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Elastic/plastic Buckling Analysis of Skew Thin Plates based on Incremental and Deformation Theories of Plasticity using Generalized Differential Quadrature Method

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ABSTRACT

In this study, generalized differential quadrature analysis of elastic/plastic buckling of skew thin plates is presented. The governing equations are derived for the first time based on the incremental and deformation theories of plasticity and classical plate theory (CPT). The elastic/plastic behavior of plates is described by the Ramberg-Osgood model. The ranges of plate geometries are $0.5 \leq a/b \leq 2.5$ and $0.001 \leq h/b \leq 0.05$ under uniaxial uniform compression or biaxial compression/tension. Generalized differential quadrature (GDQ) discretization rules in association with an exact coordinate transformation are simultaneously used to transform and discretize the equilibrium equations and the related boundary conditions. The accuracy of the results are compared with previously published results. Finally, the effects of aspect, loading and thickness ratios, skew angle, incremental and deformation theories and different types of boundary conditions on the buckling coefficient are presented. Moreover, the effect of skew angle and thickness ratio on the convergence and accuracy of the method are studied. Due to the lack of published solutions for plastic buckling of skew thin plates and the high accuracy of the present approach, the solutions obtained may serve as benchmark values for further studies.

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1. INTRODUCTION

Skew plates are extensively used in structures such as bridges and aircraft wings. The elastic buckling of skew plates has been studied by some researchers. However, to the best of authors' knowledge, the plastic buckling of skew plates is not available in open literature.

The elastic buckling behavior of skew plates has been investigated by Anderson [1] using the energy technique. Durvasula [2, 3] applied Ritzmethod to study the behavior of clamped and simply supported skew plates. Fried and Schmitt [4], Mizusawa et al. [5-7], used a finite element, lagrangian multiplier, finite strip and Ritz methods to study the elastic buckling of skew plates. Kitipornchai et al. [8] and Xiang et al. [9] investigated elastic buckling behavior of thick plates by using Ritz method. York [10] studied the buckling solution of skew plate by applying the classical plate theory. Huyton and York [11, Wang et al. [12] and Wu et al. [13] applied finite element (ABAQUS software), differential quadrature and LSFD methods to study the elastic buckling of skew plates. Some researchers studied the elastic/plastic buckling of rectangular plate based on the incremental and deformation theories of plasticity. Durban [14] found out that the incremental theory can predict more buckling load in comparison with the deformation theory, and that the experimental data have more congruence with deformation theory. Durban and Zuckerman [15] carried out the elastic/plastic buckling analysis of rectangular plates under uniaxial loading with the separation of variables solution. Wang et al. [16, 17] and Chakrabarty [18] investigated the elastic/plastic buckling of thin and thick

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plates based on deformation and incremental theories by use of separation of variables and Ritz method. They came to the conclusion that the deformation theory predicts less buckling stress factor, and as the thickness and Ramberg-Osgood constant increase, the difference between two theories increases. Tahmasebi-nejad and Shanmugam [19] studied elastic buckling behavior of uniaxially loaded skew plates by using FEM. The buckling coefficient generally increases with the skew angle of the plate with gradual and moderate increase for lower angles up to 30° and steep increase for larger angles. Jaberzadeh et al. [20] analyzed the plastic buckling of thin skew plates using the element-free Galerkin method. They used Stowell theory for the plastic buckling of skew plates with variable thickness. They concluded that plastic critical stresses increased with increasing thickness of the plate. Maarefdoust and Kadkhodayan [21, 22] analyzed thin isotropic plate subjected to free edges and uniform and linearly varying in-plane loading using incremental and deformation theories. Since the plastic buckling of skew plate has not received much attention in the literature yet, as a first attempt, the elastic/plastic buckling of skew thin plates with nonlinear material properties and subjected to different boundary conditions is studied. The generalized differential quadrature method is applied to discretize the nonlinear governing equation and the related boundary conditions which are based on the classical plate theory, and the results are compared with known solutions in the literature. Finally, the effect of aspect, loading and thickness ratios, skew angle, incremental and deformation theories and various boundary conditions on the buckling coefficient on the results are investigated.

2. GOVERNING DIFFERENTIAL EQUATION

Consider a skew thin plate of length a, oblique width b and thickness h(Figure 1). The stress is applied uniformly $\sigma_y = \sigma_c$ and $\sigma_x = -\zeta\sigma_c$. The load ratio ($\zeta$) is $\zeta = 1$ for biaxial compression/tension and $\zeta = -1$ for the equibiaxial compression.

In the Cartesian coordinates the relationship between the stress rate and strain rate in the plates are as given below:

$$\begin{Bmatrix}\dot s_x\\ \dot s_y\\ \dot t_{xy}\end{Bmatrix} = E\begin{bmatrix} a & b & c\\ & g & m\\ sym & & d\end{bmatrix}\begin{Bmatrix}\dot e_x\\ \dot e_y\\ \dot g_{xy}\end{Bmatrix}, \quad (1)$$

where E is elastic modulus and parameters α, β, γ, χ, μ and δ dependent on the plasticity theory employed. In the present study, the incremental and deformation theories of plasticity with the Prandtl-Reuss and the Hencky constitutive equations are used. The main difference between these two is that the IT depends on incremental plastic strain and DT depends on total strain. Based on the classical plate theory, the strain-displacement relations can be expressed as:

$$e_x = -z\frac{\partial^2 w}{\partial x^2},\ e_y = -z\frac{\partial^2 w}{\partial y^2},\ g_{xy} = -2z\frac{\partial^2 w}{\partial x\partial y}, \quad (2)$$

