable network by trustworthy operating radius.

Due to complexity of hub location problems, many researchers applied meta-heuristics to find a near optimal solution. Tavakkoli-Moghadam et al. [28] presented a novel multi-objective mathematical model for capacitated single allocation hub location problem and considered a vehicle capacity for their model. The authors minimized total cost and maximum travel time in their model. They used multi-objective imperialist

competitive algorithm for large instances. Ghodratnama et al. [29] compared three Meta-heuristics to solve  $p$ -hub location problem. Parvaresh et al. [30] formulated multiple allocation  $p$ -hub median problem by a bi-objective model and two objective functions. They proposed a multi objective simulated annealing algorithm to solve their model. Geramianfar et al. [31] studied a multi-objective hub covering location problem. The authors considered two objective functions to minimize total cost and total waiting time in hubs and utilized NSGA-II algorithm to obtain Pareto solutions.

Eghbali et al. [32] proposed a Multi-objective reliable hub covering location. The author considered two objective functions to minimize total cost and maximize the customer convenience. They utilized NSGA-II algorithm to obtain Pareto solutions. In this paper, we propose NSGA-II algorithm to obtain Pareto solutions of the proposed model.

Finally, a summary of previous related works are compared with our proposed model which is presented in Table 1.

**TABLE 1.** Summary of previous studies.

Authors	Year	Model structure		Objective function				Transportation					
		Hub	Covering	Cost	Travel time	Path reliability	Covering reliability	Maximal cover	Single vehicle	Multiple vehicles	Considering reliability	Hubs capability	NSGA-II
Baharet et al. [18]	2001	*			*							*	
Alder et al. [9]	2005	*		*									
Berman et al. [11]	2007	*		*									
Cunha et al. [15]	2007	*		*							*		
Tan et al. [33]	2007	*	*	*									
Takno et al. [16]	2009	*		*					*				
Kim et al. [3]	2009	*				*			*		*		
Davari et al. [26]	2010	*				*					*		
Gelareh et al. [17]	2011	*		*					*				
Zarandi et al. [27]	2011	*				*					*		
Ishfaq et al. [13]	2012	*		*					*				
mohammadi et al. [2]	2013	*	*	*	*					*			
Ghodratnama et al.[29]	2013	*		*						*			
Tavakkoli-Moghadam et al. [28]	2013	*		*	*				*				
Eghbalizarch et al. [24]	2013	*	*	*									
Yahyaei et al. [22]	2013	*		*									
Geramianfar et al. [31]	2013	*	*	*									*
Setak et al. [34]	2013	*		*									
Eghbali et al. [32]	2014	*	*	*							*		*
An et al. [19]	2014	*		*		*					*		
Azizi et al. [20]	2014	*		*		*					*		
Parvaresh et al. [30]	2014	*		*		*			*		*		
Ebrahimi et al. [35]	2014		*			*		*					*
Karimi et al. [21]	2014	*	*	*					*				
Hamidi et al. [25]	2014	*		*					*				
Tavakkoli-Moghaddam et al. [36]	2014	*		*									*
<b>This research</b>	-	*	*	*	*	*	*	*		*		*	*

The review of previous studies about the hub location problem confirms that some extensions have been developed for the hub location problem according to the real world applications. As a best of our knowledge, there is no direct study of reliable hub location problem considering hub capabilities in previous studies, so in this study we try to formulate the reliable hub location problem when hubs have different capabilities in the network.

### 3. MATHEMATICAL FORMULATION

In this section sets, parameters and variables are introduced.

**Sets:**

- $n = \{1 \dots N\}$  Set of nodes.
- $v1 = \{1 \dots V1\}$  Set of vehicles that can be used for pathway between hubs and non-hub nodes.
- $v2 = \{1 \dots V2\}$  Set of special vehicles that can be used for pathway between two hubs.
- $s = \{1 \dots S\}$  Set of sorting facilities that can be assigned to opened hubs.

**Parameters:**

- $P$  Number of hubs that should be established at the network.
- $R_{s,k}$  Reliability rate of  $k$ th hub by using  $s$ th facility in the hub.
- $d_{i,k}$  Distance between nodes  $i$  and  $k$ .
- $C1_{v1,i,k}$  Unit transportation cost between nodes  $i$  and  $k$  using vehicle  $v1$ .
- $C2_{v2,k,l}$  Unit transportation cost between hubs  $k$  and  $l$  using vehicle  $v2$ .
- $T_{v1,i,k}$  Transportation time between nodes  $i$  and  $k$  using vehicle  $v1$ .
- $T_{v2,k,l}$  Transportation time between nodes  $k$  and  $l$  using vehicle  $v2$ .
- $TS_{s,k}$  Sorting time of facility  $s$  at hub  $k$ .
- $F1_{v1,i,k}$  Risk factor at pathway  $i$  to  $k$  using vehicle  $v1$ .
- $F2_{v2,k,l}$  Risk factor at pathway  $k$  to  $l$  using vehicle  $v2$ .
- $W_{i,j}$  Flow to be sent from node  $i$  to node  $j$ .
- $FC_{v1}$  Fixed cost of using vehicle  $v1$ .
- $FC_{v2}$  Fixed cost of using vehicle  $v2$ .
- $FS_{s,k}$  Sorting cost of facility  $s$  at hub  $k$ .
- $F_{i,k}$  Fixed cost of constructing a pathway between nodes  $i$  and  $k$ .
- $F_k$  Fixed cost of opening a hub at node  $k$ .
- $\Gamma_k$  Capacity of hub  $k$ .

- $r_k$  Covering nominal radius of hub  $k$ .
- $C_{v1}$  Capacity of vehicle  $v1$ .
- $C_{v2}$  Capacity of vehicle  $v2$ .
- $NV_{\max v1}$  Maximum available number of type  $v1$  vehicles.
- $NV_{\max v2}$  Maximum available number of type  $v2$  vehicles.
- $O_i$  Total flow originating from node  $i$ .
- $D_i$  Total flow destined to node  $i$ .
- $\beta$  Maximum travel time between pair of nodes.

