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Improving the Performance of Bayesian Estimation Methods in Estimations of Shift Point and Comparison with Maximum Likelihood Estimation Approach

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ABSTRACT

A Bayesian analysis is used to detect a change-point in a sequence of independent random variables from exponential distributions. The Bayes estimators are derived for change point, the rate of exponential distribution before shift and the rate of exponential distribution after shift. Likelihood, Prior, Posterior and Marginal distribution of the change point is derived. Also, maximum likelihood estimation (MLE) method is used for determining change point. The sensitivity analysis of Bayes estimators are performed by simulation. Also, we suggest a new approach to achieve more precise results by determining correct choice for parameters of prior distribution and compared the approach with existing methods. The result of simulation shows good performance of proposed approach in comparison with existing methods. Also, a sensitivity analysis on the location of the shift is performed.

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1. INTRODUCTION

In this paper, we propose a new approach for estimating change point which occurs in any sequence of independent observations of exponential distribution. Exponential distribution plays an important role in practical applications like time to failure for special item. In some manufacturing systems, the products are subject to abrupt shifts in the failure rate function, which are observed because of maintenance actions. In such cases, determining the time of shift in failure rate of items is important and the observed point is known as shift point. When the data set of individual observations is available, a control chart can be used to detect a shift in the parameters. Also, the estimation of change points plays an important role in the analysis of markets since the rates of occurring irregular incomes are considered in the evaluation of financial risk [1].

The problem under study is about determining the time of change in data. This is called change point inference problem. Bayesian ideas may play an important role in the study of such change point problem and has been often proposed as a valid alternative to classical estimation procedure [2]

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There are many studies on shift point problem in a sequence of random variables. Hinkley [3] studied the shift point problem in a sequence of independent continuous random variables.

Lee [4] proposed a Bayesian analysis to detect a change point in a sequence of independent random variables from exponential family distributions. Perreault et al. [5] presented a Bayesian approach to characterize when and by how much a single shift has occurred in a sequence of hydrometeorological random variables. In one of other studies, Bayesian multiple change-point models are proposed for multivariate means. The models require that the data be from a multivariate normal distribution with a truncated Poisson prior for the number of change points and conjugate priors for the distributional parameters [6]. Hinkley and Hinkley [7] studied the shift point problem in a sequence of binomial variables. Worsley [8] investigated the shift point in sequences of exponential data. The problem of detecting change in Bayesian context has been studied by many authors such as Perreault, et. al. [5], Ghorbanzade and Rachid [9], Johnson [10], Angelo, et al. [11].

Kadilar and Karasoy [12] proposed an efficient estimate for the change point in the hazard function that is based on a Bayesian estimator. It is found through a simulation study that the proposed estimator is more efficient than the traditional estimators in many cases. Loschi et. al. [1] considered the change-point identification problem both in means and variances of normal data sequences. Srivastava [13] estimated shift point which occurs in any sequence of independent observations of Poisson model under Asymmetric Loss Functions in statistical process control using Bayesian inference. Prakash [14] has investigated the effects of Bayes estimations in the inverse Rayleigh model. Tourneret et al. [15] considered Bayesian off-line detection of multiple change-points. In this paper, we propose a new approach for estimating change point which occurs in any sequence of independent observations of exponential distribution. Recently, Keramatpour et al. [16] proposed a method for monitoring and change point estimation of AR(1) autocorrelated polynomial profiles. Also, Khedmati and Niaki [17] proposed a method to detect the change time of multivariate binomial processes for step changes. Ghazanfari et al. [18] proposed a novel clustering approach for estimating the time of step changes in Shewhart control charts. The proposed approach is based on Bayesian Inference that has been successfully applied in many quality control problems [19, 20]. In this research, we show that the parameters of prior distribution affect the performance of the Bayesian change point estimator and a new method is developed for change point estimation by selecting appropriate parameters for prior distributions.

In section 2, we obtain Likelihood, Prior, Posterior and Marginal distribution. Maximum likelihood estimation for change point comes in section 3. We present a simulation study in section 4. Bayesian and maximum likelihood estimation of two change point comes in section 5. Finally, we conclude the paper in section 6.

2. LIKELIHOOD, PRIOR, POSTERIOR AND MARGINAL

Let $X_1, X_2, \ldots\ldots, X_n$ be a sequence of observations that come from exponential distribution with probability density function as follows:

$$p(x) = \lambda e^{-\lambda x} \quad ; x > 0 \ , \ \lambda > 0. \tag{1}$$

Let m be the change point in the hazard rate of exponential distribution that leads to the event that two sequences $X_1, \ldots\ldots, X_m$ and $X_{m+1}, X_{m+2}, \ldots, X_n$ have different hazard rates. The probability density functions of the sequence $X_1, \ldots\ldots, X_m$ is defined as follows:

$$p_1(x) = \lambda_1 e^{-\lambda_1 x} \quad ; x > 0 \ , \ \lambda_1 > 0. \tag{2}$$

And the probability density functions of the sequence $X_{m+1}, X_{m+2}, \ldots, X_n$ is:

$$p_2(x) = \lambda_2 e^{-\lambda_2 x} \quad ; x > 0 \ , \ \lambda_2 > 0. \tag{3}$$

The likelihood functions of probability density function of the sequences are as follows:

$$L\left(\lambda_1 | \underline{x}\right) \ \alpha \ \lambda_1^{\ m} e^{-\lambda_1 \sum_{i=1}^{m} x_i}.$$
$$L\left(\lambda_2 | \underline{x}\right) \ \alpha \ \lambda_2^{\ n-m} e^{-\lambda_2 \sum_{i=m+1}^{n} x_i}. \tag{4}$$

And the joint Likelihood function is given by:

$$L\left(\lambda_1 \lambda_2 | \underline{x}\right) \ \alpha \ \lambda_1^{\ m} e^{-\lambda_1 \sum_{i=1}^{m} x_i} \ \lambda_2^{\ n-m} e^{-\lambda_2 \sum_{i=m+1}^{n} x_i}. \tag{5}$$