wheree_x and e_y are the normal strains, g_{xy} is the shear strain and w is the transverse displacement. The fundamental equation of incremental theory with Prandtl-Reuss constitutive equation is [15, 16]:

$$E\dot e_{ij} = (1+u)\dot S_{ij} + \frac{1-2u}{3}\dot s_{kk} d_{ij} + \frac{3\dot s_e}{2s_e}\left(\frac{E}{T}-1\right)S_{ij}, \quad (3)$$

where S_{ij} is stress deviator tensor, T the tangent modulus calculated through stress-strain curved and s_e the effective stress. The tangent modulus and effective stress are calculated as follows [16]:

$$T = ds_e/de_e, \quad s_e^2 = s_x^2 - s_x s_y + s_y^2 + 3t_{xy}^2, \quad (4)$$

wheree_e is the total effective strain. The parameters a, b, g, c, m, d and G in this method are defined as follows:[17]:

$$a = \frac{1}{r}\left[c_{22}c_{33} - c_{23}^2\right],\ b = \frac{1}{r}\left[c_{13}c_{23} - c_{12}c_{33}\right],$$
$$g = \frac{1}{r}\left[c_{11}c_{33} - c_{13}^2\right],\ m = \frac{1}{r}\left[c_{12}c_{13} - c_{11}c_{23}\right],$$
$$c = \frac{1}{r}\left[c_{12}c_{23} - c_{13}c_{22}\right],\ d = \frac{1}{r}\left[c_{11}c_{22} - c_{12}^2\right], \quad (5)$$
$$r = \frac{E}{T}\begin{vmatrix} c_{11} & c_{12} & c_{13}\\ c_{21} & c_{22} & c_{23}\\ c_{31} & c_{32} & c_{33}\end{vmatrix},\ G = \frac{E}{2(1+u)},$$

in which

$$c_{11} = 1 - 3\left(1-\frac{T}{E}\right)\left(\frac{s_y^2}{4s_e^2} + \frac{t_{xy}^2}{s_e^2}\right),$$
$$c_{12} = -\frac{1}{2}\left[1-(1-2u)\frac{T}{E} - 3\left(1-\frac{T}{E}\right)\left(\frac{s_x s_y}{2s_e^2} + \frac{t_{xy}^2}{s_e^2}\right)\right],$$
$$c_{13} = \frac{3}{2}\left(1-\frac{T}{E}\right)\left(\frac{2s_x - s_y}{s_e}\right)\left(\frac{t_{xy}}{s_e}\right),$$
$$c_{22} = 1 - 3\left(1-\frac{T}{E}\right)\left(\frac{s_x^2}{4s_e^2} + \frac{t_{xy}^2}{s_e^2}\right), \quad (6)$$
$$c_{23} = \frac{3}{2}\left(1-\frac{T}{E}\right)\left(\frac{2s_y - s_x}{s_e}\right)\left(\frac{t_{xy}}{s_e}\right),$$
$$c_{33} = 2(1+u)\left(\frac{T}{E}\right) + 9\left(1-\frac{T}{E}\right)\left(\frac{t_{xy}^2}{s_e^2}\right).$$

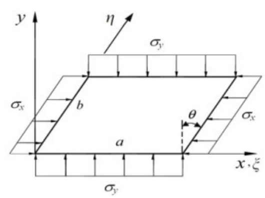


Figure 1. The skew plate under in-plane loading.

The fundamental equation of deformation theory with Hencky constitutive equation is [15, 16]:

$$E\dot{e}_{ij}=\left(\frac{3E}{2S}-\frac{1-2u}{2}\right)\dot{S}_{ij}+\frac{1-2u}{3}\dot{s}_{kk}d_{ij}+\frac{3\dot{s}_e}{2s_e}\left(\frac{E}{T}-\frac{E}{S}\right)S_{ij,} \quad (7)$$

where S is the secant modulus calculated through stress-strain curves. The parameters a, b, g, c, m and d are calculated by employing Equation (5) and G in this theory is defined as follows [17]:

$$G=\frac{E}{2+2u+3\left(\frac{E}{S}-1\right)}. \quad (8)$$

in which

$$\begin{aligned}
c_{11}&=1-3\left(1-\frac{T}{S}\right)\left(\frac{s_y^2}{4s_e^2}+\frac{t_{xy}^2}{s_e^2}\right),\\
c_{12}&=-\frac{1}{2}\left[1-(1-2u)\frac{T}{E}-3\left(1-\frac{T}{S}\right)\left(\frac{s_x s_y}{2s_e^2}+\frac{t_{xy}^2}{s_e^2}\right)\right],\\
c_{13}&=\frac{3}{2}\left(1-\frac{T}{S}\right)\left(\frac{2s_x-s_y}{s_e}\right)\left(\frac{t_{xy}}{s_e}\right),\\
c_{22}&=1-3\left(1-\frac{T}{S}\right)\left(\frac{s_x^2}{4s_e^2}+\frac{t_{xy}^2}{s_e^2}\right),\\
c_{23}&=\frac{3}{2}\left(1-\frac{T}{S}\right)\left(\frac{2s_y-s_x}{s_e}\right)\left(\frac{t_{xy}}{s_e}\right),\\
c_{33}&=3\frac{T}{S}-(1-2u)\left(\frac{T}{E}\right)+9\left(1-\frac{T}{S}\right)\left(\frac{t_{xy}^2}{s_e^2}\right).
\end{aligned} \quad (9)$$