**Variables:**

- $A_{v1,i,k}$  Number of used vehicles type  $v1$  in pathway between node  $i$  to hub  $k$ .
- $B_{v2,k,l}$  Number of used vehicles type  $v2$  in pathway between hubs  $k$  and  $l$ .
- $Y_{k,l}$  Total flow from node  $i$  that is routed via hubs  $k$  and  $l$ .
- $X_{i,k}$  Is 1 if node  $i$  is allocated to hub  $k$ ; otherwise, 0.  $X_{kk} = 1$  shows that a hub is established at node  $k$ .
- $Z_{k,l}$  Is 1 if created a direct path between hub  $k$  and  $l$ .
- $X_{k,s}$  Is 1 if sorting facility  $s$  is allocated to hub  $k$ ; otherwise, it is 0.
- $Y1_{i,k,l,j}$  Is 1 if path  $i \rightarrow k \rightarrow l \rightarrow j$  created.
- $Y2_{i,k,l,j,sk,sl}$  Is 1 if path  $i \rightarrow k \rightarrow l \rightarrow j$  created and sorting facility  $sk$  and  $sl$  assigned to hub  $k$  and  $l$ .
- $Y3_{i,k,s}$  Is 1 if node  $i$  allocated to hub  $k$  and sorting facility  $s$  assigned to hub  $k$ .
- $a \in [0, 1]$  Time discount factor between two hubs.
- $M$  A large value.

The proposed model is defined as following.

Objective functions:

$$\begin{aligned} \text{Min} \sum_{v1,i,k} A_{v1,i,k} (2C1_{v1,i,k} d_{i,k} + FC1_{v1}) X_{i,k} \\ + \sum_{v2,k,l} B_{v2,k,l} (C2_{v2,k,l} d_{k,l} + FC2_{v2}) Z_{k,l} \\ + \sum_{i,k} F_{i,k} X_{i,k} + \sum_{k,l} F_{k,l} Z_{k,l} \end{aligned} \tag{i}$$

$$\begin{aligned} + \sum_{k=1}^n F_k X_{k,k} + \sum_{k,s} FS_{s,k} X_{k,s} \\ \text{Min } \beta \end{aligned} \tag{ii}$$

$$\text{Max} \sum_{i,k,l,j,sk,sl, k \neq l} Y2_{i,k,l,j,sk,sl} R_{sk,k} R_{sl,l} \tag{iii}$$

$$Min \text{ Max } \{ (d_{i,k} Y3_{i,k,s}) / R_{s,k} \} \forall i, k, s \tag{iv}$$

Location and allocation constraints:

$$\sum_{k=1}^n X_{i,k} = 1 \forall i \tag{1}$$

$$X_{i,k} \leq X_{k,k} \forall i, k \tag{2}$$

$$\sum_{k=1}^n X_{k,k} = P \tag{3}$$

$$\sum_s Xs_{k,s} = X_{k,k} \forall k \tag{4}$$

$$d_{i,k} X_{i,k} \leq r_k \forall i, k \tag{5}$$

$$X_{i,k} + Xs_{k,s} - 1 \leq Y3_{i,k,s} \forall i, k, s \tag{6}$$

$$Y3_{i,k,s} \leq X_{i,k} \forall i, k, s \tag{7}$$

$$Y3_{i,k,s} \leq Xs_{k,s} \forall i, k, s \tag{8}$$

Capacity constraints:

$$Y_{k,l}^i = \sum_{j=1}^n Y1_{i,k,l,j} w_{i,j} \forall i, k, l \tag{9}$$

$$Y_{k,l}^i \leq M X_{k,k} \forall i, k, l \tag{10}$$

$$Y_{k,l}^j \leq M X_{l,l} \forall i, k, l \tag{11}$$

$$Y_{k,l}^i \geq 0 \forall i, k, l \tag{12}$$

$$\sum_{i=1}^n (O_i + D_i) X_{i,k} \leq \Gamma_k \forall k \tag{13}$$

$$\sum_{v1=1}^{v1} C_{v1} A_{v1,i,k} \geq \max(O_i, D_i) X_{i,k} \forall i, k, i \neq k \tag{14}$$

$$\sum_{v2=1}^{v2} C_{v2} B_{v2,k,l} \geq \sum_{j=1}^n Y_{k,l}^j \forall k, l, k \neq l \tag{15}$$

$$A_{v1,i,k} \leq NV_{\max,v1} X_{i,k} \forall i, k, v1 \tag{16}$$

$$B_{v2,k,l} \leq NV_{\max,v2} Z_{k,l} \forall k, l, v2 \tag{17}$$

Travel time constraints:

$$(T_{i,k} + \alpha T_{k,l} + T_{l,j} + TS_{s,k} + TS_{s,l}) Y2_{i,k,l,j,sk,sl} \leq \beta \tag{18}$$

$$\forall i, k, l, j, sk, sl$$

$$T_{i,k,v1} (1 + r_{i,k,v1}) X_{i,k} \leq T_{i,k} \forall i, k, v1 \tag{19}$$

$$T_{i,k,v2} (1 + r_{i,k,v2}) X_{i,k} \leq T_{i,k} \forall i, k, v2 \tag{20}$$

Linearization constraints:

$$X_{i,k} + X_{j,l} - 1 \leq Y1_{i,k,l,j} \forall i, k, l, j \tag{21}$$

$$Y1_{i,k,l,j} \leq X_{i,k} \forall i, k, l, j \tag{22}$$

$$Y1_{i,k,l,j} \leq X_{j,l} \forall i, k, l, j \tag{23}$$

$$X_{k,k} + X_{l,l} - 1 \leq Z_{k,l} \forall k, l \tag{24}$$

$$Z_{k,l} \leq X_{k,k} \forall k, l \tag{25}$$

$$Z_{k,l} \leq X_{l,l} \forall k, l \tag{26}$$

$$Y1_{i,k,l,j} + Xs_{k,sk} + Xs_{l,sl} - 2 \leq Y2_{i,k,l,j,sk,sl} \tag{27}$$

$$\forall i, k, l, j, sk, sl$$

$$Y2_{i,k,l,j,sk,sl} \leq Y1_{i,k,l,j} \forall i, k, l, j, sk, sl \tag{28}$$

$$Y2_{i,k,l,j,sk,sl} \leq Xs_{k,sk} \forall i, k, l, j, sk, sl \tag{29}$$

$$Y2_{i,k,l,j,sk,sl} \leq Xs_{l,sl} \forall i, k, l, j, sk, sl \tag{30}$$

Binary and integer constraints:

$$X_{i,k}, X_{k,sk}, Z_{k,l}, Y1_{i,k,l,j}, Y2_{i,k,l,j,sk,sl}, Y3_{i,k,s} \in \{0,1\} \tag{31}$$

$$\forall i, k, l, j, sk, sl$$

$$A_{v1,i,k}, B_{v2,k,l} \forall i, k, l, v1, v2 \text{ are integer variables} \tag{32}$$

The objective function of the proposed model consists of four parts. The part (i), minimizes total current investment costs including total transportation cost between all non-hub nodes to the hub nodes, total transportation cost between all hub links, total construction costs of pathways, total hubs establishment costs and finally total hiring cost of sorting facilities in hubs. The part (ii) of objective function, minimizes maximum traveling time between pair of nodes. The part (iii) of objective function maximizes total reliability of available paths. The part (iv) of objective function forces to allocate near nodes to more reliable hubs. Therefore, nodes will be covered by hubs with operating covering radius smaller than nominal covering radius. The mentioned operating covering radius try to appointment hub network in abnormal conditions. By utilizing objective (iv), a more reliable hub with acceptable operating covering radius is constructed to serve the nodes in most of conditions.