Suppose the marginal prior distributions of λ_1, λ_2 are Gamma priors as follows:

$$g_1(\lambda_1) \ \alpha \ \frac{b_1^{a_1}}{\Gamma(a_1)} \lambda_1^{a_1 - 1} \ e^{-b_1 \lambda_1} \ ; a_1, \ b_1 > 0.$$
$$g_2(\lambda_2) \ \alpha \ \frac{b_2^{a_2}}{\Gamma(a_2)} \lambda_2^{a_2 - 1} \ e^{-b_2 \lambda_2} \ ; a_2, \ b_2 > 0. \tag{6}$$

As Srivastava [13] proposed, we take the marginal prior distribution of m discrete uniform over the set $m = 1, \ldots\ldots, (n-1)$ then the joint prior distribution of m, λ_2, λ_1 is

$$g(\lambda_1, \lambda_2, m) \ \alpha \ \frac{b_1^{a_1} b_2^{a_2}}{\Gamma a_1 \Gamma a_2} \lambda_1^{a_1 - 1} \lambda_2^{a_2 - 1} \ e^{-b_1 \lambda_1} \ e^{-b_2 \lambda_2} \ ,$$
$$\lambda_1, \ \lambda_2 > 0 \ \& \ m = 1, \ldots\ldots, (n-1) \tag{7}$$

The Joint posterior density of m, λ_2, λ_1 say $\pi\left(\lambda_1, \lambda_2; m | \underline{x}\right)$ is determined as follows:

$$\pi\left(\lambda_1, \ \lambda_2; m | \underline{x}\right) = \frac{b_1^{a_1} b_2^{a_2}}{\Gamma(a_1)\Gamma(a_2)}$$
$$\lambda_1^{m + a_1 - 1} \ \lambda_2^{n - m + a_2 - 1} \ e^{-\lambda_1 (b_1 + \sum_{i=1}^{m} x_i)} \quad e^{-\lambda_2 (b_2 + \sum_{i=m+1}^{n} x_i)}. \tag{8}$$

Therefore, the marginal posterior distribution of change point m is:

$$\pi\left(m | \underline{x}\right) = \frac{\dfrac{\Gamma(m + a_1)}{(b_1 + \sum_{i=1}^{m} x_i)^{m + a_1}} \dfrac{\Gamma(n - m + a_2)}{(b_2 + \sum_{i=m+1}^{n} x_i)^{n - m + a_2}}}{\psi(a_1, a_2, b_1, b_2, m, n)}. \tag{9}$$

Also, the marginal posterior distribution of λ_1 is obtained as follows:

$$\pi\left(\lambda_1|\underset{-}{x}\right)=\frac{\sum_{m=1}^{n}\left(\lambda_1^{m+a_1-1}\ e^{-\lambda_1(b_1+\sum_{i=1}^{m}x_i)}\ \frac{\Gamma(n-m+a_2)}{(b_2+\sum_{i=m+1}^{n}x_i)^{n-m+a_2}}\right)}{\psi(a_1,a_2,b_1,b_2,m,n)}. \quad (10)$$

And the marginal posterior distribution of λ_2 is determined as follows:

$$\pi\left(\lambda_2|\underset{-}{x}\right)=\frac{\sum_{m=1}^{n}\left(\lambda_2^{n-m+a_2-1}\ e^{-\lambda_2(b_2+\sum_{i=m+1}^{n}x_i)}\ \frac{\Gamma(m+a_1)}{(b_1+\sum_{i=1}^{m}x_i)^{m+a_1}}\right)}{\psi(a_1,a_2,b_1,b_2,m,n)}. \quad (11)$$

where,

$$\psi\left(a_1,a_2,b_1,b_2,m,n\right)=\sum_{m=1}^{n-1}\int_0^{\infty}\int_0^{\infty} g(\lambda_1,\lambda_2,m)d\lambda_1 d\lambda_2 = \sum_{m=1}^{n-1}\frac{\Gamma\left(m+a_1\right)}{(b_1+\sum_{i=m+1}^{n}x_i)^{m+a_1}}\frac{\Gamma\left(n-m+a_2\right)}{(b_2+\sum_{i=m+1}^{n}x_i)^{n-m+a_2}}. \quad (12)$$

Using the marginal distribution of λ_2,λ_1 and m, following results are concluded,

$$E(\lambda_1)=\frac{\sum_{m=1}^{n}\left(\frac{\Gamma(m+1+a_1)}{(b_1+\sum_{i=1}^{m}x_i)^{m+1+a_1}}\frac{\Gamma(n-m+a_2)}{(b_2+\sum_{i=m+1}^{n}x_i)^{n-m+a_2}}\right)}{\psi(a_1,a_2,b_1,b_2,m,n)}. \quad (13)$$

$$E(\lambda_2)=\frac{\sum_{m=1}^{n}\left(\frac{\Gamma(m+a_1)}{(b_1+\sum_{i=1}^{m}x_i)^{m+a_1}}\frac{\Gamma(n-m+1+a_2)}{(b_2+\sum_{i=m+1}^{n}x_i)^{n-m+1+a_2}}\right)}{\psi(a_1,a_2,b_1,b_2,m,n)}. \quad (14)$$

$$E(m)=\sum_{m=1}^{n}\frac{\frac{m\ \Gamma(m+a_1)}{(b_1+\sum_{i=1}^{m}x_i)^{m+a_1}}\frac{\Gamma(n-m+a_2)}{(b_2+\sum_{i=m+1}^{n}x_i)^{n-m+a_2}}}{\psi(a_1,a_2,b_1,b_2,m,n)}. \quad (15)$$

3. MAXIMUM LIKELIHOOD ESTIMATION (MLE) OF CHANGE POINT IN EXPONENTIAL DISTRIBUTION

We consider the problem of detecting the change point m along with λ_2,λ_1 using maximum likelihood estimation in this section. In this approach, first we obtain likelihood functions for variables λ_2,λ_1 along with Likelihood estimations. Let $X_1,X_2,\ldots\ldots,X_n$ be a sequence of observations. Let m be the change point in the observation which breaks the distribution in two sequences as $X_1,\ldots\ldots,X_m$ and $X_{m+1},X_{m+2},\ldots,X_n$.
The probability mass functions of the above sequences are:

$$p_1\left(x\right)=\lambda_1 e^{-\lambda_1 x}\quad ;\ x>0\ ,\ \lambda_1>0. \quad (16)$$

$$p_2\left(x\right)=\lambda_2 e^{-\lambda_2 x}\quad ;\ x>0\ ,\ \lambda_2>0. \quad (17)$$