The total potential energy (Π) functional can be expressed as:

$$\Pi=U+V, \quad (10)$$

where U is the strain energy functional, and V is potential energy. The strain energy functional can be expressed as:

$$\begin{aligned}
U=\frac{1}{2}\int_A &\frac{aEh^3}{12}\left(\frac{\partial^2 w}{\partial x^2}\right)^2+\frac{4dEh^3}{12}\left(\frac{\partial^2 w}{\partial x\partial y}\right)^2+\\
&\frac{2bEh^3}{12}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial y^2}\right)+\frac{4cEh^3}{12}\left(\frac{\partial^2 w}{\partial x^2}\right)\left(\frac{\partial^2 w}{\partial x\partial y}\right)\\
&+\frac{gEh^3}{12}\left(\frac{\partial^2 w}{\partial y^2}\right)^2+\frac{4mEh^3}{12}\left(\frac{\partial^2 w}{\partial y^2}\right)\left(\frac{\partial^2 w}{\partial x\partial y}\right)
\end{aligned} \quad (11)$$

The potential energy for the plate subjected to uniform in-plane compressive stress is given by:

$$V=-\frac{1}{2}\int_A s_x h\left(\frac{\partial w}{\partial x}\right)^2+s_y h\left(\frac{\partial w}{\partial y}\right)^2+2t_{xy}h\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)dA. \quad (12)$$

For skew thin plates, the material points of skew plates in the physical domain can be exactly transformed into the computational domain, using the following linear transformation rules (see Figure 1):

$$x=\xi+\eta\sin\theta,\ y=\eta\cos\theta, \quad (13)$$

where x and h are natural coordinate variables of the computational domain and q is the skew angle.

Using Equations (10)-(13) and calculus of variations, the Euler-Lagrange differential equations associated with the minimization of the total potential energy functional, the equilibrium equation of elastic/plastic buckling of skew thin plate can be derived.

3. GDQ DISCRETIZED FORM OF THE GOVERNING EQUATIONS

The GDQ method has simple formulation, low computational cost and high accuracy when used for the analysis of the global behaviors of structural elements such as buckling problem. This method was introduced in 1971 by Bellman and Casti [23] as a new technique for numerically solving ordinary or partial equation. The first widespread use of this technique in the field of engineering problems was given by Bert and Malik [24]. The important components of the GDQ approximation are the weighting coefficients and the choice of grid spacing.

The benefit of accessing a new and exact solution with the least analysis burdon in comparison with others numerical solutions like finite element and boundary element gradually reveals the efficiency of this method. This method can solve higher order differential equations with selecting few grid spacing. Its other characteristics are simple application and programming and high convergence rate. The major advantage of the GDQ method is that much less computer memory is needed when compared to the finite element method. The distributions of grid spacing of Chebyshev–Gauss–Lobatto (C-G-L) have the best convergence and highest accuracy [25, 26]. In this study, the following relations are used:

$$\begin{aligned}
x_i&=\frac{1}{2}\left(1-\cos\frac{i-1}{N_x-1}p\right),\ i=1,2,\dots\dots,N_x.\\
h_j&=\frac{1}{2}\left(1-\cos\frac{j-1}{N_h-1}p\right),\ j=1,2,\dots\dots,N_h.
\end{aligned} \quad (14)$$

The GDQ discretization rules can be used to discretize the spatial derivatives in the computational domain. After these steps the GDQ discretized form of the equilibrium at each grid point (i, j) with i = 2,3,…,N_ξ-1 and j = 2,3,…,N_η-1 can be obtained. Here are the GDQ discretized form of the transverse the equilibrium equation of elastic/plastic buckling of skew thin plate:

$$\begin{aligned}
&\left[\frac{Eh^3}{12}a-2\frac{Eh^3}{12}c\tan q+\frac{Eh^3}{12}(2b+4d)\tan^2 q\right.\\
&\left.+\frac{Eh^3}{12}(g\tan q-4m)\tan^3 q\right]\sum_{k=1}^{N_x}C_{ik}^{(4)}w_{kj}+\\
&\left[-4\frac{Eh^3}{12}(b+2d)\sec q\tan q+4\frac{Eh^3}{12}c\sec q\right.
\end{aligned}$$

$$\begin{aligned}&-4\frac{Eh^3}{12}(g\tan q-3m)\sec q\tan^2 q\Bigg]\times\\
&\sum_{m=1}^{N_h}C_{jm}^{(1)}\sum_{k=1}^{N_x}C_{ik}^{(3)}w_{km}+\Bigg[2\frac{Eh^3}{12}(b+2d)\sec^2 q\\
&+6\frac{Eh^3}{12}(g\tan q-2m)\sec^2 q\tan q\\
&+2\frac{Eh^3}{12}c\cos^2 q\tan q\Bigg]\sum_{m=1}^{N_h}C_{jm}^{(2)}\sum_{k=1}^{N_x}C_{ik}^{(2)}w_{km}+\\
&\Bigg[-4\frac{Eh^3}{12}(g\tan q-m)\sec^3 q\Bigg]\sum_{m=1}^{N_h}C_{jm}^{(3)}\sum_{k=1}^{N_x}C_{ik}^{(1)}w_{km}+\\
&\Bigg[\frac{Eh^3}{12}g\sec^4 q\Bigg]\sum_{k=1}^{N_h}C_{jk}^{(4)}w_{ik}=-\left(\sum_{k=1}^{N_x}C_{ik}^{(2)}w_{kj}\right)s_x\\
&+\sec^2 q\Bigg(\sin^2 q\sum_{k=1}^{N_x}C_{ik}^{(2)}w_{kj}-\sum_{k=1}^{N_h}C_{jk}^{(2)}w_{ik}\\
&+2\sin q\sum_{m=1}^{N_h}C_{jm}^{(1)}\sum_{k=1}^{N_x}C_{ik}^{(1)}w_{km}\Bigg)s_y\\
&-2\sec q\left(\sum_{m=1}^{N_h}C_{jm}^{(1)}\sum_{k=1}^{N_x}C_{ik}^{(1)}w_{km}-\sin q\sum_{k=1}^{N_x}C_{ik}^{(2)}w_{kj}\right)t_{xy}
\end{aligned}\qquad(15)$$

Now, the GDQ analyses of general boundary conditions are developed.