Constraints (1) and (31) together ensures single allocation strategy in the network. Constraint (2) assures that nodes can be allocated to only open hubs. Constraint

(3) assures that  $P$  hubs should be established in the network. Constraints (4) and (31) together enforce that a sorting facility have to be assigned for each established hubs. Constraint (5) makes sure that node  $i$  only can be allocated to hub  $k$  if distance between  $i$  and  $k$  is less than the nominal covering radius. Constraints (6)-(8) make the product  $Y3_{i,k,s} = X_{i,k} \times X_{s,k,s}$  to a linear form. Constraint (9) computes amount of flow from node  $i$  that is routed via hubs  $k$  and  $l$ . Constraints (10) and (11) together ensure that the value of  $Y_{k,l}^i$  can be more than zero if nodes  $k$  and  $l$  are the valid hubs. Constraint (12) ensures that  $Y_{k,l}^i$  is a positive value. Constraint (13) guarantees to meet the hub capacity. Constraints (14), (15) and (32) ensures that assigned number of vehicles can satisfy maximum flow in each node. Constraint (16) ensures that the number of assigned vehicle  $v1$  to the path between node  $i$  to hub  $k$  is at most  $NV_{max_{v1}}$  if and only if the link  $i$  to  $k$  is constructed. Constraint (17) enforces that number of vehicle  $v2$  between hubs  $l$  and  $k$  should be less than  $NV_{max_{v2}}$  if the hubs are opened. Constraint (18) determines largest time between pare of nodes. Constraints (19) and (20) compute maximum duration time between nodes  $i$  and  $k$  by different vehicles and considering related risk. Constraints (21)-(23) make product  $Y1_{i,k,l,j} = X_{i,k} \times X_{j,l}$  to be in a linear form. Constraints (24)-(26) make product  $Z_{k,j} = X_{k,k} \times X_{l,j}$  to a linear form. Constraints (27)-(30) make product  $Y2_{i,k,l,j,sk,sl} = Y1_{i,k,l,j} \times X_{s,k,sk} \times X_{s,l,sl}$  to a linear form.

#### 4. SOLUTION ALGORITHM

We utilize non-dominated sorting genetic algorithm II (NSGA-II) to obtain the Pareto frontier and the results are compared with the results obtained by  $\epsilon$ -constraint method. In section 4.1, the proposed solution representation is defined, in section 4.2, the proposed method of creating the neighbor solution is defined and in section 4.3, the proposed algorithm is presented.

**4. 1. Solution Representation** We use a feasible solution that consists of two parts of location-allocation representation and transportation mode representation. To generate a random initial solution following steps should be done (suppose  $n=10, s=3$ ):

Step 1: Let us define location-allocation part representation by using a  $(3 \times n)$  matrix,  $n$  is the number of nodes. The first row shows the index of nodes  $(1 \dots n)$ . The second row is filled by uniformly generated random numbers between  $[0,100]$  as depicted in Figure 1.

Step 2: Node by number  $n$ , should be selected as a hub in the initial solution. Other  $P-1$  hubs should be selected according to higher corresponding random values, then left nodes are allocated to nearest right

hubs in the solution representation. For example in Figure 2 it is shown that nodes 10, 7 and 4 have been selected as hub nodes while nodes 7 and 4 have larger corresponding random values. Also, the left nodes have been allocated to the nearest hub in the right side. For example nodes 5 and 6 is allocated to the hub 7.

Step 3: After hub location stage and allocating nodes to them, facility type in the established hubs should be determined. This work is done randomly like above. Finally, location-allocation part of solution representation is generated as follows. For example assume that there are four types of facilities in the hub. As illustrated in Figure 3 facilities types 2, 4 and 1 are selected to be activated in the opened hubs. As an application of the proposed model, suppose that we are interested in configuring of a trade hub network between some cities of different countries under an agreement. For example, suppose that 10 cities have been selected in this agreement which have been determined in Table 2. The initial feasible solution of Figure 3 for the proposed model is represented in Figure 4. When a pathway was created, suitable vehicles should be assigned. We have to determine number of vehicles and their types. Number of vehicles and their types between nodes and hubs are determined by two matrices, while one matrix presents number and type of vehicles between the hubs. In the trade hub example, it is assumed that there are two types of vehicles between node and hubs and two different vehicles between the hubs ( $v1=v2=2$ ). Figure 5 represents number and type of vehicles in each part of trade hub network example. By this solution, we need one vehicle of type 1 for pathways  $1 \rightarrow 4, 3 \rightarrow 4$  and vice versa. Also, we need two vehicle of type 1 for pathways  $5 \rightarrow 7, 7 \rightarrow 5$  and three vehicle type 1 for pathways  $6 \rightarrow 7, 7 \rightarrow 6$ . In addition, we need one vehicle of type 2 for pathways  $8 \rightarrow 10, 9 \rightarrow 10$  and vice versa and one vehicle type 2 for pathways  $2 \rightarrow 4, 4 \rightarrow 2$ . Similarly Figure 6 represents number and type of vehicles between hubs for the example. Left number shows the type and right number represents number of vehicles between corresponding hubs. When a pathway was created, suitable vehicles should be assigned. We have to determine number of vehicles and their types. Number of vehicles and their types between nodes and hubs are determined by two matrices. While one matrix presents number and type of vehicles between the hubs. In the trade hub example, it is assumed that there are two types of vehicles between node and hubs and two different vehicles between the hubs ( $v1=v2=2$ ). Figure 5 represents number and type of vehicles in each part of trade hub network example. By this solution, we need one vehicle of type 1 for pathways  $1 \rightarrow 4, 3 \rightarrow 4$  and vice versa. Also, we need two vehicle of type 1 for pathways  $5 \rightarrow 7, 7 \rightarrow 5$  and three vehicle type 1 for pathways  $6 \rightarrow 7, 7 \rightarrow 6$ . In addition, we

need one vehicle of type 2 for pathways 8→10, 9→10 and vice versa and one vehicle type 2 for pathways 2→4, 4→2.

Similarly, Figure 6 represents number and type of vehicles between hubs for the example. Left number shows the type and right number represents number of vehicles between corresponding hubs.