The likelihood functions of p.m.f.'s of the sequences are:

$$L\left(\lambda_1|\underset{-}{x}\right)\ \alpha\ \ \lambda_1^{m}e^{-\lambda_1\sum_{i=1}^{m}x_i}. \quad (18)$$

$$L\left(\lambda_2|\underset{-}{x}\right)\ \alpha\ \ \lambda_2^{n-m}e^{-\lambda_2\sum_{i=m+1}^{n}x_i}. \quad (19)$$

The maximum likelihood estimation of λ_2,λ_1 are given as:

$$\lambda_1=\frac{m}{\sum_{i=1}^{m}x_i},\qquad \lambda_2=\frac{n-m}{\sum_{i=m+1}^{n}x_i}. \quad (20)$$

And Likelihood function is obtained as follows:

$$L\left(k\right)=\frac{k}{\sum_{i=1}^{k}x_i}e^{-k}\ \frac{n-k}{\sum_{i=k+1}^{n}x_i}\ e^{-(n-k)}. \quad (21)$$

Now, we determine the value of m that maximizes the likelihood function. The maximum Likelihood Estimation (MLE) of change point is:

$$m^*=\left\{m:L\left(m\right)=\ \mathrm{Max}\left\{\ L\left(k\right);k=1,2,\ldots,n\right\}\right\}. \quad (22)$$

The value of m that is obtained by MLE approach is the estimated change point.

4. SIMULATION STUDY

In the simulation study, we generate 20 random observations from an exponential distribution. Let the shift in sequence happen at the 10th observation. Also, the hazard rates of sequences $X_1,\ldots\ldots,X_m$ and $X_{m+1},X_{m+2},\ldots,X_n$ are $\lambda_2=3,\lambda_1=2$ respectively. If the target values of m,λ_2,λ_1 are unknown, their estimation are given by using posterior distributions of each parameter in Bayes estimation method. The Bayes estimators of λ_2, λ_1 and change point m are calculated by excel. We have repeated these steps for 100 times to calculate the respective m, λ_2, λ_1 and MSE of various Bayes estimators. As can be seen in the following tables, the MSE of estimations are relatively small, and since the standard error of estimations is obtained by equation $MSE/\sqrt{100} = MSE/10$, therefore, it is concluded that the standard error of estimations is sufficiently small. Another reason for obtaining the results in 100 iterations is the long time of runs and huge simulation studies.

4. 1. Sensitivity Analysis of Bayes Estimates We have considered the sensitivity of the Bayes estimates with respect to shifts in the parameters of prior distribution (a_1,b_1) and (a_2,b_2). The means and variances of the prior distribution are used as prior information in computing these parameters. Then, we have computed the Bayes estimates of λ_1, λ_2 and m using different set of values of (a_1,b_1) and (a_2,b_2). We assume different values of (a_1,b_1) and (a_2,b_2) based on the research in this context done by Srivastava [13] The numbers in parentheses in Table 1 shows the MSE of Bayesian estimators. It is observed that the estimates of m and λ_1 are close to their exact values but the estimate of λ_2 has a large deviation from its exact value by using the priors 1, 2, 3 and 4 that are proposed by Srivastava [13]. Also, when we use a Gamma prior distribution with parameters $(a_1 = 0.001, b_1 = 0.001)$ that is a non-informative prior [21], then the estimations of λ_1, λ_2 are close to their exact values; but, the estimation of change point has a large deviation from its actual value. To solve this problem, we proposed using a two stages Bayesian approach. If we adjust the parameters of prior distributions for λ_1, λ_2 such that their means would be close to their exact values, it is seen that the performance of proposed approach significantly improves as has been done in cases 6 and 7 of Table 1. In case 6, we have assumed that partial information existed about parameters λ_1, λ_2 and the parameters of prior distribution is adjusted so that their means would be close to their exact values and it is seen that the performance of proposed method improves. In case 7, we have assumed that the exact values of parameters λ_1, λ_2 are known and the parameters of prior distribution for λ_1, λ_2 are adjusted so that their means would be equal to their exact values and we see the performance of proposed approach significantly improves. Therefore, following two stages Bayesian approach is suggested. In the first stage of the proposed approach, we use a suitable prior in the Bayesian updating procedure and after estimating the approximate value of the shift point, m, in the second stage, we use Gamma priors with parameters, $(a_1 = 1, b_1 = \overline{X}_1)$ and $(a_2 = 1, b_2 = \overline{X}_2)$ for λ_1, λ_2 where $\overline{X}_1$ is the average of observations $X_1, \ldots\ldots, X_m$ and $\overline{X}_2$ is the average of observations $X_{m+1}, X_{m+2}, \ldots, X_n$ respectively. Suitable prior should be selected by using historical data and experience of decision maker. For example, in this simulation study, we obtain the value of m that is close to its exact value of change point by using the first prior mentioned in Table 1. Thus, first we use a suitable prior for change point estimation in the first stage of Bayesian approach. Then, after determining the approximate value of shift point, we use proposed priors for parameter of exponential distributions in the second stage of Bayesian approach and the Bayesian technique will be repeated again. Table 2 shows the results of estimation by applying this approach.