-Simply supported edges

for $x = 0,\ x = a$

$$\begin{aligned}&w_{1j}=w_{N_xj}=0,\\
&a\sum_{m=1}^{N_x}C_{im}^{(2)}w_{mj}+b(\tan^2 q\sum_{m=1}^{N_x}C_{im}^{(2)}w_{mj}-2\sec q\tan q\\
&\times\sum_{m=1}^{N_h}C_{jm}^{(1)}\sum_{k=1}^{N_x}C_{ik}^{(1)}w_{km}+\sec^2 q\sum_{n=1}^{N_h}C_{jn}^{(2)}w_{in})=0.\\
&\qquad i=1,\dots\dots,N_x,\ j=1,\dots\dots,N_y.
\end{aligned}\qquad(16)$$

-Clamped edges

for $x = 0,\ x = a$

$$\begin{aligned}&w_{1j}=w_{N_xj}=0,\quad j=1,\dots\dots,N_y.\\
&\sum_{m=1}^{N_x}C_{im}^{(1)}w_{mj}=0,\sum_{m=1}^{N_x}C_{N_xm}^{(1)}w_{mj}=0,\quad i=1,\dots\dots,N_x.
\end{aligned}\qquad(17)$$

Assume that s_x = -$z s_c$, σ_y = s_c and t_{xy}= 0. It is easily seen that the final equations of matrices, Equations (15-17), are a set of nonlinear eigen value equations with the size of $(N_x)^2\times(N_y)^2$. Equation (15) yields the buckling coefficient (the lowest eigen value) by solving the generalized following eigen value problem:

$$[M][w]=\frac{12 s_c a^2}{Eh^2}[N][w],\qquad(18)$$

where M and N are matrices derived from the governing Equation(15). Now, the non-dimensional buckling coefficient, K can be defined as:

$$K=\frac{s_c hb^2}{p^2 D},\qquad(19)$$

where $D = Eh^3/12(1\text{-}u^2)$ is the flexural rigidity. After discretizing the equilibrium equation and the boundary conditions, one obtains a nonlinear generalized eigen value equation. Since parameters a, b, g, c, m and d depend on the unknown load, the direct iteration procedure is used for obtaining the solution to nonlinear eigen value Equation (14). A computer program EBISTP (Elastic/plastic Buckling of Isotropic Skew Thin Plates) is developed based on the above mentioned formulation to generate elastic/plastic buckling coefficient of plate. According to aforementioned, a complete algorithm for the elastic/plastic buckling of skew thin plate combined with the consistent GDQ method can be obtained, Figure2.

4. MECHANICAL PROPERTIES OF MATERIAL

The material used in this study is AL 7075-T6. Here the Ramberg-Osgood elastic/plastic stress-strain relationship is used as [15]:

$$e=\frac{s_e}{E}+k\left(\frac{s_e}{E}\right)^n\qquad(20)$$

where$e$ is the total strain and n and k are material parameters. The tangent and secant moduli used in the equation are calculated as follows [15]:

$$\frac{E}{T}=1+nk\left(\frac{s_e}{E}\right)^{n-1},\frac{E}{S}=1+k\left(\frac{s_e}{E}\right)^{n-1}\qquad(21)$$

The characteristics of this material is obtained by means of Equation (20), E = 72.4 MPa, Ramberg-Osgood parameters n = 10.9, $k = 3.94\times10^{21}$ and Poisson's ratio n=0.32 [15]. Figure 3 shows the Ramberg-Osgood stress–strain relation for the material AL 7075-T6 described by Equation (20).

5. RESULTS AND DISCUSSIONS

In this section, firstly, the grid spacing problem and convergence rate studies are carried out. In Table 1, the results for different number of grid points at various thickness ratios and skew angles are shown. Converged results are obtained using thirteen grid points ($N_x = N_h$ = 13). Notice that nine grid points is sufficient to obtain results with acceptable accuracy. In all cases, the fast rate of convergence of the method is quite evident. With increasing the thickness ratio and skew angle, the convergence results take place in more grid spacing (Table 2).In order to validate the presented formulation and the efficiency of the method in analyzing the elastic/plastic buckling of skew thin plate, the results for skew plates with different skew angle and boundary condition are shown in Table 3. It is seen that in all cases the results are in very good agreement with those of the other methods and have a closer agreement with those of Kitipornchai et al. [8] and Wu et al. [13]. A comparison between these results and experimental data for rectangular plates under uniaxial compression are presented in Figure 4. It is seen that the results attained by deformation theory are close to the experimental ones. Now, some parametric studies are carried out for the elastic/plastic buckling of skew thin plates.

5.1. Effect of Aspect Ratio on the Buckling Coefficient

The influences of aspect ratio (a/b) in uniaxial compression loading on buckling coefficient for different thickness ratios and both incremental and deformation theories are presented in Figure 5. The results are prepared for the three thickness ratios and three boundary conditions at$q$ = 30°. It is seen that by increasing the thickness ratio, the differences between the results obtained by the incremental and deformation theories increase. In addition, the maximum variation of buckling coefficient occurs at0.5 ≤a/b≤ 1.5. The buckling coefficient decreases as the plate thickness ratio increases. It is clear that for the same plate thickness and aspect ratio, the boundary condition plays an important role on the buckling coefficient of plates. The buckling coefficient increases with increasing the clamped boundary condition, Figures 5a and 5c. Moreover, the effects of different loading types on the discrepancy between IT and DT results can be observed. In equibiaxial loading, the agreement between IT and DT results is more than that of uniaxial loading, Figures 5 and 6. In equibiaxial loading, the buckling coefficient decreases monotonically with increasing the aspect ratio.