**4. 2. Generating Neighbor Solutions** Two common operators called mutation and crossover are used in the genetic algorithm to generate neighbor solution. They are illustrated more for the proposed model in the following sections.

**4. 2. 1. Mutation Operator** In the mutation operator of genetic algorithm, one character of solution is changed randomly. The mutation operator in our problem has been designed to change one of three main characteristics in a solution which are selected hub nodes, allocation structure and hub capability. The mutation is performed in a solution by swapping allocation of two nodes, swapping opened hubs, opening a new hub with closing one of opened hubs, swapping capabilities of two hubs and finally adding a new capability to a hub with removing a capability from that hub. Each of mentioned states occur with equal probabilities. For example, consider a solution represented in Figure 3, we implement all of four states of mutation operator in mentioned solution.

Swapping allocation of nodes to hubs:

Select two random nodes and swap their allocation to hubs. For example, nodes 1 and 9 have been selected in the previous solution in Figure 3 and after swapping their allocation, the new solution is depicted in Figure 7.

Swapping hubs:

Select two random hubs of a solution and swap them. For example in Figure 3, suppose that hubs 10 and 4 have been chosen randomly, the final solution is depicted in Figure 8.

Insertion of a hub:

One of opened hubs is chosen randomly to be closed while the next node is opened as a new hub. For example in Figure 3, suppose that hub 4 is selected to be closed and node 2 is selected randomly as a new opened hub. Figure 9 represents the result of insertion of a hub. Swapping hub capabilities:

It is look like swapping hubs, so firstly two hubs are selected randomly and their capabilities are swapped. For example suppose that hubs 7 and 10 are selected. Figure 10 represents the final result of swapping the hubs capabilities in previous example:

Insertion of a hub capability:

One of opened hub is chosen randomly to be remove its capability while the next capability added as a new capability. For example in Figure 3, suppose that hub 7 is selected to be remove its capability and capability 3 is

selected randomly as a new capability. Figure 11 represent the result of insertion of a capability.

**TABLE 2.** Selected cities for the trade hub network in the example

Node number	City name	Node number	City name
1	Ankara	6	Yazd
2	Shaantil	7	Esfahan
3	Baghdad	8	Ashghabad
4	Tabriz	9	Kabol
5	Shiraz	10	Mashhad

1	2	3	4	5	6	7	8	9	10
31	39	56	86	49	80	99	45	12	15

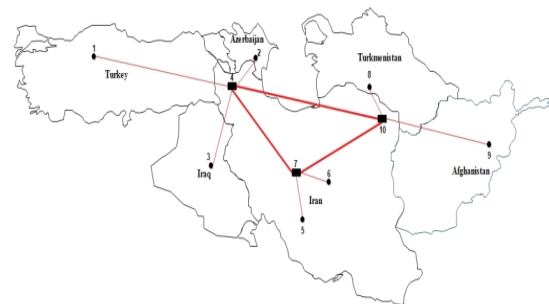
**Figure 1.** Generating random numbers to create feasible first part of a random solution

1	2	3	4	5	6	7	8	9	10
4	4	4	4	7	7	7	10	10	10

**Figure 2.** Locating hubs and allocation of nodes to opened hubs in the initial first part of a random solution

1	2	3	4	5	6	7	8	9	10
4	4	4	4	7	7	7	10	10	10
2	2	2	2	4	4	4	1	1	1

**Figure 3.** A complete random generated initial solution including location, allocation and type of facility in the hubs



**Figure 4.** Schematic view of a solution representation in an example of a trade hub network

0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	2	0	0	0	0	0	0
0	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
1	1	1	0	0	0	0	0	0	0	1	2	1	0	0	0	0	0	0	0
0	0	0	0	0	2	0	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	0	0	3	0	0	0	0	0	0	0	0	0	1	0	0	0
0	0	0	0	2	3	0	0	0	0	0	0	0	0	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	2
0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	2
0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	2	2	0

**Figure 5.** Number and type of vehicles (from set V1) for pathway between hubs and nodes  
a - Number of vehicles      b - Type of vehicles

Index of hubs	4	7	10
4	0	1, 4	1, 5
7	1, 4	0	2, 7
10	1, 5	2, 7	0

Figure 6. Number and type of vehicles (from set V2) for pathway between hubs

9	2	3	4	5	6	7	8	1	10
4	4	4	4	7	7	7	10	10	10
2	2	2	2	4	4	4	1	1	1

Figure 7. Swapping allocation for the example solution

1	2	3	4	5	6	7	8	9	10
10	10	10	4	7	7	7	4	4	10
1	1	1	2	4	4	4	2	2	1

Figure 8. Swapping hubs for the example solution

1	2	3	4	5	6	7	8	9	10
2	2	2	2	7	7	7	10	10	10
2	2	2	2	4	4	4	1	1	1

Figure 9. Result of insertion of a hub for the example solution

1	2	3	4	5	6	7	8	9	10
4	4	4	4	7	7	7	10	10	10
2	2	2	2	1	1	1	4	4	4

Figure 10. Swapping hubs capability for the example solution

1	2	3	4	5	6	7	8	9	10
4	4	4	4	7	7	7	10	10	10
2	2	2	2	3	3	3	1	1	1

Figure 11. Result of insertion of a capability for example solution

**4. 2. 2. Crossover Operator**

By the crossover, two new solutions are generated according to their corresponding parents' features. During the crossover operation, first row of parent 1 is merged by 2<sup>nd</sup> and 3<sup>rd</sup> row of parent 2 which has been shown in Figure 12. Then, repairing of the solution is done to achieve feasible solutions. By looking at child 1, it is clear that it is an infeasible solution, because node 2 is a hub node while it has not been allocated to itself. The repaired feasible solution for both child 1 and 2 are depicted in Figure 13.

**4. 3. Non-dominated Sorting Genetic Algorithm (NSGA-II)**

Non-dominate sorting genetic algorithm II (NSGA-II) was introduced by Deb et al. [37] in 2002.

The operators of genetic algorithm are used in NSGA-II as well, but in the NSGA-II more than one objective functions are available and solutions will be ranked in NSGA-II. NSGA-II is a subset of smart evolutionary methods, therefore after generating solutions in each iteration, all objective functions have to be computed. In the genetic algorithm, the value of the objective function is an important measure, but in the multi objective problems rank of a solution in Pareto sets is the decision measure. Sorting algorithm is utilized in the NSGA-II for ranking the solutions. Sorting algorithm compare all pair of solutions and rank them to the number of dominance. Solutions that have rank 1 are the Pareto frontier. We prepare a pseudo-code of the proposed NSGA-II algorithm for the reliable p-hub covering problem with hub capabilities. The proposed NSGA-II algorithm parameters whose values need to be set prior the algorithm execution are as follows:

- N – Number of generations.
  - Npop– Number of populations.
  - CN -Number of crossover operations.
  - MN - Number of mutation operations.
  - Dn(i) – Number of domination for i<sup>th</sup> solution.
  - Ds(i) – Set of solution that dominate solution i.
- The pseudo-code of proposed NSGA-II algorithm is defined as follow.