TABLE 1. Bayes Estimates of m, λ_1, λ_2 and their respective MSE

No.	(a_1,b_1)	(a_2,b_2)	$\widehat{m}$	$\widehat{\lambda}_1$	$\widehat{\lambda}_2$
1	(1.5, 1.75)	(1.8, 2.0)	10.566 (2.89)	1.813 (0.20)	1.869 (1.38)
2	(1.75, 2.0)	(2.0, 2.25)	10.598 (4.40)	1.762 (0.25)	1.842 (1.40)
3	(2.0, 2.25)	(2.20, 2.5)	10.772 (4.30)	1.791 (0.20)	1.778 (1.56)
4	(2.25,2.50)	(2.40,2.75)	10.646 (3.50)	1.675 (0.24)	1.719 (1.70)
5	(0.001,0.001)	(0.001,0.001)	18.842 (9.67)	2.9 (3.085)	3.01 (7.36)
6	(1,0.59)	(1,0.54)	13.682 (2.394)	2.364 (0.284)	1.249 (1.719)
7	(1,0.5)	(1,0.33)	9.8438 (0.024)	2.049 (0.219)	3.086 (1.066)

TABLE 2. Bayes Estimates of m, λ_1, λ_2 and their respective MSE

(λ_1,λ_2)	(a_1,b_1)	(a_2,b_2)	$\widehat{m}$	$\widehat{\lambda}_1$	$\widehat{\lambda}_2$
(2,3)	$(1,\overline{X}_1)$	$(1,\overline{X}_2)$	10.21 (1.84)	2.45 (0.25)	3.65 (1.28)
(2,4)	$(1,\overline{X}_1)$	$(1,\overline{X}_2)$	10.31 (2.1)	2.7 (0.35)	4.45 (1.67)
(2,5)	$(1,\overline{X}_1)$	$(1,\overline{X}_2)$	10.72 (2.3)	2.9 (0.65)	5.7 (2.17)
(2,2.5)	$(1,\overline{X}_1)$	$(1,\overline{X}_2)$	10.12 (1.2)	2.1 (0.12)	2.54 (1.16)

By comparing the results of Tables 1 and 2, we come to this point that applying two stages Bayesian approach will significantly improve the performance of Bayesian estimation technique. We see that estimation of change point and parameter of exponential distribution is close to their exact values. Also, it is seen that MSE of estimations has decreased significantly. It is necessary to mention that selection of optimal parameter prior to exponential distribution depends on application of optimal parameter. In general, the effect of optimal parameter would be meaningless; when there is no selection of parameters prior to distribution affected on the final results. Therefore, selecting appropriate parameter prior to distribution is important; especially, in the cases when small number gathered by observations. Moreover, comparing the statistical performances of the Bayes estimators of a quantity of interest under different prior distributions has no sense. Indeed, the prior distribution should not be chosen on the basis of its statistical performances and the prior distribution should be selected depending on the prior information available to the analyst, and not on the basis of its statistical performances.

4. 2. Simulation Study for Maximum Likelihood Estimation (MLE) We estimated the values of m, λ_2, λ_1 by MLE approach mentioned in Section 3 where the results are shown in Table 3. The maximum likelihood estimation λ_2, λ_1 and change point m are calculated by 'Matlab' software. MSE of estimations are given for estimated values of m, λ_2, λ_1 where they are obtained based on 100 runs of the program.

In general, it is concluded that although the maximum likelihood estimation (shown in Table 3) and the Bayes estimation of parameters (shown in Table 1) were not very accurate in determining the change point and parameter of exponential distribution, but the results of two stages Bayesian process in our solution is more precise (Table 2) and MSE of estimations has decreased. Therefore, we concluded that Bayes estimation of parameters is more accurate in the proposed approach. Also, we obtained the confidence interval for estimation of change point in the Bayesian approach. The results are shown in Table 3.

TABLE 3. The results of MLE approach for estimation of m, λ_2, λ_1 and their respective MSE

(λ_1, λ_2)	$\widehat{m}$	$\widehat{\lambda}_1$	$\widehat{\lambda}_2$
(2,3)	11.64(2.68)	1.73(0.07)	2.04(0.91)
(2,4)	10.75(2.7)	1.7(0.12)	2.23(1.2)
(2,5)	12.23(3.2)	1.6(1.3)	3.7(4.1)
(2,2.5)	9.6(1.8)	1.8(0.06)	1.85(0.5)

TABLE 4. Confidence interval for estimation of change point

L	**Bayesian approach** $p(\lvert\hat{m}-m\rvert \le L)$	**two stages approach** $p(\lvert\hat{m}-m\rvert \le L)$
1	0.06	0.11
2	0.15	0.21
3	0.31	0.5
4	0.42	0.6
5	0.73	0.8

TABLE 5. Sensitivity Analysis on the location of shift points

Location / (λ_1, λ_2)	**7**	**10**	**13**	**16**
(2,3)	6.65 (1.65)	8.87 (3.50)	11.97 (3.62)	17.71 (1.68)
(2,4)	8.35 (2. 82)	9.86 (3.59)	14.27 (3.05)	18.16 (5.65)
(2,5)	9.45 (6.18)	12.87 (5.94)	10.19 (7.21)	18.69 (8.69)
(2,2.5)	7.23 (0.38)	9.75 (0.95)	13.87 (1.32)	14.98 (2.56)

As can be seen in Table 4, Bayesian approach does not perform very well in point estimations. For example, the deviation between the results of Bayesian approach and the exact value of shift point in 6 percent of runs has been less than one. As shown in Table 4, the proposed two stages approach can improve the results of point estimations. Since in the literature of statistical process control, the location of shift also affects the performance of change point estimator, thus we evaluate the performance of the Bayesian estimator in the locations 7, 10, 13, and 16. The results are shown in Table 5. It is seen from Table 5 that even though the location of shift effects on the performance of proposed methodology and also the MSE of estimations in different locations are substantially differs with each other, but the estimations are generally acceptable. Also, it is seen that when the location of shift is a very small or a very large number in sequence of data, then the MSE of Bayes estimator increases. Since increasing the number of observations before and after shift affects the performance of estimators, hence, this result was expected.

Also we compared the Bayesian change point estimator with MLE under different magnitudes of shifts in the parameter of exponential distribution. Assume that location of shift is at the tenth observation. The results of one stage Bayesian inference is shown in Table 5, that of the MLE method in Table 3 and the result of two stages Bayesian inference in Table 2. By comparing the results, it is concluded that the result of one stage Bayesian inference is slightly better than MLE approach, but two stages Bayesian inference totally outperforms other methods.