5. 2. Effect of Skew Angle on the Buckling Coefficient

The effect of skew angle (θ°) on the buckling coefficient of skew thin plates for uniaxial and equibiaxial loadings are shown in Figures 7 and 8. It is seen that the effect is more significant when the thickness ratio decreases. Generally, by increasing the skew angle the buckling coefficient increases. It can be seen that increasing the clamped boundary conditions at the edges and increasing the skew angle, increases the discrepancy between IT and DT results, Figures 7 and 8. In addition, the minimum discrepancy between IT and DT results occurs in equibiaxial loading, Figure 8. In the same condition, the buckling coefficient of equibiaxial loading is lower than that of uniaxial one and the difference decreases when the thickness ratio increases.

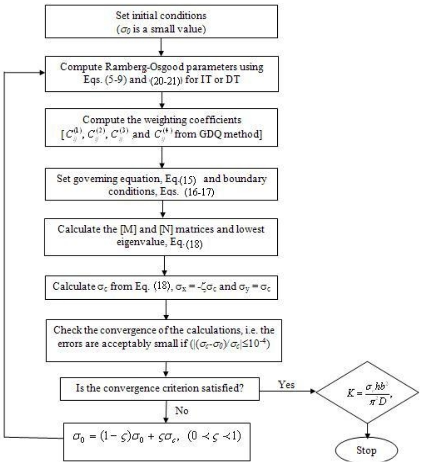


Figure 2. Flow chart of the solution strategy.

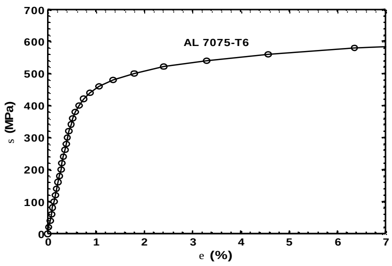


Figure 3. The stress – strain curve for AL 7075-T6.

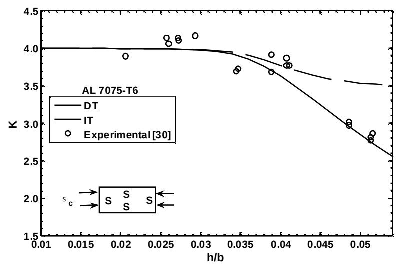


Figure 4. Comparison of buckling coefficient obtained by IT and DT with test results (Al 7075-T6, q = 0).

TABLE 1. Convergence rate of the results for skew plate

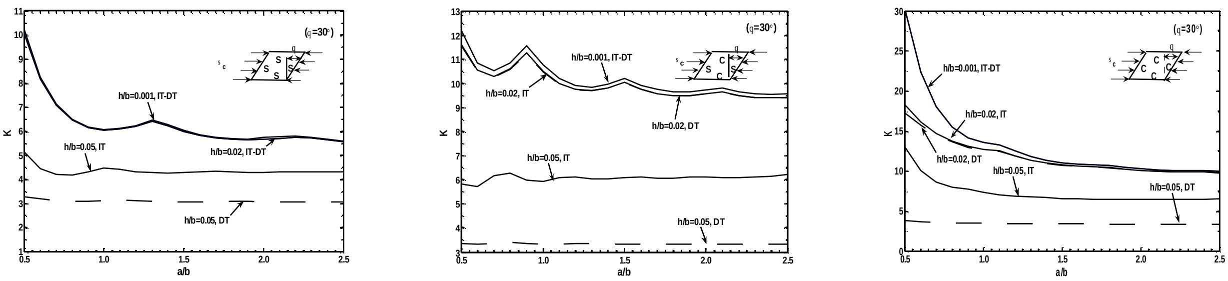
B.C	q	h/b	$N_x = N_h$ 7	9	11	13	15
SSSS uniaxial	45°	0.001	10.7156	10.4612	10.4309	10.4520	10.4520
		0.05	3.4388	3.4038	3.3935	3.3912	3.3912
CCCC biaxial	30°	0.001	6.9759	6.8548	6.8547	6.8547	6.8547
		0.05	2.9047	2.8949	2.8948	2.8948	2.8948
SCSC uniaxial	15°	0.001	8.2687	8.3750	8.3537	8.3546	8.3546
		0.05	3.2095	3.2089	3.2074	3.2071	3.2071

TABLE 2. Convergence rate of buckling ratio of SCSC skew plate under uniaxial loading (*h/b* = 0.02).

	$N_x = N_h$					
q	**7**	**9**	**11**	**13**	**15**	**17**
15°	4.9792	4.9758	4.9751	4.9751	4.9751	4.9751
30°	5.9401	5.9399	5.9394	5.9394	5.9394	5.9394
45°	8.9842	8.9350	8.9084	8.9047	8.9047	8.9047
60°	14.1204	13.8649	13.7654	13.7586	13.7586	13.7586