- Do:
- Step 0.** Generate initial feasible population and compute their objective function values and let n=1.
  - Step 1.** Sort and rank populations according their objective function values.
  - For (i=1 to Npop)
    - For (j=1 to Npop)
      - If solution i dominates solution j, then Dn(j)=Dn(j)+1 and Ds(j)={Ds(j), i}.
  - Step 2.** Select elite number of parents from the population by elite strategy as a new generation.
  - Step 3.** Do crossover for randomly selected parents from the population.
    - For (cn=1 to CN)
      - a) Select two solutions randomly and do crossover them to bring two new solutions.
      - b) Check the feasibility of new solutions.
      - c) Add each of feasible solution to collection of crossover solutions.
  - Step 4.** Utilize mutation operation for some solutions.
    - For (mn=1 to MN)
      - Select a solution and implement mutation operation.
  - Step 5.** Combine elite and mutated and crossover solutions.
  - Step 6.** Select Npop number of combined solutions and let n=n+1.
  - Step 7.** If n≤N, go to step 1, else stop.
- End.

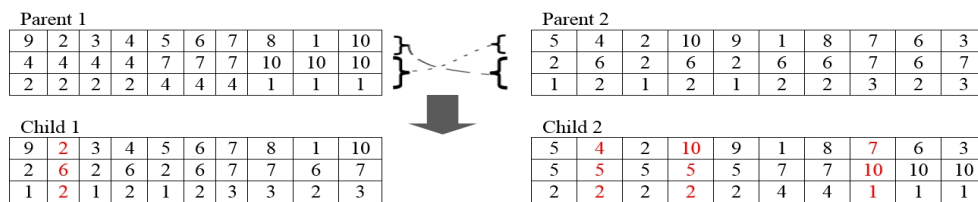


Figure 12. Result of crossover operator for the example solution



Child 1										Child 2									
9	2	3	4	5	6	7	8	1	10	5	4	2	10	9	1	8	7	6	3
2	2	2	6	2	6	7	7	6	7	5	5	5	10	5	7	7	7	10	10
1	1	1	2	1	2	3	3	2	3	2	2	2	1	2	4	4	4	1	1

Figure 13. Repaired solution of Figure 12

5. COMPUTATIONAL RESULTS

As mentioned before, the proposed model is solved by non-dominated sorting genetic algorithm (NSGA-II) and the results are compared with  $\epsilon$ -constraint algorithm solved by Gams software. Because there is no data set about considering proposed model, the numerical examples are generated randomly. Value and distribution of random instances prepared in Table 3. However, we modified Turkish network data set and obtained the Pareto solution. Modification mechanism of Turkish network was prepared in Table 4. First, comparison is about classic model and proposed model to check the effect of considering hub capabilities and reliability parameters. Because the classic model could not consider such parameters, after optimizing the classic model, the parameters are added manually, now classic model and proposed model can be compared. Table 3 shows the cost function and transportation time value of instance 1 that is solved by two models.

Table 4 represent some change in Turkish network for utilize in proposed model, another characters generated according Table 3. Reported results in Table 5 confirms that the proposed model is more efficient than the classic one while the classic model can't consider hub capabilities. The proposed model can consider the capability of facilities and allocate suitable facilities to established hub by optimum value of objective functions. Figure 14 represents difference of the proposed and classic models in choosing of hubs capabilities. Part (a) in Figure 14 represents solution of classic model. As mentioned in part a, classic model could not choose optimum capabilities.

As mentioned in section 3, we should try to allocate nodes to near hubs with more reliability. Figure 15 part a, represents result of classic model and Figure 15 part b represents result of proposed model in compare nominal covering radius and operating covering radius and hubs reliability. As mentioned in Figure 15, utilizing objective function (iv) in the proposed model, ensure nodes to be allocated to near more reliable hubs. Therefore, in abnormal (failure) conditions, the proposed network will survive hub network activities by operating covering radius.

Another sensitive analysis is about parameters such as nominal radius and number of hubs, so we utilize  $r'_k = \delta r_k$  instead of  $r_k$  in the following sensitive analysis. As mentioned in Table 6, by increasing of  $\delta$ , difference of nominal radius and operating radius is

decreased. Therefore, the proposed model could prepared more efficient network in abnormal conditions by operating radius. In Table 7, by decreasing of P, the proposed model prepared more reliable network than the classic one. As hub location problems size are large enough in real world applications, we try to solve the proposed model for large instances. Then, we try to consider the performance of the proposed solution approach.

TABLE 3. Value and distribution of random instances

Parameter	Value and distribution	Parameter	Value and distribution
a	U~[0,1]	$r^2_{v1,k,l}$	U~[0,1]
$r^2_{v2,k,l}$	U~[0,1]	$d_{i,k}$	U~[1,10]
$F_k$	U~[40,50]	$W_{ij}$	U~[2,10]
$\Gamma_k$	U~[1900,2000]	$r_k$	U~[8,10]
$NV_{max,v1}$	U~[50,100]	$NV_{max,v2}$	U~[50,100]
$F_{i,k}$	U~[1,9]	$C^1_{v1,i,k}, v1=1$	U~[10,16]
$C^1_{v1,i,k}, v1=2$	U~[13,20]	$T_{v1,i,k}, v1=1$	U~[0.6,6]
$T_{v1,i,k}, v1=2$	U~[0.5,5]	$FC_{v1}, v1=1$	U~[80,100]
$FC_{v1}, v1=2$	U~[40,50]	$C_{v1}, v1=1$	U~[10,20]
$C_{v1}, v1=2$	U~[40,50]	$C^1_{v2,i,k}, v2=1$	U~[1,7]
$C^1_{v2,i,k}, v2=2$	U~[3,9]	$T_{v2,i,k}, v2=1$	U~[0.6,6]
$T_{v2,i,k}, v2=2$	U~[0.5,5]	$FC_{v2}, v2=1$	U~[400,500]
$FC_{v2}, v2=2$	U~[150,200]	$C_{v2}, v2=1$	U~[30,40]
$C_{v2}, v2=2$	U~[60,70]	$R_{s,k}, s=1$	U~[0.5,0.6]
$R_{s,k}, s=2$	U~[0.7,0.8]	$R_{s,k}, s=3$	U~[0.8,0.9]
$FS_{s,k}, s=1$	U~[10,20]	$FS_{s,k}, s=2$	U~[20,30]
$FS_{s,k}, s=3$	U~[30,40]	$TS_{s,k}, s=1$	U~[15,20]
$TS_{s,k}, s=2$	U~[10,15]	$TS_{s,k}, s=3$	U~[1,9]