5. BAYESIAN AND MAXIMUM LIKELIHOOD ESTIMATION OF TWO CHANGE POINTS IN EXPONENTIAL DATA

In this section, we try to estimate two change points m' and m which occur in any sequence of independent observations $X_1,\ldots\ldots,X_m ; X_{m+1},\ldots\ldots,X_{m'} ; X_{m'+1},\ldots\ldots,X_n$ of exponential data. Sometimes we encounter a sudden variation in real applications for a short period of time and then the system returns to its initial setting. Finding the time of sudden variation and the time of omitting this sudden variation is investigated in this paper. In this article, we have developed a Bayesian method for estimating two change points in the exponential data.

It is assumed that a sudden variation has occurred in the hazard rate of data at m_{th} observation that changes the value of hazard rate form λ_1 to λ_2 and then the process comes back to its initial state at m'_{th} observation. It means that the value of hazard rate changes from λ_2 to λ_1. The Bayes estimator are derived for two change points m', m and λ_2, λ_1. Also we presented a new method for decreasing MSE of estimators for two change points m', m.

5. 1. Likelihood, Prior, Posterior and Marginal

Let $X_1, X_2, \ldots, X_n (n \geq 3)$ be a sequence of observations. Let observations $X_1, X_2, \ldots, X_n$ follow exponential distribution with probability density function as follows:

$$p(x) = \lambda e^{-\lambda x} \quad ; x > 0 \, , \, \lambda > 0. \tag{23}$$

Let m and m' be two change points in the observation which divides the sequence into three parts as follows:

Part 1: $X_1,\ldots\ldots,X_m$

Part 2: $X_{m+1},\ldots\ldots,X_{m'}$

Part 3: $X_{m'+1},\ldots\ldots,X_n$

The probability distribution functions of the part 1 and part 3 are as follows:

$$p_1(x) = \lambda_1 e^{-\lambda_1 x} \quad ; x > 0 \, , \, \lambda_1 > 0. \tag{24}$$

And the probability distribution functions of the part 2 is:

$$p_2(x) = \lambda_2 e^{-\lambda_2 x} \quad ; x > 0 \, , \, \lambda_2 > 0. \tag{25}$$

The likelihood functions of the sequences are:

$$L\left(\lambda_1 | \underline{x}\right) \;\alpha\; \lambda_1^{m+n-m'} e^{-\lambda_1 \sum_{i=1}^{m} x_i} \, e^{-\lambda_1 \sum_{i=m'+1}^{n} x_i} .$$
$$L\left(\lambda_2 | \underline{x}\right) \;\alpha\; \lambda_2^{m'-m} e^{-\lambda_2 \sum_{i=m+1}^{m'} x_i} . \tag{26}$$

And the joint Likelihood function is given by:

$$L\left(\lambda_1, \lambda_2 | \underline{x}\right) \;\alpha\; \lambda_1^{m+n-m'} e^{-\lambda_1 (\sum_{i=1}^{m} x_i + \sum_{i=m'+1}^{n} x_i)} \, \lambda_2^{m'-m} e^{-\lambda_2 \sum_{i=m+1}^{m'} x_i} . \tag{27}$$

Suppose the marginal prior distributions of λ_2, λ_1 are natural conjugate Gamma prior distribution as follows:

$$g_1(\lambda_1) \;\alpha\; \frac{b_1^{a_1}}{\Gamma(a_1)} \lambda_1^{a_1-1} e^{-b_1 \lambda_1} \quad ; a_1, \, b_1 > 0.$$
$$g_2(\lambda_2) \;\alpha\; \frac{b_2^{a_2}}{\Gamma(a_2)} \lambda_2^{a_2-1} e^{-b_2 \lambda_2} \quad ; a_2, \, b_2 > 0. \tag{28}$$

To perform the mathematical computations easily, the prior distribution is preferred to be a conjugate prior. Since a posterior distribution of a conjugate prior distribution is a member of the same conjugate family, thus the successive applications of Bayes' theorem can be easily performed. Since posterior distribution function of the parameter of exponential distribution is a Gamma distribution, therefore Gamma prior for parameter of exponential distribution is a conjugate prior. Also, Gamma prior can be used as a non-informative prior distribution. The Gamma prior can represent a wide variety of states of prior information, including the non-informative prior, by changing the values of parameters a and b. When b tends to zero, then the prior variance of parameter of exponential distribution tends to infinity, which adequately denotes the inspector's vague knowledge about this parameter [21]. Similar to work by Srivastava [13], we take the marginal prior distribution of m discrete uniform over the set $\{1, 2, 3, \ldots\ldots (n-1)\}$ and the marginal prior distribution of m' discrete uniform over the set $\{m+1, 2, 3, \ldots\ldots n\}$. It is obvious that if $m' = m + 1$ then it means that no shift is occurred in the hazard rate of process. Therefore, the joint prior distribution of λ_2, λ_1 and change points m', m are as follows:

$$g\left(\lambda_1, \lambda_2, m, m'\right) \;\alpha\; \frac{b_1^{a_1} b_2^{a_2}}{\Gamma(a_1)\Gamma(a_2)} \lambda_1^{a_1-1} \lambda_2^{a_2-1} \; e^{-b_1 \lambda_1} \; e^{-b_2 \lambda_2} , \tag{29}$$

where $\lambda_1, \, \lambda_2 > 0$, $m = 1, \ldots, n-1, m' = m+1, \ldots, n.$ The Joint posterior density of λ_2, λ_1 and m', m say

$\pi\left(\lambda_{1,}\lambda_2;m,m'|\underset{-}{x}\right)$ is obtained as Equation (30). The marginal posterior distribution of change points m', m is obtained as Equation (31). Also following is obtained Equation (32). Also, The marginal posterior distribution of λ_1 is as Equation (33). And the marginal posterior distribution of λ_2 is as Equation (34) where, (see Equation (35)) using the above posterior distribution functions, the estimation of λ_1 is obtained as, Equation (36). Other estimators are obtained by similar mathematical calculations as Equations (37), (38) and (39).