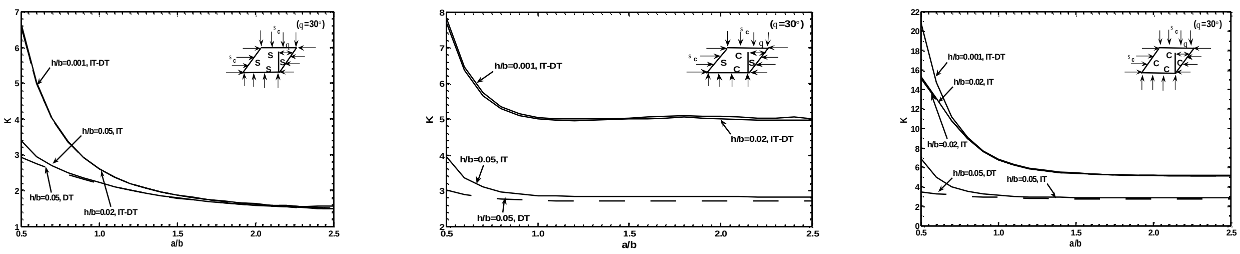
TABLE 3. Comparison of elastic buckling coefficient of SSSS and CCCC skew plate under uniaxial loading (a/b = 1, h/b = 0.001).

			q°			
B.C	**Sources**	**Method**	**0**	**15**	**30**	**45**
SSSS	Anderson [1]	Energy	-----	-----	6.74	11.46
	Duruasula [3]	Ritz method	4.0000	4.4801	6.4106	12.3
	Fried and Schmitt [4]	FEM	4.0000	-----	5.9093	10.2
	Mizusawa et al. [5]	Lagrangian-multiplier	4.0000	4.3397	5.6106	8.64
	Kitipornchai et al.[8]	Ritz method	4.0000	4.3937	5.9217	10.104
	Huyton and York [11]	FEM	4.0000	-----	5.85	9.67
	Wang et al. [12]	DQ	4.0000	-----	5.83	9.39
	Wu et al. [13]	LSFD	4.0000	4.3924	5.8611	9.7164
	Kennedy and Prabhakara [27]	Double-Fourier	4.0000	4.33	5.53	8.47
	Present study	GDQ	4.0000	4.4343	5.9880	10.4520
CCCC	Duruasula [2]	Galerkin	10.08	10.87	13.58	20.44
	Huyton and York [11]	FEM	-----	10.8	13.5	20.1
	York [10]	Lagrangian-multipler	10.07	10.85	13.58	20.21
	Wang et al. [12]	DQ	10.07	10.83	13.54	20.10
	Wu et al. [13]	LSFD	10.0735	10.8325	13.535	20.0974
	Guest [28]	Lagrangian-multiplier	-----	-----	13.53	20.7
	Wittrick [29]	Rayleigh-Ritz	-----	-----	13.64	21.64
	Reddy and Palaninathan [30]	FEM	-----	10.76	13.64	20.62
	Tham and Sezto [31]	Spline Finite Strip	-----	10.84	13.60	20.60
	Present study	GDQ	10.073	10.8325	13.5356	20.0974



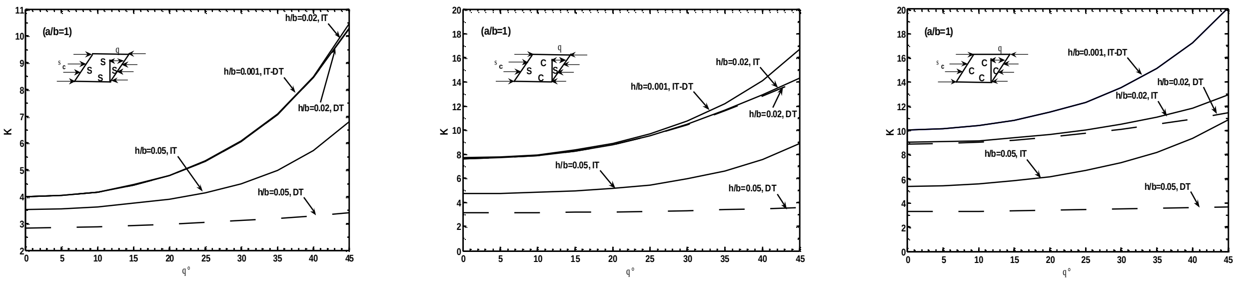
(a) (b) (c)

Figure 5. Variation of buckling coefficient with aspect ratio for various thickness ratios and boundary conditions and subjected to uniaxial compression loading.



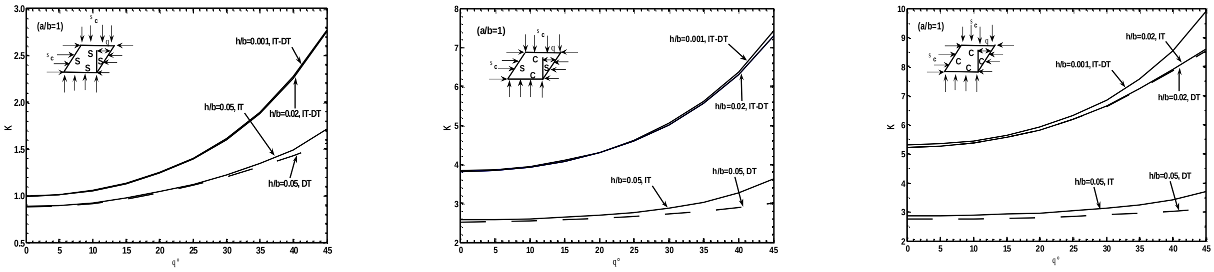
(a) (b) (c)

Figure 6. Variation of buckling coefficient with aspect ratio for various thickness ratios and boundary conditions and subjected to equibiaxial compression loading.



(a) (b) (c)

Figure 7. Variation of buckling coefficient with skew angle for various thickness ratios and boundary conditions and subjected to uniaxial compression loading.