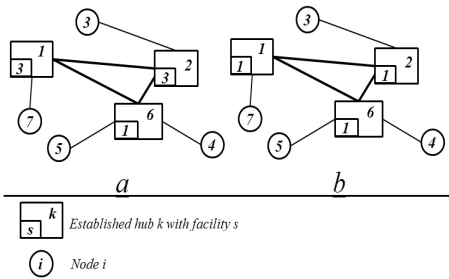
N (Number of nodes) are assumed as: 5, 7, 10, 50, 70

V1 (Number of vehicle type v1) is 2

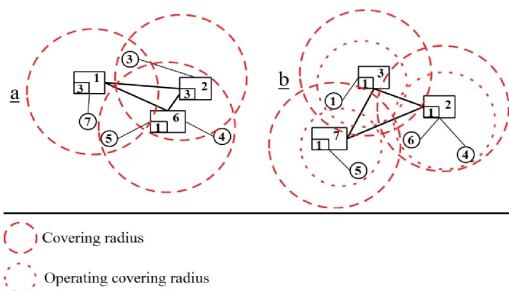
V2 (Number of vehicle type v2) is 2

P (Number of hubs should be established): 3

S (Number of available facilities) : 3



**Figure 14.** Comparing of the proposed model with classic one in minimizing the total cost.



**Figure 15.** Effect of objective function (iv) in the hub covering network

**TABLE 4.** Modification of Turkish network parameters.

$T_{v1,i,k}, v1=1$	$T \times (1+0.1)$	$T_{v1,i,k}, v1=2$	$T \times (1+0.05)$
$T_{v2,i,k}, v1=1$	$T \times (1-0.1)$	$T_{v2,i,k}, v1=2$	$T \times (1-0.05)$
$FC_{v1}, v1=1$	$U \sim [8,10] \times 10^3$	$FC_{v2}, v2=1$	$U \sim [4,5] \times 10^4$
$FC_{v1}, v1=2$	$U \sim [4,5] \times 10^3$	$FC_{v2}, v2=2$	$U \sim [150,200] \times 10^3$
$\Gamma_k$	$U \sim [900,1200]$	$\Gamma_k$	$U \sim [50,60] \times 10^6$

T: Travel time in Turkish network.

**TABLE 5.** Objective function values of instance 1 solved by two models in Gams software

	Value of cost function		Value of time function	
	Classic model	Proposed model	Classic model	Proposed model
Minimizing cost:	1967.64	1908.640	39.6	45.6
Minimizing time:	560254	532054	39.2	13.6

Effect of decreasing number of P in the proposed model is investigated and the results were reported. We use proposed operators in the GA algorithm to solve single objective form of the model then we try to solve

the multi-objective form by NSGA-II algorithm. As mentioned in Table 8, by increasing of the size of instances, the problem will be more complex, therefore efficiency of the solution algorithm will be an important factor. Moreover, results of Table 8 confirms that the performance of genetic algorithm is acceptable while the gap is small enough.

For solving the problem with multiple objectives, some indices are utilized for measuring quality of Pareto solutions. Distribution and convergence of Pareto solutions have to be considered simultaneously. Okabe et al. [38] provided an overview of the various Performance Indices. We utilize generational distance (GD) proposed by Veldhuizen et al. [39] and Veldhuizen et al. [40] to calculate the average distance of NSGA-II's solutions to  $\epsilon$ -constraint' solutions. we also utilize spacing (SP) proposed by Schott [41] to calculate diversity of solutions.

The model in small size can be solved exactly by Gams software, but when number of nodes is increased to 10 nodes the computational time of exact solution will be very long. The proposed algorithm is used for solving different instances and are compared with result of the exact solution. Table 9 shows the computational results and confirms efficient performance of the proposed algorithm for the p-hub covering location problem with hub capabilities. Also, some of Pareto solution for modified Turkish network is described in Table 10.

**TABLE 6.** Sensitivity analysis of increasing the nominal radius

$\delta$	R	NR	OR	NR-OR
0.7	0.8	6.4	6.24	0.16
1	0.86	8	6.56	1.44
1.3	0.82	10.4	6.24	4.16
1.6	0.82	12.8	6.24	6.56
2	0.82	16	6.24	9.76

R: Maximum reliability of available facility in the network.

NR: Maximum of available nominal radius in hub nodes.

OR: Maximum of operating radius in hub nodes.

NR-OR: Difference between nominal and operating radius in the network.

**TABLE 7.** Sensitivity analysis of increasing number of hubs

P	R	NR	OR	NR-OR
3	0.86	8	6.56	1.44
4	0.9	8	4.81	3.91
5	0.86	8	7.87	0.13
6	0.86	8	1.98	6.02

**TABLE 8.** Comparing exact method and genetic algorithm for different instances.

	Gams		GA		%Gap		Gams		GA		%Gap
	T	V	T	V			T	V	T	V	
	Minimizing cost	5*5	8.28	1908.64			8.26	1908.64	0%	8.2	
7*7		70.95	2148.84	29.6	2246.86	0.04	72.36	14.76	48	15.2	0.03
10*10		1000.06	3371	282.82	3707.12	0.01	579.72	12	347	12.12	0.01
50*50		-	-	298.04	77697	-	-	-	11763.48	29.2	-
70*70		-	-	1154.3	160516.6	-	-	-	45638	23.2	-
Maximizing path reliability	5*5	26.33	12.353	0.88	12.353	0%	8.13	4.363	1.45	4.363	0%
	7*7	1000.03	24.53	1.87	24.53	0%	81.91	5.344	2.59	5.344	0%
	10*10	1000.36	21.04	2.52	21.76	0.03	940.05	5	26.4	5.07	0.01
	50*50	-	-	1001.36	1230.91	-	-	-	380.66	11.34	-
	70*70	-	-	4145	2458.87	-	-	-	1542.8	11.23	-