$$\pi\left(\lambda_1,\ \lambda_2;m,\ m'|\underset{-}{x}\right) = \frac{b_1^{a_1}b_2^{a_2}}{\Gamma(a_1)\Gamma(a_2)}\lambda_1^{m+n-m'+a_1-1}\lambda_2^{m'-m+a_2-1}e^{-\lambda_1(\sum_{i=1}^{m}x_i+\sum_{i=m'+1}^{n}x_i)}e^{-\lambda_2(b_2+\sum_{i=m+1}^{m'}x_i)}. \tag{30}$$

$$\pi\left(m|\underset{-}{x}\right)=\frac{\sum_{m'=m+1}^{n}\frac{\Gamma(n+m-m'+a_1)}{(b_1+\sum_{i=1}^{m-1}x_i+\sum_{i=m'}^{n}x_i)^{n+m-m'+a_1}}\frac{\Gamma(m'-m+a_2)}{(b_2+\sum_{i=m}^{m'-1}x_i)^{m'-m+a_2}}}{\psi\left(a_1,a_2,b_1,b_2,m,m',n\right)}. \tag{31}$$

$$\pi\left(m'|\underset{-}{x}\right)=\frac{\sum_{m=1}^{m'-1}\frac{\Gamma(n+m-m'+a_1)}{(b_1+\sum_{i=1}^{m-1}x_i+\sum_{i=m'}^{n}x_i)^{n+m-m'+a_1}}\frac{\Gamma(m'-m+a_2)}{(b_2+\sum_{i=m}^{m'-1}x_i)^{m'-m+a_2}}}{\psi\left(a_1,a_2,b_1,b_2,m,m',n\right)}. \tag{32}$$

$$\pi\left(\lambda_1|\underset{-}{x}\right)=\frac{\sum_{m=1}^{n}\sum_{m'=m+1}^{n}\left(\lambda_1^{n+m-m'+a_1}e^{-\lambda_1(b_1+\sum_{i=1}^{m-1}x_i+\sum_{i=m'}^{n}x_i)}\frac{\Gamma(m'-m+a_2)}{(b_2+\sum_{i=m}^{m'-1}x_i)^{m'-m+a_2}}\right)}{\psi\left(a_1,a_2,b_1,b_2,m,m',n\right)}. \tag{33}$$

$$\Pi\left(\lambda_2|\underset{-}{x}\right)=\frac{\sum_{m=1}^{n}\sum_{m'=m+1}^{n}\left(\lambda_2^{m'-m+a_2-1}e^{-\lambda_2(b_2+\sum_{i=m}^{m'-1}x_i)}\frac{\Gamma(n+m-m'+a_1)}{(b_1+\sum_{i=1}^{m-1}x_i+\sum_{i=m'}^{n}x_i)^{n+m-m'+a_1}}\right)}{\psi\left(a_1,a_2,b_1,b_2,m,m',n\right)}. \tag{34}$$

$$\psi\left(a_1,a_2,b_1,b_2,m,n\right) = \sum_{m'=2}^{n}\sum_{m=1}^{m'-1}\int_0^\infty\int_0^\infty g(\lambda_1,\lambda_2,m)d\lambda_1 d\lambda_2 = \sum_{m=1}^{n}\sum_{m'=m+1}^{n}\frac{\Gamma(n+m-m'+a_1)}{(b_1+\sum_{i=1}^{m-1}x_i+\sum_{i=m'}^{n}x_i)^{n+m-m'+a_1}}\frac{\Gamma(m'-m+a_2)}{(b_2+\sum_{i=m}^{m'-1}x_i)^{m'-m+a_2}}. \tag{35}$$

$$E(\lambda_1) = \int_0^\infty \lambda_1\pi(\lambda_1|\underline{x})\,d\lambda_1 = \sum_{m=1}^{n}\sum_{m'=m+1}^{n}\frac{\Gamma(m'-m+a_2)}{(b_2+\sum_{i=m}^{m'-1}x_i)^{m'-m+a_2}}\left\{\int_0^\infty\lambda_1\lambda_1^{n+m-m'+a_1}e^{-\lambda_1(b_1+\sum_{i=1}^{m-1}x_i+\sum_{i=m'}^{n}x_i)}d\lambda_1\right\} =$$
$$\frac{\sum_{m=1}^{n}\sum_{m'=m+1}^{n}\left(\frac{\Gamma(m'-m+a_2)}{(b_2+\sum_{i=m}^{m'-1}x_i)^{m'-m+a_2}}\frac{\Gamma(n+m-m'+1+a_1)}{(b_1+\sum_{i=m'}^{n}x_i+\sum_{i=1}^{m-1}x_i)^{n+m-m'+1+a_1}}\right)}{\psi\left(a_1,a_2,b_1,b_2,m,m',n\right)}. \tag{36}$$

$$E(\lambda_2)=\int_0^\infty \lambda_2 \pi(\lambda_2|\underline{x})d\lambda_2=\frac{\sum_{m=1}^{n}\sum_{m'=m+1}^{n}\left(\frac{\Gamma(n+m-m'+a_1)}{(b_1+\sum_{i=m'}^{n}x_i+\sum_{i=1}^{m-1}x_i)^{n+m-m'+a_1}}\frac{\Gamma(m'-m+1+a_2)}{(b_2+\sum_{i=m}^{m'-1}x_i)^{m'-m+1+a_2}}\right)}{\psi(a_1,a_2,b_1,b_2,m,m',n)}. \quad (37)$$

$$E(m)=\sum_{m=1}^{n-1}\frac{m\sum_{m'=m+1}^{n}\frac{\Gamma(n+m-m'+a_1)}{(b_1+\sum_{i=1}^{m-1}x_i+\sum_{i=m'}^{n}x_i)^{n+m-m'+a_1}}\frac{\Gamma(m'-m+a_2)}{(b_2+\sum_{i=m}^{m'-1}x_i)^{m'-m+a_2}}}{\psi(a_1,a_2,b_1,b_2,m,m',n)}. \quad (38)$$

$$E(m')=\sum_{m'=m+1}^{n}\frac{m'\sum_{m=1}^{m'-1}\frac{\Gamma(n+m-m'+a_1)}{(b_1+\sum_{i=1}^{m-1}x_i+\sum_{i=m'}^{n}x_i)^{n+m-m'+a_1}}\frac{\Gamma(m'-m+a_2)}{(b_2+\sum_{i=m}^{m'-1}x_i)^{m'-m+a_2}}}{\psi(a_1,a_2,b_1,b_2,m,m',n)}. \quad (39)$$