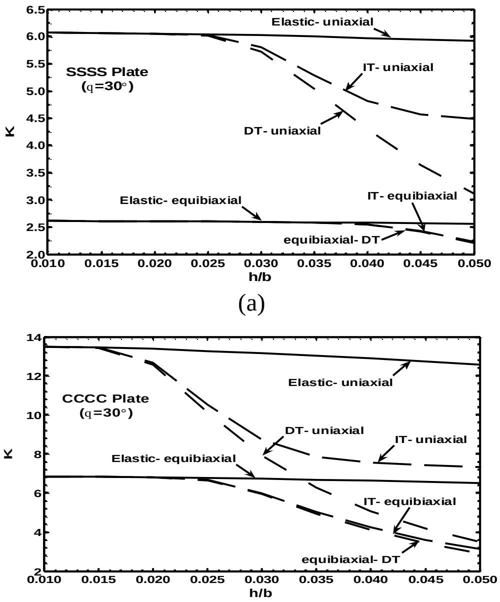
(a) (b) (c)

Figure 8. Variation of buckling coefficient withskew angle for various thickness ratios and boundary conditions and subjected to equibiaxial compression loading.

5. 3. Effect of Thickness Ratio on the Buckling Coefficient With increasing the thickness ratio, (h/b) the buckling coefficient decreases in uniaxial and equibiaxial compression loadings when IT or DT is used (Figure 9). Moreover, the agreements between the results obtained by IT and DT for equibiaxial compression loading are more than those of uniaxial ones, Figure 9. It is interesting to note that by increasing the thickness ratio ($h/b \geq 0.025$), the differences between the elastic and plastic buckling coefficients in uniaxial and equibiaxial loadings increase. In all cases, the discrepancy between the IT and DT results is more in uniaxial compression than that of equibiaxial loading (Figures 9 and 10). In addition, the deformation theory predicts lower buckling coefficient compared to incremental theory and the discrepancy between IT and DT increases with increasing thickness ratio, skew angle and clamped boundary condition(Figure 10). Moreover, it is seen that the effects of skew angle on the buckling coefficient with increasing thickness ratio in plates under biaxial compression are less than those of the uniaxial compression.

5. 4. Effect of Loading ratio on the Buckling Coefficient The variations of the buckling coefficient against the loading ratio, z, thickness ratio and IT and DT theories are shown in Figure 11 for SSSS skew plates. When h/b = 0.03, $z \leq$ 0.2 there is a good congruence between two theories of plasticity. However, for h/b = 0.05 there is no agreement between them, and also with increasing the loading ratio, the differences for biaxial tension/compression loading increase in the range of $0 < z \leq 1.5$. Moreover, the difference between IT and DT results increases with increasing the thickness ratio(Figure 11).

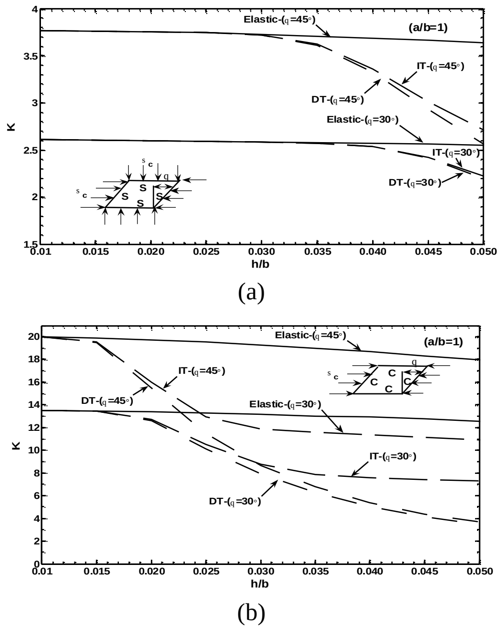
5. 5. Analysis of Plastic Buckling Mode Shapes Figure 12 shows the shape contours of the buckled plate in each mode for various aspect ratios and boundary conditions under uniaxial and equibiaxial compression loadings. In uniaxial loading the number of half-sine waves in the buckling mode shape increases as the aspect ratio increases (Figure 5). The locations of kinks at the aspect ratios of coincident buckling modes are not the same for different plate thicknesses. For DT results subjected to uniaxial compression, however, the buckling coefficients decrease slightly when the thickness and aspect ratios increase in various boundary conditions. Despite the existence of mode shape shift in this case, it is hard to distinguish the shift points (Figure 5).For equibiaxial compression, the buckling coefficient decreases monotonically as the aspect ratio increases, Figure 6. In some case, the graph is smooth which is due to unchanged buckling mode and no mode shape shift is observed, Figure 6. These kinks locations depend on the plasticity theories used. For example, the buckling mode comprises one half sine waves for IT while the plate buckles the shape of two half sine waves for DT in SCSC skew plate by equibiaxial loading and h/b = 0.05, a/b = 2 and q = 45°(Figure 12).



(a)

(b)

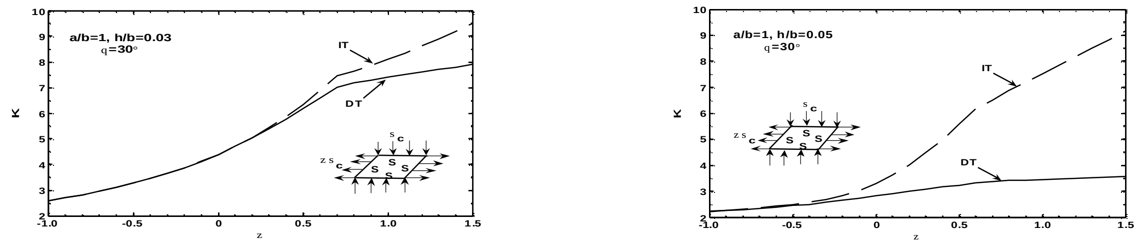
Figure 9. Variation of buckling coefficient with thickness ratio for various boundary conditions and subjected to uniaxial and equibiaxial compression loading, (q = 30°).