T: Computational Time, V: Objective function value

**TABLE 9.** Computational results and comparing quality of solutions

IN	Performance Indices	Solving Method		Convergence: GD(S,P)
		$\epsilon$ -constraint	NSGA-II	
5*5	Diversity : SP(S)	30.5932	35.3139	
	Computational time (minutes)	3	1.5235	37.4166
	Number of solutions in the Pareto front	3	5	
7*7	Diversity : spacing	7.0510	30.1172	43.4955
	Computational time (minutes)	58	4.9683	
	Number of earned solution	6	7	
10*10	Diversity : spacing	-	70.8178	
	Computational time (minutes)	-	19.72	-
	Number of earned solution	-	5	
50*50	Diversity : spacing	-	3323.26	
	Computational time (minutes)	-	141.68	-
	Number of earned solution	-	6	
70*70	Diversity : spacing	-	3905.7	
	Computational time (minutes)	-	538.6062	-
	Number of earned solution	-	5	
Turkish network: 81*81	Diversity : spacing	-	20428000	
	Computational time (minutes)	-	44324	-
	Number of earned solution	-	17	

**TABLE 10.** Some of Pareto solutions for Turkish network

Solution number	Index of established hub	Utilized facility	Cost function	Time function	Path reliability	Covering reliability
1	Node 11	Type 2	343429686	2103.4	2216.46	1946.57
	Node 50	Type 2				
	Node 81	Type 2				
2	Node 40	Type 1	306067879.8	2376.6	1870.3	3009.4
	Node 50	Type 3				
	Node 81	Type 3				
3	Node 17	Type 3	355565790.0	2557.2	2335.10	2064.29
	Node 38	Type 2				
	Node 81	Type 2				

**6. CONCLUSION**

Considering of previous studies shows that hubs are considered similar while in most of real cases hubs are not similar to each other. For example, in the operating hubs, hub capabilities differ from one hub to another one. In this paper, a model was presented to establish a hub network with considering of capabilities inside of hub facilities. Some numerical examples with different sizes were solved and the comparison confirms that more efficient hub network can be constructed by the

proposed model considering of the path and covering reliabilities. We showed that by the proposed network a more reliable network with less operating covering radius can be achieved. Moreover, a non-dominated sorting genetic algorithm-II was presented equipped by some operators to solve the problem in large instances. The analysis confirms that the presented solution algorithm is efficient in finding the Pareto solutions. Extending the problem with other structures such as hierarchical topology with considering of hub capabilities is suggested as a future study.

## 7. REFERENCES

- Alumar, S., Kara, B. Y., "Network hub location problems: The state of the art", *European Journal of Operational Research*, Vol. 45, No. 1, (2008), 1-21.
- Mohammadi, M., Jolai, F., R. Tavakkoli-Moghaddam, "Solving a new stochastic multi-mode p-hub covering location problem considering risk by a novel multi-objective algorithm", *Applied Mathematical Modelling*, Vol. 37, No. 24, (2013), 10053-10073.
- Kim, H., M. E. O'Kelly, "Reliable p-Hub Location Problems in Telecommunication Networks", *Geographical Analysis*, Vol. 41, No. 3, (2009), 283-306.
- O'Kelly, M. E., "The location of interacting hub facilities", *Transportation Science*, Vol. 20, No.2, (1986), 92-106.
- Campbell, J. F., "Continuous and discrete demand hub location problems", *Transportation Research Part B: Methodological*, Vol. 27, No. 6, (1993), 473-482.
- Campbell, J. F., "Integer programming formulations of discrete hub location problems", *European Journal of Operational Research*, Vol. 72, No. 2, (1994), 387-405.
- TOH, R.S., "The Impact of Hub and Spoke Network Centralization and Route Monopoly on Domestic Airline Profitability", *Transportation Journal*, Vol. 24, No. 4, (1985), 16-27.
- Shaw, S.L., "Hub structures of major US passenger airline", *Journal of Transport Geography*, Vol. 1, No. 1, (1993), 47-58.
- Alder, N., Hashai, N., "Effect of open skies in the Middle East region", *Transportation Research*, Vol. 39, No. 20, (2005), 878-894.
- Hakimi, S. L., "Optimum location of switching centers and the absolute centers and medians of a graph", *Operations Research*, Vol. 12, No. 3, (1964), 450-459.
- Berman, O., Drezner, Z., Wesolowsky, G. O., "The transfer point location problem", *European Journal of Operational Research*, Vol. 179, No. 3, (2007), 978-989.
- Wang, J. J., Cheng, M. C., "From a hub portcity to a global supply chain management center: A case study of Hong Kong", *Journal of Transport Geography*, Vol. 18, No. 1, (2010), 104-115.
- Ishfaq, R., Sox, C. R., "Design of intermodal logistics networks with hub delays", *European Journal of Operational Research*, Vol. 220, No. 3, (2012), 629-641.
- Don, T., Harit, S., English, J. R., Whicker, G., "Hub and spoke networks in truck load trucking: Configuration testing and operational concerns", *Logistics and Transportation*, Vol. 31, (1995), 209-237.
- Cunha, C.B., Silva, M. R., "A genetic algorithm for the problem of configuring a hub-and-spoke network for a LTL trucking company in Brazil", *European Journal of Operational Research*, Vol. 179, No. 3, (2007), 747-758.
- Takano, T., Arai, M., "A genetic algorithm for the hub-and-spoke problem applied to containerized cargo transport", *Journal of Marine Science and Technology*, Vol. 14, No. 2, (2009), 256-274.
- Gelareh, S., Nickel, S., "Hub location problems in transportation networks", *Transportation Research Part E*, Vol. 47, No. 6, (2011), 1092-1111.
- Bahar Y. Kara, B.T., "The Latest Arrival Hub Location Problem", *Management Science*, Vol. 47, No. 10, (2001), 1408-1420.
- An, Yu., Zeng, Bo., Zhang, Yu., Zhao, Long., "Reliable p-median facility location problem: two-stage robust models and algorithms", *Transportation Research Part B: Methodological*, Vol. 64, No. 2, (2014), 54-72.
- Azizi, N., Chauhan, Satyaveer, Salhi, Said, Vidyarthi, Navneet, "The impact of hub failure in hub-and-spoke networks: Mathematical formulations and solution techniques", *Computers & Operations Research*, online published, (2014).
- Karimi, M., Eydi, A.R., Korani, E., "Modeling of the capacitated single allocation hub location problem with a hierarchical approach", *International Journal of Engineering, Transactions A: Basics*, Vol. 27, No. 4, (2014), 573-586.
- Yahyaei, M., Bashiri, M., Garmeyi, Y., "Multi-criteria Logistic Hub Location by Network Segmentation under Criteria Weights Uncertainty (RESEARCH NOTE)", *International Journal of Engineering*, Vol. 27, No. 8, (2013), 1205-1214.
- Zanjirani Farahani, R., Hekmatfar, M., Arabani, A., Nikbakhs, E., "Hub location problems: A review of models, classification, solution techniques, and applications", *Computers & Industrial Engineering*, Vol. 64, No. 4, (2013), 1096-1109.
- EghbaliZarch, M., Abedzadeh, M., Setak, M., "Differential evolution algorithm for multi-commodity and multi-level of service hub covering location problem", *International Journal of Industrial Engineering Computations*, Vol. 4, No. 1, (2013), 127-138.
- Hamidi, M., Gholamian, M., Shahanaghi, K., "Developing prevention reliability in hub location models", *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, Vol. 228, No. 4, (2014), 337-346.
- Davari, S., Zarandi, M.H.F., Turksen, I.B., "The fuzzy reliable hub location problem. in Fuzzy Information Processing Society (NAFIPS)", *Annual Meeting of the North American*, (2010).
- Zarandi, M.H.F., Davari, S., Sisakht, A.H., "Design of a reliable hub-and-spoke network using an interactive fuzzy goal programming. in Fuzzy Systems (FUZZ)", *IEEE International Conference on 2011*, (2011).
- Tavakkoli-Moghaddama, Gholipour-Kanani, Y., R., Shahramifarc, M., "A Multi-objective Imperialist Competitive Algorithm for a Capacitated Single-allocation Hub Location Problem", *International Journal of Engineering*, Vol. 26, No. 6, (2013), 605-620.
- Ghodratnama, A., Tavakkoli-Moghaddam, R., Baboli, A., "Comparing three proposed meta-heuristics to solve a new p-hub location-allocation problem", *International Journal of Engineering*, Vol. 26, No. 9, (2013), 1043-1058.
- Parvaresh, F., Hussein, S. M. M., Golpayegany, S. A. H., "Hub network design problem in the presence of disruptions", *Journal of Intelligent Manufacturing*, Vol. 25, No. 4, (2014), 755-774.
- Geramianfar, R., Pakzad, M., Golhashem, H., Tavakkoli-Moghaddam, R., "A multi-objective hub covering location problem under congestion using simulated annealing algorithm", *Uncertain Supply Chain Management*, Vol. 1, No. 3, (2013), 153-164.
- Eghbali, M., Abedzadeh, M., Setak, M., "Multi-objective reliable hub covering location considering customer convenience using NSGA-II", *International Journal of System Assurance Engineering and Management*, Vol. 5, No. 3, (2014), 450-460.
- Tan, Z., Kara, Y., "A hub covering model for cargo delivery systems", *Networks*, Vol. 49, No. 1, (2007), 28-39.
- Setak, M., Karimi, H., Rastani, S., "Designing Incomplete Hub Location-routing Network in Urban Transportation Problem", *International Journal of Engineering*, Vol. 26, No. 9, (2013), 997-1006.
- Ebrahimi Zade, A., Sadegheih, A., Lotfi, M., "A modified NSGA-II solution for a new multi-objective hub maximal covering problem under uncertain shipments", *Journal of Industrial Engineering International*, Vol. 10, No. 4, (2014), 185-197.
- Tavakkoli-Moghaddam, R. Sedehzadeh, S., "A multi-objective imperialist competitive algorithm to solve a new multi-modal tree hub location problem", *Nature and Biologically Inspired Computing (NaBIC), Sixth World Congress on. 2014*, (2014), 202 - 207.
- Deb, K., Pratap, A., Agarwal, S., Meyarivan, T., "A fast and elitist multiobjective genetic algorithm: NSGA-II", *Transaction Evolutionary Computation*, Vol. 6, No. 2, (2002), 182-197.