5. 2. Maximum Likelihood Estimation (MLE) of Two Change Points in Exponential Distribution The likelihood functions of p.m.f.'s of the sequences are

$$L(\lambda_1|\underline{x})\ \alpha\ \lambda_1^{n-m'+m}e^{-\lambda_1(\sum_{i=1}^{m}x_i+\sum_{i=m'+1}^{n}x_i)},$$
$$L(\lambda_2|\underline{x})\ \alpha\ \lambda_2^{m'-m}e^{-\lambda_2\sum_{i=m+1}^{m'}x_i}. \quad (40)$$

thus,the values of λ_2,λ_1 are given by

$$\lambda_1=\frac{n-m'+m}{\sum_{i=1}^{m}x_i+\sum_{i=m'+1}^{n}x_i},$$
$$\lambda_2=\frac{m'-m}{\sum_{i=m+1}^{m'}x_i}. \quad (41)$$

And Likelihood function is obtained as:

$$L(m,m')=\frac{n+m-m'}{\sum_{i=1}^{m}x_i+\sum_{i=m'+1}^{n}x_i}e^{-(n+m-m')}$$
$$\frac{m'-m}{\sum_{i=m+1}^{m'}x_i}e^{-(m'-m)}. \quad (42)$$

Maximum Likelihood Estimation (MLE) is given by:

$$(m^*,m'^*)=(m,m'):L(m,m')=$$
$$Max\left\{L(k,k');k=1,2,\ldots,n\text{-}1\ \&\ k'=k+1,\ldots,n\right\} \quad (43)$$

Now in the next section, numerical examples along with solution algorithms are provided in order to show the application of proposed methodology.

5. 3. Sensitivity Analysis of Bayes Estimators In this section, the means of the posterior distribution of variables m',m,λ_2,λ_1 are evaluated in order to analyze the performance of Bayesian inference in change point estimation. The value of hazard rates is assumed to be $\lambda_2=3,\lambda_1=2.$ We have generated sample of 20 observations from exponential data where two change points exist between 1 and 20. Assume the first change point is 6 and second change point is 14. The Bayes estimators of λ_2,λ_1 and change points m',m are calculated by excel software. The respective m',m,λ_2,λ_1 and MSE are determined based on the 100 runs of program. Because of small value of standard errors that are obtained by equation $MSE/\sqrt{100}=MSE/10$ and long time of simulation study, it is concluded that 100 runs of program is sufficient for estimation of parameters. We have performed the sensitivity analysis of the Bayes estimates with respect to shifts in the parameters of prior distribution (a_1,b_1) and (a_2,b_2). The means and variances of the prior distribution are used as prior information in computing these parameters. Then, with these parameter values, we have computed the Bayes estimates of m',m,λ_2,λ_1 using different set of values of (a_1,b_1) and (a_2,b_2). The numbers in parentheses show the MSE of Bayesian estimators in Table 6. As can be seen from Table 6, the MSE of Bayes estimators are very large, that means the proposed Bayes estimator cannot estimate the value of change points precisely. The reason behind this variation is the existence of large difference between parameters of prior distribution for λ_2,λ_1 and the actual values of hazard rates. In Srivastava's study [13], Bayes estimates of the parameters become robust with correct choice of prior parameters and sample size. But, in this paper we suggest another way to get more precise estimators; we applied a two stages Bayesian approach. In the first stage of proposed approach, we use a suitable priors in the Bayesian updating procedure and after evaluating the approximate value of m', m, second stages will be applied. Suitable prior should be selected by historical data and experience of decision maker. In the second stage, we use Gamma priors with parameters $\left(a_1 = 1, b_1 = \overline{X_1} + \overline{X_3}\right)$ for λ_1 and Gamma prior with parameters $\left(a_2 = 1, b_2 = \overline{X_2}\right)$ for λ_2 where $\overline{X_3}, \overline{X_2}, \overline{X_1}$ are average of observations $X_1, \ldots, X_m$, $X_{m+1}, \ldots, X_{m'}$ and $X_{m'+1}, \ldots, X_n$ respectively.

TABLE 6. Bayes Estimates of $m', m, \lambda_2, \lambda_1$ and their respective MSE

(a_1, b_1)	(a_2, b_2)	$\widehat{m}$	$\widehat{m'}$	$\widehat{\lambda_1}$	$\widehat{\lambda_2}$
(1.5, 1.75)	(1.8, 2.0)	7.66 (2.79)	14.166 (1.277)	1.87 (0.563)	1.59 (2.108)
(1.75, 2.0)	(2.0, 2.25)	7.76 (4.498)	14.18 (1.271)	1.81 (0.532)	1.45 (2.438)
(2.0, 2.25)	(2.20, 2.5)	7.85 (4.784)	14.40 (1.830)	1.67 (0.684)	1.31 (2.915)
(2.25,2.50)	(2.40,2.75)	7.74 (3.903)	14.65 (1.579)	1.45 (0.758)	1.29 (3.256)
(0.001,0.001)	(0.001,0.001)	7.68 (4.495)	13.41 (2.932)	4.63 (0.819)	6.43 (3.301)
(1,0.58)	(1,0.53)	7.546 (1.352)	14.439 (1.967)	1.673 (0.587)	1.839 (2.037)
(1,0.5)	(1,0.33)	7.294 (0.987)	13.908 (0.0397)	1.826 (0.658)	2.375 (1.962)

TABLE 7. Bayes Estimates $m', m, \lambda_2, \lambda_1$ and their respective MSE in two stages Bayesian approach

(a_1, b_1)	(a_2, b_2)	$\widehat{m}$	$\widehat{m'}$	$\widehat{\lambda_1}$	$\widehat{\lambda_2}$
$(1, \overline{X}_1 + \overline{X}_3)$	$(1, \overline{X}_2)$	7.708 (0.80)	14.276 (0.68)	2.438 (0.64)	3.292 (1.97)