(a)

(b)

Figure 10. Variation of buckling coefficient with thickness ratio for various boundary conditions and skew angles subjected to uniaxial and equibiaxial compression loadings.



(a)

(b)

Figure 11. Comparison of buckling coefficient obtained by incremental and deformation theories for simply supported skew plates for various loading ratios (a) h/b = 0.03, (b) h/b = 0.05.

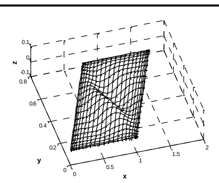
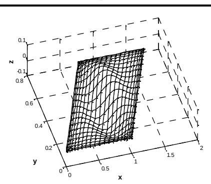
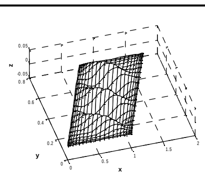
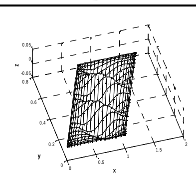
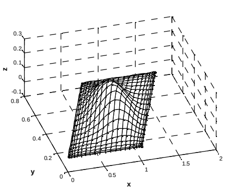
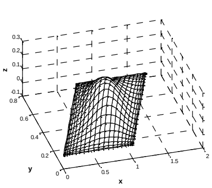
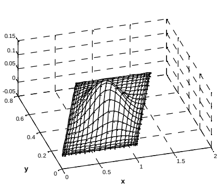
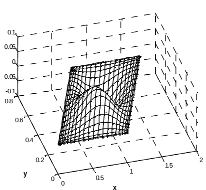
B.C	(*a/b* = 1)		(*a/b* = 2)	
	IT	DT	IT	DT
SCSC uniaxial				
	K=8.9043	K=3.5626	K=9.4655	K=3.5689
SCSC equibiaxial				
	K=3.6334	K=3.0031	K=3.2867	K=2.9617

Figure 12. Buckling mode shapes of skew plates under uniaxial and biaxial compression loadings and various boundary conditions, (h/b = 0.05, q = 45°).

6. CONCLUSION

In the present paper, the equilibrium and stability equations of elastic/plastic buckling of skew thin plates are derived. Derivations are based on the classical plate theory and incremental and deformation theories of plasticity. The generalized differential quadrature method as an efficient and accurate numerical method is employed to discretize the geometrically and physically nonlinear differential equations and the related boundary conditions. After demonstrating the fast rate of convergence and accuracy of the method, parametric study is performed to represent the effect of aspect, thickness and loading ratios, skew angle, incremental and deformation theories and various boundary conditions on the results. The followings are concluded:

- Buckling coefficient increases with the increase of skew angle q.
- The discrepancy between IT and DT results decreases in equibiaxial compression loading.
- The influence of skew angle is higher in IT than DT.
- Variations of plastic buckling mode shapes obtained for uniaxial loading for various aspect ratios are generally greater than those of biaxial loading.
- The discrepancy between IT and DT results increases with the increase of thickness of plate, while it increases with the increase of loading ratio in biaxial compression/tension.
- Buckling coefficients obtained by deformation theory generally do not show significant changes with increasing aspect ratio and plate thickness.
- The discrepancy between the two theories of plasticity increases with the increase of skew angle in various boundary conditions.
- By increasing the skew angle and decreasing the thickness ratio, the convergence rate decreases and the program run time increases.
- The results indicate that GDQ method can yield very accurate results for all cases considered. It was also found that the convergence rate of GDQ method is excellent.

Most buckling coefficients of skew thin plates are believed to be novel and could be used for testing other newly developing methods or even analytical numerical data.

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Elastic/plastic Buckling Analysis of Skew Thin Plates based on Incremental and Deformation Theories of Plasticity using Generalized Differential Quadrature Method

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چکیده

در این مقاله تحلیل یک چهارم تفاضلی تعمیم یافته کمانش الاستیک-پلاستیک صفحات نازک اریب مورد بحث قرار گرفته است. معادلات حاکم بر کمانش الاستیک–پلاستیک صفحات نازک مورب برای اولین بار به وسیلهٔ تئوری پلاستیسیته تغییرشکل و نموی و تئوری صفحات کلاسیک استخراج شده است. رفتار الاستیک–پلاستیک صفحه به وسیله مدل رامبرگ ازگود مدل‌سازی شده است. بار به صورت فشاری محوری یا کششی/ فشاری دومحوری وارد می‌شود و هندسه مورد نظر صفحه $0.001 \le h/b \le 0.05$ و $0.5 \le a/b \le 2.5$ می‌باشد. معادلات استخراجی در سیستم دکارتی به سیستم اریب انتقال داده شده و در سیستم یک چهارم تفاضلی تعمیم یافته نوشته و نتایج حاصل با کارهای پیشین مقایسه شده است. اثرات نسبت ابعادی، زاویه اریب، ضریب ضخامت صفحه، ضریب بار، تئوری تغییرشکل و تئوری نموی و شرایط مرزی مختلف بر تعیین ضریب کمانشی بررسی شده و نتایج حاصل ارائه گردیده است. همچنین، اثر زاویه اریب و ضریب ضخامت بر همگرایی و صحت روش مورد استفاده، سنجیده شده است. با توجه به این که نتایج کمانش پلاستیک صفحات نازک اریب به دست آمده دارای دقت و صحت خوبی می‌باشند می‌توانند برای کارهای آیندگان مورد استفاده قرار گیرند.

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