38. Okabe, T., Jin, Y., Bernhard, S., "A critical survey of performance indices for multi-objective optimisation", *Evolutionary Computation*, Vol. 2, (2003), 878-885.
39. Veldhuizen, D. A. V., Lamont, G. B., "Evolutionary Computation and Convergence to a Pareto Front" Late Breaking Papers on the Genetic Programming Conference, (1998), 221-228.
40. Veldhuizen, D.A.V., "Multiobjective Evolutionary Algorithms: Classifications, Analyses, and New Innovations", Faculty of the Graduate School of Engineering of the Air Force Institute of Technology, Air University, Dissertation, (1999).
41. Schott, J.R., "Fault Tolerant Design Using Single and Multicriteria Genetic Algorithm Optimization", Master Thesis, Boston, MA: Department of Aeronautics and Astronautics, Massachusetts Institute of Technology, (1995), 199-200.

## A Reliable Multi-objective $p$ -hub Covering Location Problem Considering of Hubs Capabilities

M. Bashiri, M. Rezanezhad

Department of Industrial Engineering, Shahed University, Tehran, Iran

### PAPER INFO

### چکیده

#### Paper history:

Received 01 November 2014

Received in revised form 18 December 2014

Accepted 29 January 2015

#### Keywords:

$P$ -Hub Covering

Hub Capability

Reliability

Multi Objective

همواره کاهش هزینه حمل و نقل کل و کاهش زمان از اهداف متداول در مسائل مکانیابی می‌باشند. هدف اصلی در مسئله مکانیابی پوششی  $P$ -محور، جایابی محورها و تخصیص گره‌ها به محورهای تاسیس شده می‌باشد به صورتی که در شعاع پوششی محورها قرار بگیرند. در این مقاله، هدف ما تعیین نوع قابلیت‌های محور تاسیس شده برای رسیدن به یک شبکه با قابلیت اطمینان بیشتر می‌باشد. همچنین مدل پیشنهادی به تشکیل شبکه محور قابل اطمینانی می‌پردازد که در آن به تعیین شعاع پوششی اجرایی بجای استفاده از شعاع پوششی اسمی پرداخته شده است. برای نشان دادن تاثیر پارامترهای مدل پیشنهادی، تحلیل حساسیتی روی این پارامترها صورت گرفته است. مدل چند هدفه پیشنهادی توسط روش  $\epsilon$ -constraint برای مثالهایی با اندازه‌های کوچک حل شده و برای مثالهای بزرگ از الگوریتم NSGA-II برای بدست آوردن جوابهای پارتو استفاده شده و جوابهای بدست آمده با نتایج روش  $\epsilon$ -constraint مقایسه شده است. مدل و روش حل توسط مثال‌های زیادی از جمله داده‌های Turkish network تحلیل شده و برخی تحلیل حساسیت‌ها صورت گرفته است. نتایج نشان می‌دهد که شبکه بدست آمده توسط مدل پیشنهادی از کارایی بیشتری نسبت به شبکه کلاسیک برخوردار است.

.doi: 10.5829/idosi.ije.2015.28.05b.10