TABLE 8. Results of MLE approach for estimation $m', m, \lambda_2, \lambda_1$ and their respective MSE

	$\widehat{m}$	$\widehat{m'}$	$\widehat{\lambda_1}$	$\widehat{\lambda_2}$
Estimates	9.2(10.24)	11.7(5.29)	2.55(0.305)	1.49(2.27)

In the first prior of Table 6, the values of m', m have least deviation from their exact values, thus we used first priors to estimate the parameters m', m in the first stage of change points estimation. Then, we use Gamma priors for parameter of exponential distributions in the second stage of Bayesian approach and estimation technique will be repeated again. Table 7 shows the results of estimation with these prior distributions. It is concluded from the results in Table 7 that the MSE of Bayes estimators have decreased in proposed approach which makes the estimations more precise. Therefore, the results of two stages Bayesian process in our solution are better than the results of single stage Bayesian estimation method (Table 6).

Also, the result of likelihood estimation technique is denoted in Table 8. The maximum likelihood estimation λ_2, λ_1 and change points m', m are calculated by 'Matlab' software.

The values of m', m and λ_2, λ_1 and their MSE we have obtained based on 100 runs of program. We compared the Bayesian change point estimator with MLE. Assume that location of shift is at tenth observation. The results of one stage Bayesian inference comes in Table 6, that of MLE method in Table 8 and the result of two stages Bayesian inference in Table 7. By comparing the results, it is concluded that the result of one stage Bayesian inference is a little better than MLE approach, but two stages Bayesian inference totally outperforms other methods. In general, it is concluded that although the results of maximum likelihood estimation (shown in Table 8) and the Bayes estimates of parameters (shown in Table 6) were not very accurate in determining two change points and parameter of exponential distribution, but the results of two stages Bayesian process in our solution is more precise (Table 7) and MSE of estimations have decreased. Therefore, we concluded that the method of Bayes estimation of parameters in two stages is more accurate in the proposed approach.

TABLE 9. Confidence interval for estimation of two change points

L	**Bayesian approach** $p(\lvert\widehat{m}-m\rvert \le L)$	**Two stages approach** $p(\lvert\widehat{m}-m\rvert \le L)$	**Bayesian approach** $p(\lvert\widehat{m}'-m'\rvert \le L)$	**Two stages approach** $p(\lvert\widehat{m}'-m'\rvert \le L)$
1	0.11	0.20	0.27	0.35
2	0.29	0.41	0.36	0.45
3	0.41	0.66	0.59	0.75
4	0.62	0.89	0.74	0.90
5	0.84	0.92	0.93	0.96

TABLE 10. Sensitivity analysis on the location of two shift points

location	$\widehat{m}$	$\widehat{m'}$	$\widehat{m}$	$\widehat{m'}$
	5	**16**	**9**	**18**
(2,3)	4.234 (1.92)	17.657 (2.00)	11.198 (5.62)	17.357 (1.97)
(2,4)	5.657 (1. 89)	15.824 (0.85)	8.295 (1.83)	18.197 (0.85)
(2,5)	6.457 (4.65)	12.876 (6.14)	11.194 (3.81)	19.498 (4.69)
(2,2.5)	5.294 (1.36)	16.752 (2.05)	9.893 (2.76)	15.983 (6.02)

Also, we obtained the confidence interval for estimation of change point in the Bayesian approach. The results are shown in Table 9. As can be seen in Table 9, Bayesian approach does not perform very well in point estimations. For example, the deviation between the results of Bayesian approach and the exact value of first shift point in 11 percent of runs has been less than one. As shown in Table 9, the proposed two stages approach can improve the results of point estimations. Since in the literature of statistical process control, the location of shift also affects the performance of change point estimator, thus we evaluate the performance of the Bayesian estimator in the locations $m=5, m=16$ and $m=9, m=18$. The results are shown in Table 10. It is seen from Table 10 that even though the location of shift affects on the performance of proposed methodology, and also the MSE of estimations in different locations are substantially different with each other, but the estimations are generally acceptable. Also, it is concluded that the values of λ_2, λ_1 affect the performance of Bayes estimators and the values of MSE substantially differs with variation of these parameters.

6. CONCLUSION

The paper considers determining the change point in any sequence of independent observations of exponential distribution. The Bayes estimator of change point is derived based on posterior probability distribution of change point. The sensitivity analysis on the location of shift and parameters of prior distribution is performed in a simulation study. Also, we suggested a two stage Bayesian process to improve the estimation of change point. We have developed a Bayesian method for estimating two change points in the exponential data as well. As a future research, we suggest to consider cost of over-estimating or under-estimating the quantity of interest along with applying different loss functions.

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Improving the Performance of Bayesian Estimation Methods in Estimations of Shift Point and Comparison with Maximum Likelihood Estimation Approach

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چکیده

در این تحقیق، تحلیل بیزی برای تخمین نقطه تغییر در دنباله‌ای از متغیرهای تصادفی با توزیع نمایی استفاده شده‌است. برآوردگرهای بیز را برای نقطه تغییر پارامتر توزیع نمایی قبل و بعد از تغییر به دست آورده‌ایم. توابع درست‌نمایی، پیشین، پسین و توزیع حاشیه‌ای نقطه تغییر نیز ارائه شده‌است. همچنین، روش تخمین درست‌نمایی برای تعیین نقطه تغییر ارائه شده‌است. تحلیل حساسیت برآوردگرهای بیز را نیز با شبیه‌سازی انجام داده‌ایم. در این مقاله یک روش جدید برای یافتن نتایج دقیق‌تر با تعیین انتخاب صحیح پارامترهای توزیع پیشین ارائه کرده‌ایم و این روش جدید را با سایر روش‌های موجود مقایسه می‌کنیم. نتایج شبیه‌سازی عملکرد خوب روش پیشنهادی را در مقایسه با سایر روش‌ها تایید می‌کند. همچنین، یک تحلیل حساسیت بر روی نقطه مکان تغییر هم انجام شده است.

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